Symbol	Meaning	Value
$\zeta(3)$	Apery's constant	1.202...
B	Backhouse's constant	1.456...
e	Base of the natural logarithm	2.718...
K	Catalan's constant	0.915...
π	Circle constant	3.141...
γ	Euler-Mascheroni constant	0.577...
α	Feigenbaum's alpha	2.502...
δ	Feigenbaum's delta	4.669...
ξ	Foias constant	1.187...
F	Fransén-Robinson constant	2.807...
G	Gauss's constant	0.834...
A	Glaisher-Kinkelin constant	1.282...
ϕ	Golden ratio	1.618...
η	Grossman's constant	0.737...
ρ	Kepler-Bouwkamp constant	0.114...
K_0	Khinchin's constant	2.685...
q	Komornik-Loreti constant	1.787...
K_{LR}	Landau-Ramanujan constant	0.764...
L	Lemniscate constant	2.622...
M_3	Madelung constant	-1.747...
M	Meissel-Mertens constant	0.261...
C	Niven's constant	1.705...
Ω	Omega constant	0.567...
σ_p	Paper folding constant	0.850...
μ	Ramanujan-Soldner constant	1.451...
ψ	Reciprocal Fibonacci constant	3.359...
σ	Somos' quadratic recurrence constant	1.661...
Π_2	Twin primes constant	0.660...
P	Universal parabolic constant	2.295...

Constant Definitions

0.1 Backhouse's Constant, B

Let p_k be the kth prime. Let

$$P(x) = 1 + \sum_{k=1}^{\infty} p_k x^k$$

and in turn let

$$\begin{aligned} Q(x) &= \frac{1}{P(x)} \\ &= \sum_{k=0}^{\infty} q_k x^k. \end{aligned}$$

Then Backhouse's constant is given by

$$B = \lim_{k\to\infty} \left| \frac{q_{k+1}}{q_k} \right|.$$

0.2 The Base of the Natural Logarithm, e

The base of the natural logarithm.

0.3 Catalan's Constant, K

$$K = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2}$$

0.4 The Circle Constant, π

The ratio of a circle's circumference to its diameter.

0.5 The Euler-Mascheroni Constant, γ

$$\gamma = \lim_{n\to\infty} \left(-\ln(n) + \sum_{k=1}^{n} \frac{1}{k} \right)$$

0.6 Feigenbaum's α and δ Constants

Consider a species where the population of one generation depends on the population of the prior generation according to the recurrence relation

$$P_n = rP_{n-1}\left(1 - P_{n-1}\right)$$

where r is a constant parameter. As n goes to ∞, you might expect the population to settle on an equilibrium value. The value of the parameter r controls what this equilibrium value is. For many values of r, we see the population settle on a single value. However, for other values, the population may cycle back and forth between two or three (or more) values with successive generations. This behavior can be visualized in a bifurcation plot, like Figure 1.

When r is less than one, the population drops every generation, settling on 0, as seen in the plot. When r is about 2, the population settles near 0.5. When r is 3.2, the population will vacillate between two different sizes with successive generations. At 3.5, we cycle between four different sizes. If we pay attention to the values of r at which these splits (bifurcations) happen, we can find the δ constant.

If three consecutive bifurcations occur at r values of r_n, r_{n+1}, and r_{n+2}, then let the lengths

