Dynamics of Short and Long Capillary-Gravity Waves

Ben Segal
bsegal@uw.edu
www.bsegal.org

University of Washington, Seattle, WA

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- Bernard Deconinck, University of Washington

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Foundations - NLS and KdV

- Nonlinear Schrödinger equation (NLS)

$$iu_t + u_{xx} + \tau_1 |u|^2 u = 0$$

Foundations - NLS and KdV

- Nonlinear Schrödinger equation (NLS)

$$iu_t + u_{xx} + \tau_1 |u|^2 u = 0$$

- Korteweg-de Vries equation (KdV)

$$v_t + vv_x + \tau_2 v_{xxx} = 0$$

System of equations

It is natural to consider the hypothetical system

$$
\begin{aligned}
iu_t + u_{xx} + \tau_1 |u|^2 u &= -\alpha u v, \\
v_t + v v_x + \tau_2 v_{xxx} &= -\frac{\alpha}{2}(|u|^2)_x.
\end{aligned}
$$

- Hamiltonian structure
- Globally well-posed
- Solitary waves are stable

See (Albert and Bhattarai 2013) and (Corcho and Linares 2007). Also see next talk.

Constructing solutions

Periodic traveling wave ansatz

Goal: Find all periodic traveling wave solutions to the system

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Transform to a moving frame,

$$(x, t) \to (z = x - \nu t, t = t).$$

Assume the ansatz,

$$\begin{aligned} u(x, t) &= e^{i\omega t} e^{i\theta(z)} r(z), \\ v(x, t) &= g(z). \end{aligned}$$

Periodic traveling wave ansatz

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$$\begin{aligned} u(x,t) &= e^{i\omega t} e^{i\theta(z)} r(z), \\ v(x,t) &= g(z). \end{aligned}$$

After plugging in,

$$-\omega r + \nu\theta' r - i\nu r' + i\theta'' r - (\theta')^2 r + 2i\theta' r' + r'' + \tau_1 r^3 = -\alpha rg,$$

$$-\nu g' + g''' + \tau_2 gg' = -\alpha rr'.$$

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$$-\nu g' + g''' + \tau_2 g g' = -\alpha r r'.$$

Taking real and imaginary parts of the first equation we find,

$$-\omega r + \nu\theta' r - (\theta')^2 r + r'' + \tau_1 r^3 = -\alpha r g,$$

$$-\nu r' + \theta'' r + 2\theta' r' = 0.$$

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We integrate the second equation to find,

$$\frac{d\theta}{dz} = \frac{c}{r^2} + \frac{\nu}{2}.$$

Periodic traveling wave ansatz

$$-\omega r + \nu\theta' r - (\theta')^2 r + r'' + \tau_1 r^3 = -\alpha r g,$$
$$-\nu g' + g''' + \tau_2 g g' = -\alpha r r'.$$

Letting $s = r^2$, the equations become,

$$\alpha g - \omega + \frac{\nu^2}{4} - \frac{c^2}{s^2} + \frac{s''}{2s} - \frac{(s')^2}{4s^2} + \tau_1 s = 0,$$
$$-\nu g' + g''' + \tau_2 g g' + \frac{\alpha}{2} s' = 0.$$

Assume the ansatz,

$$s(z) = A_1 \operatorname{sn}^2(\lambda z, k) + B_1,$$
$$g(z) = A_2 \operatorname{sn}^2(\lambda z, k) + B_2.$$

Periodic traveling wave solutions

- After plugging in, equate powers of sn^2,
- Need condition that $6\alpha - \tau_2 \neq 0$.

Arrive at solution

$$\begin{aligned} s(z) =& \frac{2k^2\lambda^2}{\tau_1\tau_2}\,(6\alpha-\tau_2)\,\mathrm{sn}^2(\lambda(x-\nu t),k) + B_1, \\ g(z) =& -\frac{12k^2\lambda^2}{\tau_2}\,\mathrm{sn}^2(\lambda(x-\nu t),k) \\ &+ \frac{1}{12\tau_1\tau_2}\left[\alpha(6\alpha-\tau_2) + 12\tau_1(4(1+k^2)\lambda^2+\nu)\right]. \end{aligned}$$

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\end{aligned}
$$

So,

$$
\begin{aligned}
u(x,t) =& e^{i\omega t}e^{i\theta(x-\nu t)}\sqrt{\frac{2k^2\lambda^2}{\tau_1\tau_2}\left(6\alpha - \tau_2\right)\mathrm{sn}^2(\lambda(x-\nu t), k) + B_1}, \\
v(x,t) =& -\frac{12k^2\lambda^2}{\tau_2}\mathrm{sn}^2(\lambda(x-\nu t), k) \\
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\end{aligned}
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Periodic traveling wave solutions

