Discrete Fourier Transform

The discrete Fourier transform, or DFT, is the primary tool of digital signal processing. The foundation of the product is the fast Fourier transform (FFT), a method for computing the DFT with reduced execution time. Many of the toolbox functions (including Z-domain frequency response, spectrum and cepstrum analysis, and some filter design and implementation functions) incorporate the FFT.

The MATLAB® environnement provides the functions *fft* and *ifft* to compute the discrete Fourier transform and its inverse, respectively. For the input sequence *x* and its transformed version *X* (the discrete-time Fourier transform at equally spaced frequencies around the unit circle), the two functions implement the relationships :

$$X(k+1) = \sum_{n=0}^{N-1} x(n+1) W_N^{kn}$$

and

$$x(n+1) = \frac{1}{N} \sum_{n=0}^{N-1} X(k+1) W_N^{-kn}$$

In these equations, the series of subscripts begin with 1 instead of 0 because of the MATLAB vector indexing scheme, and $W_N = e^{-\mathrm{j}2\pi/N}$.

Note The MATLAB convention is to use a negative *j* for the *fft* function. This is an engineering convention; pyhsics and pure mathematics typically use a positive *j*.

fft, with a single input argument, *x*, computes the DFT of the input vector or matrix. If x is vector, *fft* conputes the DFT of the vector; if *x* vector is a rectangular array, *fft* computes the DFT of each array column.

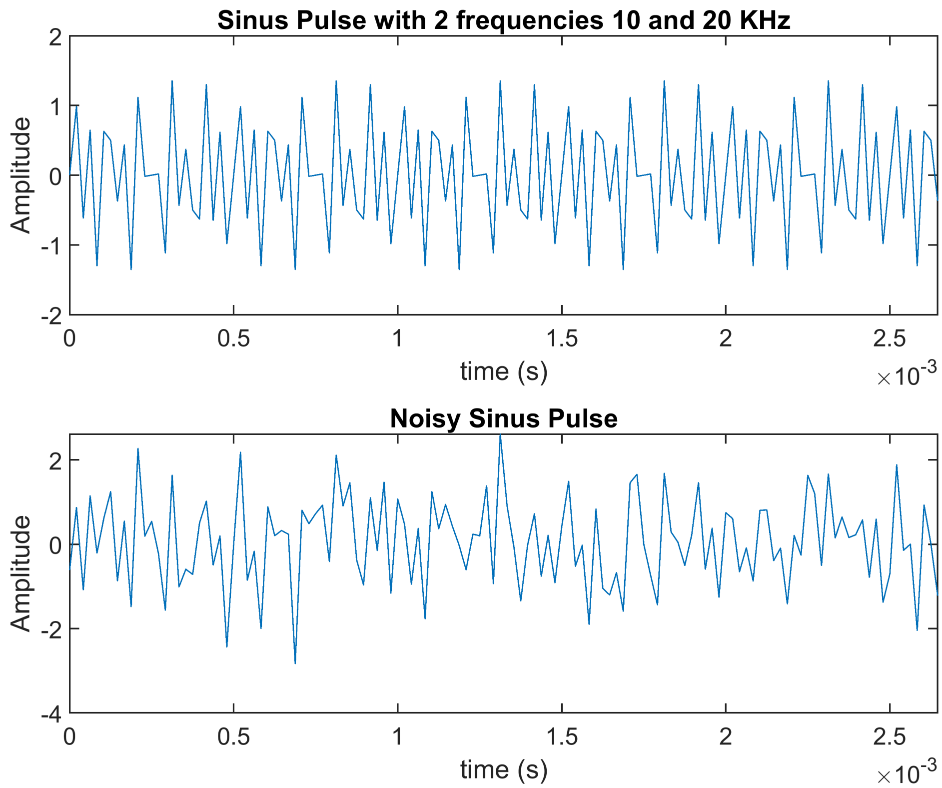
For example, create a time vector and signal :

```
clear
clear                                   % Clear all data stored in variables
Am = 1;                                 % 1V Amplitude
samplingFrequency = 48000;              % 48 KHz
f0 = 20e+3;                             % 20 KHz
f1 = 10e+3;                             % 10 KHz
duration = (127/samplingFrequency);     % ~ 2.6 ms
t = 0:1/samplingFrequency:duration;     % Time Vector
```

```
sinusPulse = Am*sin(2*pi*f0*t) + Am/2*sin(2*pi*f1*t);
noisySinus_3dB = awgn(sinusPulse,3);        % Add noise to sinusPulse with SNR = 3 dB
```