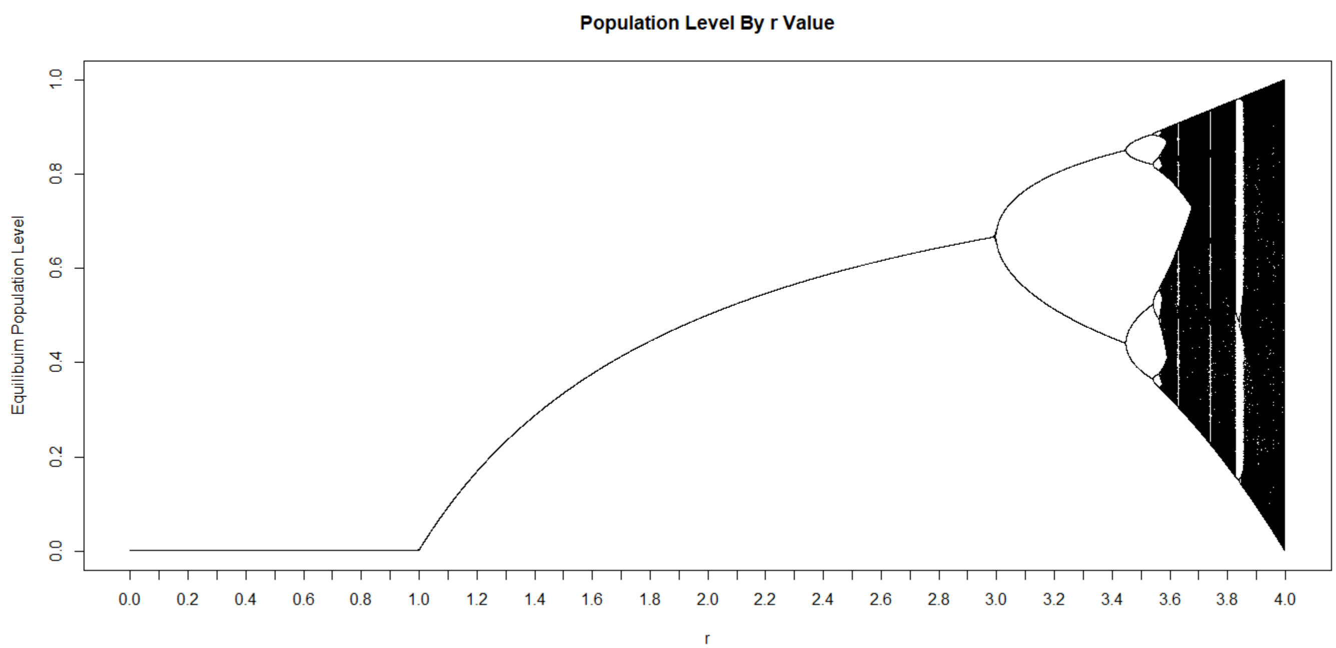
$$L_n = r_{n+1} - r_n \text{ and}$$
$$L_{n+1} = r_{n+2} - r_{n+1}.$$

We then get

$$\delta = \lim_{n\to\infty} \frac{L_n}{L_{n+1}}.$$

The α constant is found similarly. When a bifurcation occurs, two "tines" arise. If we then increase r until just before the next bifurcation occurs, we can record the distance between the tips of those two tines. The ratio of these distances for successive bifurcations converges to the α constant.

Figure 1: Bifurcation diagram for the logistic map



0.7 Foias Constant, ξ

Let $x_1 > 0$ and

$$x_{n+1} = \left(1 + \frac{1}{x_n}\right)^n.$$

Then there is exactly one value of $x_1 = \xi$ such that $x_n \to \infty$.

0.8 The Fransén-Robinson Constant, F

$$F = \int_0^\infty \frac{dx}{\Gamma(x)}$$

Where

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt$$

0.9 Gauss's Constant, G

$$G = \frac{1}{\operatorname{agm}(1, \sqrt{2})}$$

Where $\operatorname{agm}(a, b)$ is the arithmetic-geometric mean of a and b.

0.10 The Golden Ratio, ϕ

The greater of the two solutions to $x^2 = x + 1$, namely

$$\frac{1 + \sqrt{5}}{2}.$$

0.11 Grossman's Constant, η

Define the sequence $a_0 = 1$, $a_1 = x$, and

$$a_n = \frac{a_{n-2}}{1 + a_{n-1}}.$$

This sequence converges for exactly one value of $x = \eta$.

