Interest Rates under Falling Stars

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January 31, 2025

Intro

- Connection between Macroeconomic Variables and Term Structure of Interest Rates
- The long run mean of macroeconomic series and interest rates are time varying.
- Research Question: Will accounting for time-varying long run trends help understand treasury yield and predict excess returns?

Trend

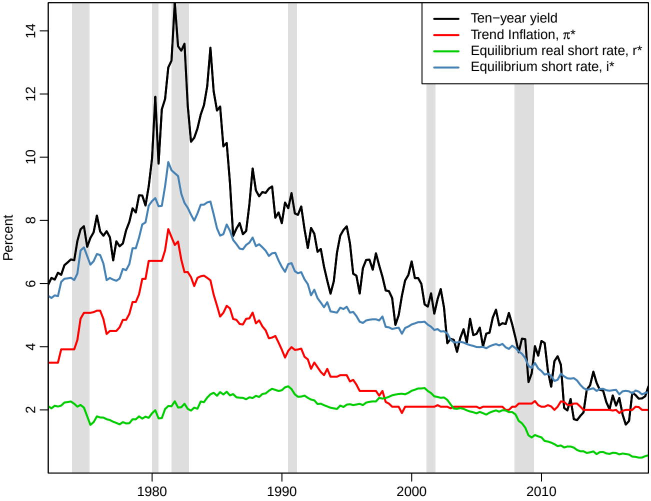

Figure: Ten-Year Yield and Macroeconomic Trends

Modelling the Term Structure (Preliminary)

- Let $y_t^{(n)}$ denote the zero-coupon bond yield with maturity n at time t
- $rx_{t+1}^{(n)}$ is the 1-period excess return on the bond with maturity n. $p_t^{(n)}$ is the log-price of bond with maturity n at time t.

$$rx_{t+1}^{(n)} = p_{t+1}^{(n-1)} - p_t^{(n)} - y_t^{(1)} \tag{1}$$

- Decomposition of yields:

$$y_t^{(n)} = \underbrace{\mathbb{E}_t \left[\frac{1}{n} \sum_{i=0}^{n-1} y_{t+i}^{(1)} \right]}_{\text{Expected path of short rates}} + \mathrm{TP}_t^{(n)} \tag{2}$$

- Expectation Theory suggests that TP is zero (strong form).

Stylized Facts

- Interest rate is drifting: [**?**]
- Bond excess returns are stationary
- Standard (no-arbitrage) models assume that interest rates are stationary. e.g.: [**?**], [**?**], [**?**]
- Standard ATSM models tend to generate term premia that are a-cyclical and parallel to the secular trend in yields: [**?**]
- Consequence: Low-frequency variation in interest rate must be captured by term premia.
- It is better to model a highly persistent process as unit root process. [**?**]
- Stationary assumption and random walk will give drastically different implications: [**?**]