With the ansatz, we needed,

$$\frac{d\theta}{dz} = \frac{c}{r^2} + \frac{\nu}{2},$$

$$c^2 = \frac{1}{2\tau_1\tau_2(6\alpha - \tau_2)}\left[B_1(2\lambda^2(6\alpha - \tau_2) + B_1\tau_1\tau_2)(2k^2\lambda^2(6\alpha - \tau_2) + B_1\tau_1\tau_2)\right],$$

$$\begin{aligned}\omega = \frac{1}{12\tau_1\tau_2(6\alpha - \tau_2)}\Bigg[&36\alpha^4 - 12\alpha^3\tau_2 + 72\alpha^2\nu\tau_1 + 288\alpha^2\tau_1\lambda^2 + \alpha^2\tau_2^2\\ &+ 18\alpha\nu^2\tau_1\tau_2 - 12\alpha\nu\tau_1\tau_2 - 120\alpha\tau_1\tau_2\lambda^2 + 72\alpha B_1\tau_1^2\tau_2 - 18B_1\tau_1^2\tau_2^2\\ &+ 288\alpha^2k^2\tau_1\lambda^2 - 120\alpha k^2\tau_1\tau_2\lambda^2 + 12k^2\tau_1\tau_2^2\lambda^2 - 3\nu^2\tau_1\tau_2^2 + 12\tau_1\tau_2^2\lambda^2\Bigg],\end{aligned}$$

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- What we assumed to be real, must be real,
- What we assumed to be positive, must be positive.

Solution class

$$
\begin{aligned}
u(x,t) =& e^{i\omega t} e^{i\theta(x-\nu t)} \sqrt{\frac{2k^2\lambda^2}{\tau_1\tau_2}\left(6\alpha-\tau_2\right)\operatorname{sn}^2(\lambda(x-\nu t),k)+B_1},\\
v(x,t) =& -\frac{12k^2\lambda^2}{\tau_2}\operatorname{sn}^2(\lambda(x-\nu t),k)\\
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With free parameters $B_1, \nu, \lambda,$ and k.

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With free parameters B_1, ν, λ, and k.

- B_1 - offset

Solution class

$$u(x,t) = e^{i\omega t} e^{i\theta(x-\nu t)} \sqrt{\frac{2k^2\lambda^2}{\tau_1\tau_2}\left(6\alpha - \tau_2\right)\operatorname{sn}^2(\lambda(x-\nu t),k) + B_1},$$

$$\begin{aligned} v(x,t) = &-\frac{12k^2\lambda^2}{\tau_2}\operatorname{sn}^2(\lambda(x-\nu t),k) \\ &+ \frac{1}{12\tau_1\tau_2}\left[\alpha(6\alpha-\tau_2) + 12\tau_1(4(1+k^2)\lambda^2+\nu)\right]. \end{aligned}$$

With free parameters B_1, ν, λ, and k.

- B_1 - offset
- $\nu \in \mathbb{R}$ - velocity

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With free parameters B_1, ν, λ, and k.

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With free parameters B_1, ν, λ, and k.

- B_1 - offset
- $\nu \in \mathbb{R}$ - velocity
- $\lambda > 0$ - scaling
- $k \in [0,1]$ - elliptic modulus, relates to the period

Cases for solutions

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- For NLS, we have two cases
  - Defocusing NLS ($\tau_1 < 0$)
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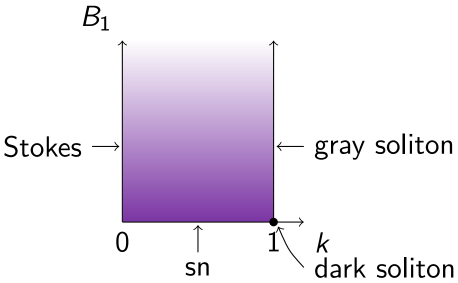
- For NLS, we have two cases
  - Defocusing NLS ($\tau_1 < 0$)
  - Focusing NLS ($\tau_1 > 0$)
- In both cases we need $\alpha > \tau_2/6$

Defocusing NLS ($\tau_1 < 0$)

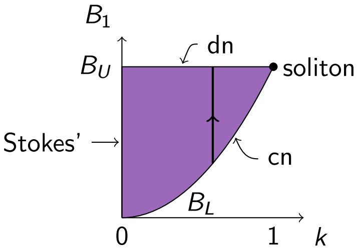
values of k and B_1	Solution type	$r(z)$, where $u = e^{i\omega t} e^{i\theta(z)} r(z)$
$k = 0,\ B_1 \geq 0$	Stokes wave	$r = \sqrt{B_1}$
$k \in (0, 1),\ B_1 = 0$	sn-type	$r = \sqrt{\frac{2k^2\lambda^2(6\alpha - \tau_2)}{\tau_1\tau_2}}\,\mathrm{sn}(\lambda z, k)$
$k = 1,\ B_1 = 0$	dark soliton	$r = \sqrt{\frac{2k^2\lambda^2(6\alpha - \tau_2)}{\tau_1\tau_2}}\,\tanh(z)$