0.12 The Kepler-Bouwkamp Constant, ρ

$$\rho = \prod_{k=3}^{\infty} \cos\left(\frac{\pi}{k}\right)$$

0.13 Khinchin's Constant, K_0

The coefficients of the continued fraction expansion of a real number almost always have a finite geometric mean which converges to this constant. That is, if

$$x = a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \cdots}},$$

then it is almost always true that

$$\lim_{n \to \infty} (a_1 a_2 \ldots a_n)^{1/n} = K_0.$$

0.14 The Komornik-Loreti Constant, q

The Thue-Morse sequence is defined by the recurrence relation

$$\begin{aligned} t_0 &= 0 \\ t_{2n} &= t_n \\ t_{2n+1} &= 1 - t_n \end{aligned}$$

The Komornik-Loreti constant, q, is the solution to

$$1 = \sum_{n=1}^{\infty} \frac{t_n}{q^n}$$

0.15 The Meissel-Mertens Constant, M

The value of the limit

$$\lim_{n\to\infty} \left(-\ln\left(\ln(n)\right) + \sum_{p\le n} \frac{1}{p} \right)$$

for primes p.

0.16 Niven's Constant, C

If the prime factorization of an integer m is written as

$$m = (p_1^{a_1})(p_2^{a_2})(p_3^{a_3})\ldots,$$

then we define the function H as

$$H(m) = \max(a_1,\ a_2,\ a_3, \ldots).$$

Niven's constant is then given by the limit

$$C = \lim_{n\to\infty} \frac{1}{n} \sum_{m=1}^{n} H(m).$$

So C is the "average" greatest number of times a prime occurs in the prime factorization of an integer.

0.17 The Ω Constant

The solution to

$$xe^x = 1.$$

0.18 The Paper Folding Constant, σ_p

Take a strip of paper and fold it in half, right over left, infinitely often. Upon unfolding it, assign each "valley" crease a 1, and each "mountain" crease a 0. That sequence begins:

$$a = \{1, 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, \ldots\}$$

Then,

$$\sigma_p = \sum_{n=1}^{\infty} \frac{a_n}{2^n}$$

0.19 The Ramanujan-Soldner Constant, μ

The Ramanujan-Soldner constant, μ, is the unique positive solution to

$$\int_0^x \frac{dt}{\ln(t)} = 0$$

0.20 The Reciprocal Fibonacci Constant, ψ

The value of the summation

$$\sum_{n=1}^{\infty} \frac{1}{F_n}$$

where F_n is the nth Fibonacci number.

0.21 Somos' Quadratic Recurrence Constant, σ

$$\begin{aligned}
\sigma &= \sqrt{1\sqrt{2\sqrt{3\cdots}}} \\
&= \prod_{n=1}^{\infty} \left(\frac{1+n}{n}\right)^{2^{-n}}
\end{aligned}$$

0.22 The Universal Parabolic Constant, P

The latus rectum of a parabola is the line segment whose end points lie on the parabola and which passes through its focus parallel to its directrix. The focal parameter of a parabola is the distance from its directrix to its focus. Let A be the arc length of the parabolic segment formed by the latus rectum of a parabola and p be the focal parameter of that parabola. The the universal parabolic constant is

$$\begin{aligned}
P &= \frac{A}{p} \\
&= \ln(1+\sqrt{2}) + \sqrt{2}.
\end{aligned}$$

1 January

$$1.01 = \sqrt[K]{e - C}$$
$$1.02 = -\left(\frac{M_3}{C}\right)$$
$$1.03 = \csc(\gamma\pi)$$
$$1.04 = \mu^{\rho}$$
$$1.05 = \sqrt[\sigma]{\frac{1}{K}}$$
$$1.06 = \cosh\left(\sqrt[M]{K_{LR}}\right)$$
$$1.07 = \frac{1}{\gamma\phi}$$
$$1.08 = \ln(\cosh(M_3))$$
$$1.09 = \frac{1}{K}$$
$$1.10 = \gamma^{\ln(G)} \text{ or } \tan(G)$$
$$1.11 = P\log_G(K)$$
$$1.12 = \sqrt{P - \ln(F)} \text{ or } \gamma - \ln(\gamma) \text{ or } \cosh^{-1}(C)$$
$$1.13 = \ln^{\gamma}\left(\pi + \frac{1}{\pi}\right)$$
$$1.14 = \ln(\phi\cosh(A)) \text{ or } \sinh(\tanh(P))$$
$$1.15 = \left(1 + \sqrt[(e-\pi)]{e}\right)^{\phi}$$
$$1.16 = \frac{e\sqrt[\phi]{\phi}}{\pi}$$
$$1.17 = \sec(\ln\gamma)$$
$$1.18 = \phi - \frac{1}{2\ln\pi}$$
$$1.19 = ***$$
$$1.20 = ***$$