Trend

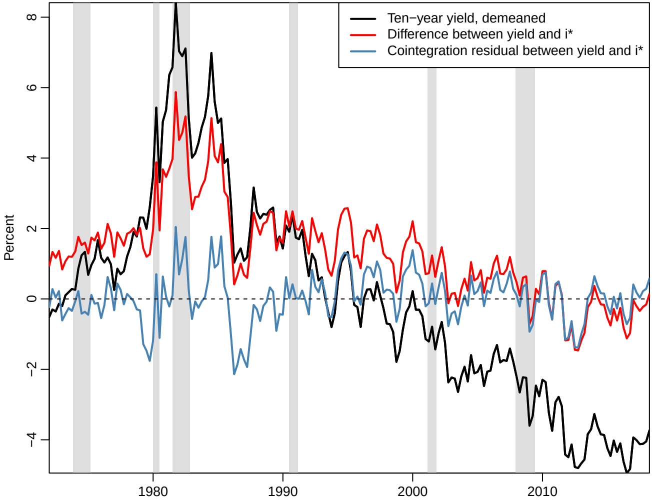

Figure: Detrending the Ten-Year Yield

Co-integration Test

Table 1—Cointegration Regressions and Tests

	Yield	(1)	(2)	(3)	(4)	(5)
Constant	6.48	0.65	−0.22	−1.82	−0.64	−2.17
	(0.55)	(0.54)	(0.33)	(0.43)	(0.39)	(0.30)
π_t^*		1.65	1.26	1.53	1.47	
		(0.11)	(0.08)	(0.07)	(0.07)	
r_t^*			0.99	1.76	1.18	
			(0.15)	(0.16)	(0.13)	
i_t^*						1.67
						(0.06)
R^2		0.85	0.93	0.96	0.96	0.95
Memo: r^*			Filtered	Real-time	Moving average	Real-time
SD	2.94	1.31	1.09	0.70	0.87	0.70
$\hat{\rho}$	0.97	0.88	0.85	0.65	0.75	0.64
Half-life	26.4	5.6	4.3	1.6	2.5	1.5
ADF	−1.13	−2.60	−3.94	−5.32	−4.33	−5.37
PP	−3.11	−18.32	−26.73	−68.47	−46.30	−70.30
LFST	0.00	0.03	0.16	0.72	0.23	0.71
Johansen $r = 0$		13.34	33.08	46.83	45.49	30.69
Johansen $r = 1$		1.29	5.92	11.57	9.14	0.73
ECM $\hat{\alpha}$		−0.11	−0.18	−0.44	−0.49	−0.45
		(0.03)	(0.05)	(0.08)	(0.09)	(0.08)

Figure: Co-integration Tests

Stylized Facts

- Variation in the macro trends accounts for the persistence of interest rates
- Yield trend component moves more than one-for-one with i_t^*
- The ten-year term premium must contain a trend component positively related to i_t^*
- Changes in i_t^* , properly scaled, can fully capture the trend in the ten-year Treasury yield
- Treasury yields are better modeled as having a stochastic trend
- Yield curve contains useful information for predicting bond excess return: [**?**], [**?**]

Interest Rate Trends

- Consider the following decomposition

$$y_t^n = \frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}_t[i_{t+j}] + \mathrm{TP}_t^{(n)} = i_t^* + \frac{1}{n}\sum_{j=0}^{n-1}\mathbb{E}_t[i_{t+j}^c] + \mathrm{TP}_t^{(n)} \quad (3)$$

  where i_t is the nominal interest rate. And $i_t^c = i_t - i_t^*$.
- The authors define i_t^* as the Beveridge-Nelson trend in the short-term nominal interest rate:

$$i_t^* = \lim_{j\to\infty}\mathbb{E}_t[i_{t+j}] \quad (4)$$

- If this trend is time-varying, then yields are non-stationary with a common trend. a.k.a "shifting endpoint" [**?**]

Interest Rate Trends

- The Fisher equation suggests

$$i_t = r_t + \mathbb{E}_t \pi_{t+1} \Rightarrow i_t^* = r_t^* + \pi_t^* \tag{5}$$

  where r_t is the real short rate, π_t is inflation. And the trends in r_t and π_t are defined analogously to i_t^*.
- Time varying π_t^* can be viewed as the perceived inflation target of the central bank.
- Proxy of r_t^*: (1) long-run real rate trends from time series models identified with Bayesian methods ; (2) New Keynesian macro models and use the Kalman filter to infer the neutral real interest rate (3) ...

Predicting Excess Return with Macro Trends

- One-period (log) excess return on bond:

$$rx_{t+1}^{(n)} = -(n-1)y_{t+1}^{(n-1)} + ny_t^{(n)} - y_t^{(1)} \tag{6}$$

- Does current yield curve contain *all* the information predicting bond excess return?
- [**?**]: Adding a proxy for the inflation trend significantly improved predictive power
- [**?**]: The trend inflation proxy remains a relatively robust predictor after correcting for the small-sample econometric distortions and thus is unspanned by the yield curve
- [**?**]: Yield curve *slope* predicts positive excess returns
- This paper: *Level* of the yield curve is a powerful predictor.