Focusing NLS ($\tau_1 > 0$)

values of k and B_1	Solution type	$r(z)$, where $u = e^{i\omega t}e^{i\theta(z)}r(z)$
$k = 0,\ B_1 \in [0, B_U]$	Stokes wave	$r = \sqrt{B_1}$
$k \in (0,1),\ B_1 = B_L(k)$	cn-type	$r = \sqrt{\frac{-2k^2\lambda^2(6\alpha-\tau_2)}{\tau_1\tau_2}}\,\mathrm{cn}(\lambda z, k)$
$k \in (0,1),\ B_1 = B_U$	dn-type	$r = \sqrt{\frac{-2\lambda^2(6\alpha-\tau_2)}{\tau_1\tau_2}}\,\mathrm{dn}(\lambda z, k)$
$k = 1,\ B_1 = B_U$	bright soliton	$r = \sqrt{\frac{-\lambda^2(6\alpha-\tau_2)}{\tau_1\tau_2}}\,\mathrm{sech}(z)$

Table: $B_L(k) = \frac{-2k^2\lambda^2(6\alpha-\tau_2)}{\tau_1\tau_2}$ and $B_U = \frac{-\lambda^2(6\alpha-\tau_2)}{\tau_1\tau_2}$.



Other solutions?

Other elliptic solutions

- We have a large class of elliptic solutions found by an ansatz.
- We would like to verify that these are all elliptic solutions.
- We employ the method of Conte and Musette (Conte and Musette 2008)

Outline of the method of Conte and Musette

An efficient algorithm for computing all elliptic solutions to an ODE system:

Outline of the method of Conte and Musette

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Outline of the method of Conte and Musette

An efficient algorithm for computing all elliptic solutions to an ODE system:

1. Find Laurent series expansion for each solution u and v to coupled ODE system,
2. Derive the decoupled first order ODEs which each Laurent series satisfies,
3. Integrate the resulting ODEs F(u',u)=0 (we use the Maple package "algcurves"),

Summary of results from method

We find that all elliptic solutions are of the form

$$u = e^{i\omega t} e^{i\theta(z)} \sqrt{\gamma_1 + \gamma_2 \wp(\zeta z, 1, g_3)},$$
$$v = \delta_1 + \delta_2 \wp(\zeta z, 1, g_3).$$

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- With explicit expressions for $\gamma_1, \gamma_2, \zeta, g_3, \delta_1$, and δ_2 in terms of four free parameters.

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- With explicit expressions for $\gamma_1, \gamma_2, \zeta, g_3, \delta_1$, and δ_2 in terms of four free parameters.
- **These solutions are in one-to-one correspondence with our ansatz solutions.**
- We use the Jacobi sn solutions instead of these because their physical interpretation is more straightforward.

Stability results

Spectral stability

- All solitary wave solutions to focusing NLS are stable.
- All solitary wave solutions to KdV are stable.

All solitary wave solutions to NLS-KdV (focusing NLS only) are stable (Albert, Bhattarai 2010)

Spectral stability

- All solitary wave solutions to focusing NLS are stable.
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All solitary wave solutions to NLS-KdV (focusing NLS only) are stable (Albert, Bhattarai 2010)

- All sn-type solutions to defocusing NLS are spectrally stable (Bottman, Deconinck, Nivala 2011)
- All cn-type solutions to KdV are spectrally and orbitally stable (Bottman, Deconinck 2009; Deconinck, Kapitula 2010)

Spectral stability

- A stationary solution of a nonlinear problem is spectrally stable if there are no exponentially growing modes of the corresponding linearized problem
- We use the Fourier-Floquet-Hill method (Deconinck and Kutz 2006) to determine spectral stability numerically.
- This method converges spectrally (Curtis, Deconinck, 2009; Johnson, Zumbrun 2013)
- Using a Floquet decomposition and Fourier series truncation we get a matrix eigenvalue problem for the spectrum of the operator
- Any spectral elements with nonzero real part correspond to instability (the system is Hamiltonian)

Spectral stability - movies

- Following the method of (Deconinck and Kutz 2006) movies are created with
  - $N = 20$ (number of Fourier modes)
  - $D = 500$ (number of Floquet values)

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- Moving through parameter space we use
  - k=0.8 (elliptic modulus)
  - $\lambda = K(k)$ (for 2π-periodic solutions)
  - $\tau_1 = -2$ (for defocusing NLS)
  - $\tau_2 = -6$
  - $B_1 = 0$
  - $\nu = 0$

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  - $\tau_2 = -6$
  - $B_1 = 0$
  - $\nu = 0$
- With these values we need $\alpha > -1$
- $\alpha = 0$ is the case with no coupling (stable)

Positive values of coupling α

We let α range from 0 to 3.

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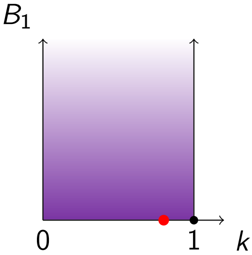

Negative values of coupling α

We let α range from 0 to -1.

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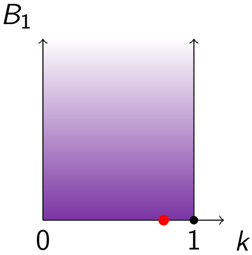

Summary

- We find all bounded traveling-waves solutions to the KdV-NLS system
- We hoped to find spectral stability numerically for a nontrivial set of parameters with coupling
  - A representative set of parameters are shown here
  - Any amount of coupling results in all period solutions losing stability numerically

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Thank you

- Ben Segal
- bsegal@uw.edu
- www.bsegal.org