$$1.21 = \frac{1}{G \tanh\left(\frac{P}{K}\right)}$$

$$1.22 = ***$$

$$1.23 = ***$$

$$1.24 = ***$$

$$1.25 = ***$$

$$1.26 = ***$$

$$1.27 = ***$$

$$1.28 = A$$

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$$1.30 = ***$$

$$1.31 = ***$$

2 February

$$2.01 = ***$$
$$2.02 = ***$$
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$$2.07 = e^{\gamma^{\gamma}}$$
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$$2.17 = ***$$
$$2.18 = ***$$
$$2.19 = \frac{1}{B-1}$$
$$2.20 = \sqrt{\pi \ln \delta}$$
$$2.21 = \pi^{\ln(2)}$$
$$2.22 = \cosh^{-1}(\delta) \text{ or } \phi^{\sigma}$$
$$2.23 = (1+\Omega)^{q} \text{ or } \log_{\mu}(P)$$
$$2.24 = \sqrt[\sigma]{\frac{1}{M}}$$
$$2.25 = \sqrt{\alpha + K + \sigma}$$
$$2.26 = \frac{e}{\zeta(3)}$$
$$2.27 = q^{\sqrt{2}}$$
$$2.28 = K_0^G$$
$$2.29 = P$$

3 March

$$3.01 = \phi^{\alpha K}$$
$$3.02 = \phi^{B+\sin(1)}$$
$$3.03 = \frac{1}{\rho - \cos(q)}$$
$$3.04 = \frac{\pi}{\ln F}$$
$$3.05 = \delta - \phi$$
$$3.06 = \phi + \frac{1}{\ln 2} \text{ or } \pi^{\sqrt[4]{K}}$$
$$3.07 = **$$
$$3.08 = **$$
$$3.09 = P^{\frac{e}{2}}$$
$$3.10 = \pi \ln K_0$$
$$3.11 = \log_{\mu}(L + \Omega)$$
$$3.12 = -\frac{\ln \pi}{\ln(\ln 2)}$$
$$3.13 = K_0^{\frac{\pi}{e}}$$
$$3.14 = **$$
$$3.15 = **$$
$$3.16 = C + B$$
$$3.17 = \frac{\alpha}{C - K}$$
$$3.18 = e^{\sin^{-1}(K)} \text{ or } \sqrt{\pi} + \sqrt{2}$$
$$3.19 = P\sqrt{\sinh\sqrt{2}} \text{ or } K_0\sqrt[4]{2}$$
$$3.20 = \sqrt[e]{\frac{e}{\rho}} \text{ or } \frac{1}{e\rho}$$
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$$5.12 = \sigma^{\delta/\mu} \text{ or } \pi + \Omega + \sqrt{2}$$
$$5.13 = -\tan\left(\frac{1}{\Omega}\right)$$
$$5.14 = K_0^{\cosh^{-1}(e)} \text{ or } e^{\sqrt{K_0}}$$
$$5.15 = \alpha^q \text{ or } \frac{\tan(\mu)}{\phi} \text{ or } \delta\tan(G)$$
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$$7.25 = **$$
$$7.26 = **$$
$$7.27 = \pi^{\rho+\phi}$$
$$7.28 = e^{\sqrt{K_0}}\sqrt{2}$$
$$7.29 = \frac{G}{\tanh(\rho)}$$
$$7.30 = \sqrt{\sinh(\delta)}$$
$$7.31 = \alpha^{\sigma} + e \text{ or } Ce^{B}$$