Figure below shows sinusPulse and its noisy version :

```
subplot(2,1,1)
plot(t,sinusPulse)
xlim([0 max(t)])
title('Sinus Pulse with 2 frequencies 10 and 20 KHz')
ylabel('Amplitude')
xlabel('time (s)')
subplot(2,1,2)
plot(t,noisySinus_3dB)
xlim([0 max(t)])
title('Noisy Sinus Pulse')
ylabel('Amplitude')
xlabel('time (s)')
```



Compute the DFT of the *noisySinus* and the magnitude of its DFT.

```
noisySinus_FFT = fft(noisySinus_3dB);       % Compute the DFT
magnitude = abs(noisySinus_FFT);            % Magnitude
```

Calculate the frequency vector and plot the single-sided amplitude spectrum.

```
f = (0:length(noisySinus_3dB)/2-1)*samplingFrequency/length(noisySinus_3dB);
```

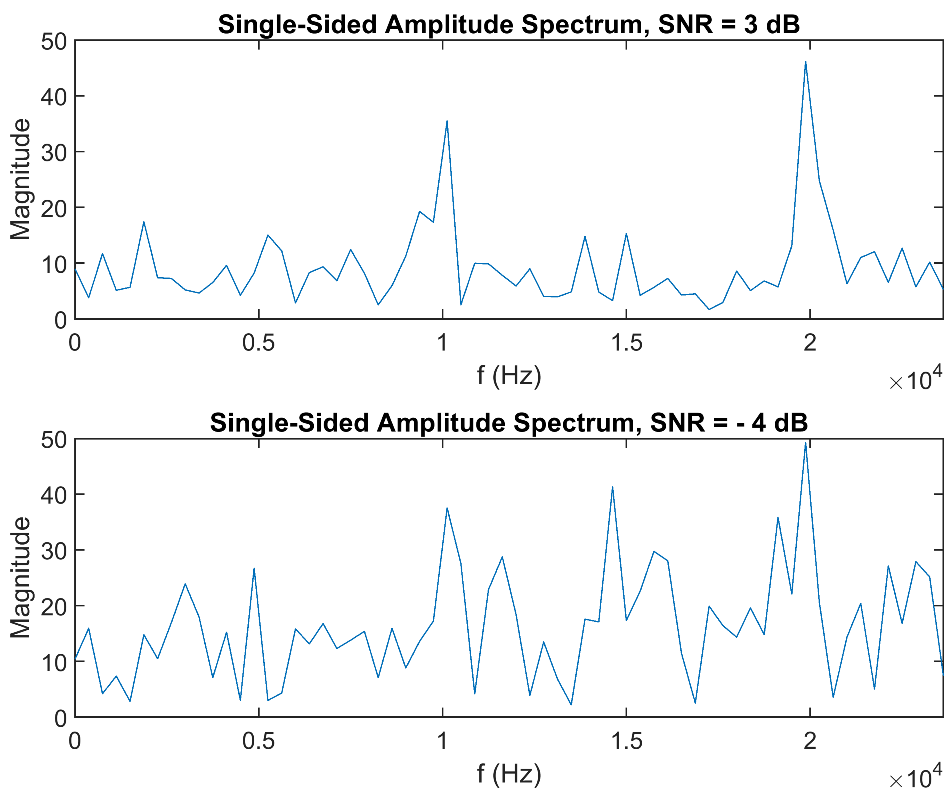
```
% Frequency Vector
magnitude_Single_Sided  = magnitude(1:round(length(noisySinus_3dB)/2));

figure
subplot(2,1,1)
plot(f,magnitude_Single_Sided)
title('Single-Sided Amplitude Spectrum, SNR = 3 dB')
xlabel('f (Hz)')
ylabel('Magnitude')
xlim([0 max(f)])


noisySinus_4dB = awgn(sinusPulse,-4);             % Add noise with snr = -4 dB
magnitude_4db = abs(fft(noisySinus_4dB));        % Compute the DFT and Magnitude

magnitude_4db_S_S = magnitude_4db(1:round(length(noisySinus_4dB)/2));

subplot(2,1,2)
plot(f,magnitude_4db_S_S)
title('Single-Sided Amplitude Spectrum, SNR = - 4 dB')
xlabel('f (Hz)')
ylabel('Magnitude')
xlim([0 max(f)])
```



As we see in the figure above, the DFT capable to detect the frequencies *f0 = 20Khz* and *f1 = 10Khz* of the noisy signal with a snr >= 3 dB and can not detect f0 and f1 with a snr < 3 dB.

So we summarize this DFT article with a fundamental idea that proves the relationship between the DFT (Discrete Fourier Transform) and SNR (Signal-to-Noise Ratio) it is proportional with each other, so as we increase the SNR, the DFT detect the frequencies with high precision.

Referecens :

- https://www.mathworks.com/help/signal/ug/discrete-fourier-transform.html
- https://en.wikipedia.org/wiki/Discrete_Fourier_transform