Predictive Regressions

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A. Full sample, 1971:IV–2018:I						
PC1	0.08	0.98	1.39	2.38	2.04	2.47
	(0.17)	(0.26)	(0.39)	(0.67)	(0.56)	(0.61)
PC2	0.43	0.47	0.43	0.67	0.68	0.70
	(0.17)	(0.17)	(0.17)	(0.15)	(0.15)	(0.15)
PC3	−2.37	−1.79	−1.92	−0.92	−0.90	−0.86
	(1.34)	(1.27)	(1.22)	(1.39)	(1.43)	(1.35)
π_t^*		−1.95	−2.21	−4.40	−3.89	
		(0.44)	(0.47)	(1.10)	(0.92)	
		[0.00]	[0.00]	[0.00]	[0.00]	
r_t^*			−1.19	−3.89	−2.71	
			(0.59)	(1.47)	(1.04)	
			[0.14]	[0.07]	[0.04]	
i_t^*						−4.50
						(1.05)
						[0.00]
R^2	0.09	0.16	0.18	0.21	0.20	0.21
Memo: r^*			filtered	real-time	moving average	real-time
Panel B. Subsample, 1985:I–2018:I						
PC1	0.25	0.59	1.67	2.65	2.38	1.93
	(0.16)	(0.22)	(0.47)	(0.57)	(0.51)	(0.47)
PC2	0.41	0.50	0.49	0.53	0.65	0.58
	(0.15)	(0.16)	(0.16)	(0.15)	(0.15)	(0.15)
PC3	−1.09	−0.97	0.14	1.74	2.11	0.56
	(1.14)	(1.12)	(1.30)	(1.48)	(1.55)	(1.19)
π_t^*		−1.05	−1.95	−3.44	−3.34	
		(0.73)	(0.75)	(0.87)	(0.83)	
		[0.38]	[0.10]	[0.01]	[0.01]	
r_t^*			−2.03	−5.80	−4.11	
			(0.82)	(1.54)	(1.08)	
			[0.07]	[0.01]	[0.01]	
i_t^*						−3.08
						(0.91)
						[0.02]
R^2	0.08	0.10	0.14	0.19	0.18	0.16
Memo: r^*			filtered	real-time	moving average	real-time

Figure: Predictive Regressions: Yields and Macro Trends

No-Arbitrage Model with a Stochastic Trend

- The state variables of yield dynamics are $\mathbf{P}_t$. They used $N = 3$ such yields factors.
- The key feature is the stochastic trend τ_t:

$$\mathbf{P}_t = \bar{\mathbf{P}} + \gamma\tau_t + \tilde{\mathbf{P}}_t, \quad \tau_t = \tau_{t-1} + \eta_t, \quad \tilde{\mathbf{P}}_t = \mathbf{\Phi}\tilde{\mathbf{P}}_{t-1} + \tilde{\mathbf{u}}_t \tag{7}$$

- η_t i.i.d. $\mathbf{\Phi}$ mean reversion matrix with modulus less than 1.
- long-run trend components of are

$$\mathbf{P}_t^* = \lim_{j\to\infty} \mathbb{E}_t \mathbf{P}_{t+j} = \bar{\mathbf{P}} + \gamma\tau_t \tag{8}$$

- The short rate

$$i_t = \delta_0 + \delta_1' \mathbf{P}_t \tag{9}$$

- Absence of arbitrage: There exists risk neutral measure $\mathbb{Q}$. Also, $\mathbf{P}_t$ is stationary under $\mathbb{Q}$:

$$\mathbf{P}_t = \mu^{\mathbb{Q}} + \mathbf{\Phi}^{\mathbb{Q}} \mathbf{P}_{t-1} + \mathbf{u}_t^{\mathbb{Q}} \tag{10}$$

Yield and Normalization

- The yield is affine in factors:

$$\mathbf{Y}_t = \mathbf{A} + \mathbf{B}\mathbf{P}_t \tag{11}$$

- To identify model parameters and state variables, two types of normalizations are required.
- Affine transformation - $\mathbb{Q}$ dynamics is determined by a scalar $k^{\mathbb{Q}}$ and N-vector $\lambda^{\mathbb{Q}}$ containing the eigenvalues of $\mathbf{\Phi}^{\mathbb{Q}}$ (real, distinct, less than 1).
- One linear constraint on each of $\bar{\mathbf{P}}$, γ: $\delta_0 + \delta_1' \bar{\mathbf{P}} = 0$, $\delta_1' \gamma = 1$
- The long run trend is *unspanned* by the yield curve - the cross section of interest rates at time t is not deterministically related to i_t^*

Data

- Yields: $J = 17$ bonds, quarterly. [**?**] with maturities from 1 to 15 years; three-month and six-month Treasury bill rates from the Federal Reserve's H.15 release
- π_t^*: the mostly survey-based PTR (perceived target rate for inflation) measure from the Federal Reserve
- r_t^*: an average of all filtered and real-time estimates: [**?**] and other estimated.

Estimation Methods

- 'Observed Shifting Endpoint' (OSE) model: adds data that can directly help pin down the trend estimate.
- 'Estimated Shifting Endpoint' (ESE) model: Use MCMC algorithm to simulate draws from the joint posterior distribution of the latent state variables and parameters
- 'Fixed Endpoint' (FE) model: separately simulate i_t^* from a random walk process using the OSE parameters.