8 August

$$8.01 = **$$
$$8.02 = **$$
$$8.03 = \ln(2)\cosh(\pi)$$
$$8.04 = e^{\sqrt{\phi K_0}}$$
$$8.05 = \frac{\alpha K_0}{G}$$
$$8.06 = \sinh\left(\frac{\psi}{\sqrt{B}}\right)$$
$$8.07 = \sinh\sqrt{\sigma\delta}$$
$$8.08 = **$$
$$8.09 = **$$
$$8.10 = B^{\frac{B}{M}}$$
$$8.11 = \phi^{\frac{1}{2\rho}} \text{ or } \alpha^{A+1}$$
$$8.12 = \frac{\alpha^{\psi}}{K_0} \text{ or } \delta^{\frac{e}{2}}$$
$$8.13 = -\frac{K_0 M_3}{\gamma} \text{ or } -\tan(1+\ln(2))$$
$$8.14 = e^P - q$$
$$8.15 = **$$
$$8.16 = **$$
$$8.17 = e^{\frac{B}{\ln(2)}}$$
$$8.18 = K_0 \log_B(\pi) \text{ or } \left(\sinh^{-1} P\right)^{\delta}$$
$$8.19 = 2^{\frac{\sinh\alpha}{2}} \text{ or } \sigma^{\psi} + K_0$$
$$8.20 = \cosh(F) - \rho \text{ or } \frac{\log_{\phi} F}{M}$$
$$8.21 = \alpha^P \text{ or } K + \sinh(K_0)$$
$$8.22 = **$$
$$8.23 = \left(\frac{\pi}{2}\right)^{\delta} \text{ or } e^{(K+F)^{\Omega}}$$
$$8.24 = B^{q\pi} \text{ or } e\log_{\sigma}(\delta)$$
$$8.25 = \sinh(F) \text{ or } \log_K\left(\frac{1}{e}\right) - \pi$$

$$8.26 = \frac{Ke}{\rho L} \text{ or } \psi - FM_3$$

$$8.27 = \delta\sqrt{\pi} \text{ or } e^{\mu B} \text{ or } \frac{-\log(\rho)}{M}$$

$$8.28 = \frac{q - G}{\rho}$$

$$8.29 = **$$

$$8.30 = **$$

$$8.31 = \cosh(F) \text{ or } K - \tan(C) \text{ or } \psi^{-M_3}$$

9 September

$9.01 = \cosh(B)^{\gamma\delta}$ or $\pi^{\phi}\sqrt{2}$

$9.02 = \pi F^{G^{(\alpha-L)}}$ or ψK_0

$9.03 = C^{P^C}$

$9.04 = \sqrt[\Omega]{eA}$ or $\sigma^{\frac{\alpha}{\gamma}}$ or $\psi^{\frac{e+K}{2}}$

$9.05 = **$

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$9.13 = **$

$9.14 = \delta + K_0 + q$ or $\ln(\psi)\sinh(e)$

$9.15 = B + \sqrt[M]{C}$ or $\dfrac{F}{1-\ln(2)}$ or $e\,(\sigma + C)$

$9.16 = K_0\,(e + \ln 2)$ or $\phi + \sinh(e)$

$9.17 = \phi(1+\delta)$ or $\sigma\delta + \sqrt{2}$ or $Fe\zeta(3)$

$9.18 = \cosh(B+\mu)$

$9.19 = \mu\sqrt[G]{\delta}$ or $\mu\tan\left(\sqrt{2}\right)$

$9.20 = \sqrt[\pi]{\rho} + \dfrac{1}{\rho}$ or $(\mu+\pi)^B$

$9.21 = \delta + \phi F$ or $\sqrt[M]{q}$

$9.22 = \psi^{\sqrt{\psi}}$ or $\dfrac{\pi\psi}{\ln\pi}$

$9.23 = \dfrac{\cosh^{-1}(\phi)}{\rho}$ or $\dfrac{1}{\rho} + \ln C$

$9.24 = \dfrac{1}{\rho} + \sinh^{-1}\Omega$

$9.25 = \sinh(\pi) - P$ or $\sqrt[q]{\sinh(\delta)}$

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10 October

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