ESE

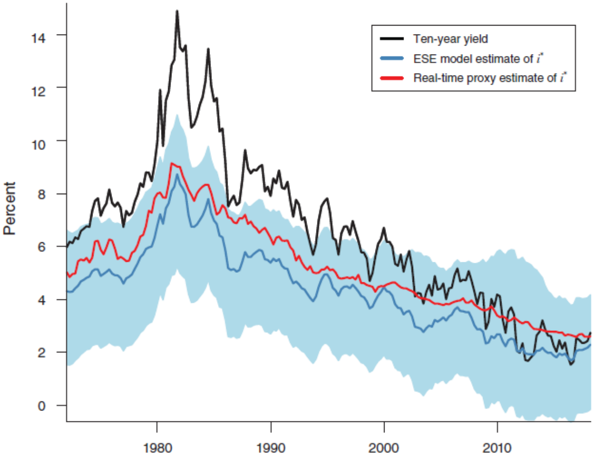

Figure: Model-Based Estimate of Equilibrium Interest Rate

Implications for Trend Components of Yields

- We can also decompose yield into trend and cycle components

$$\mathbf{Y}_t = \mathbf{Y}_t^* + \tilde{\mathbf{Y}}_t = \mathbf{A} + \mathbf{B}\bar{\mathbf{P}} + \mathbf{B}\gamma i_t^* + \mathbf{B}\tilde{\mathbf{P}}_t, \quad \mathbf{Y}_t^* = \lim_{j \to \infty} \mathbb{E}_t \mathbf{Y}_{t+j} \tag{12}$$

- The implied loadings of $\mathbf{Y}_t$ on i_t^* is $\mathbf{B}\gamma$
- Gradually rise from unity at the short end to around 1.7 at the long end
- Coefficients above 1 indicate that the term premium positively responds to changes in i_t^*

ESE

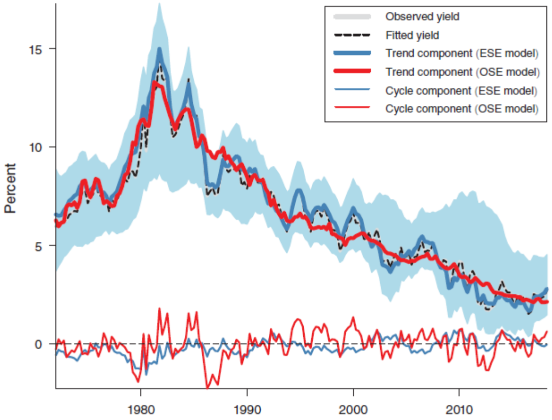

FIGURE 5. TREND AND CYCLE COMPONENTS OF TEN-YEAR YIELD

Figure: Trend and Cycle Components of Ten-Year Yield

ESE

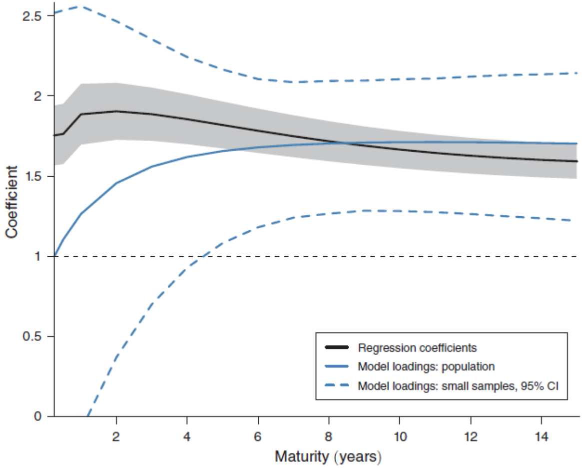

Figure: Loadings of Yields on the Equilibrium Interest Rate

Implied Excess Return Predictability

	R^2 PCs only	R^2 with i_t^*	ΔR^2
Data	0.09	0.21	0.12
FE model	0.09	0.10	0.01
	[0.04, 0.17]	[0.04, 0.17]	[0.00, 0.04]
OSE model	0.10	0.19	0.09
	[0.04, 0.17]	[0.13, 0.26]	[0.02, 0.18]
ESE model	0.07	0.14	0.08
	[0.02, 0.13]	[0.05, 0.27]	[0.00, 0.20]

Notes: The R^2 of predictive regressions for quarterly excess bond returns, averaged across maturities of 2 to 15 years. The R^2 in the data correspond to the full-sample estimates in Table 2 (first and last columns). The model-implied R^2 are based on 5,000 simulations of artificial datasets of the same size as the full sample. The table reports means and 95 percent Monte Carlo intervals (in square brackets) of the R^2 of predictive regressions estimated in these simulated data.

Figure: Model-Implied Predictability of Excess Bond Returns

Term Premium

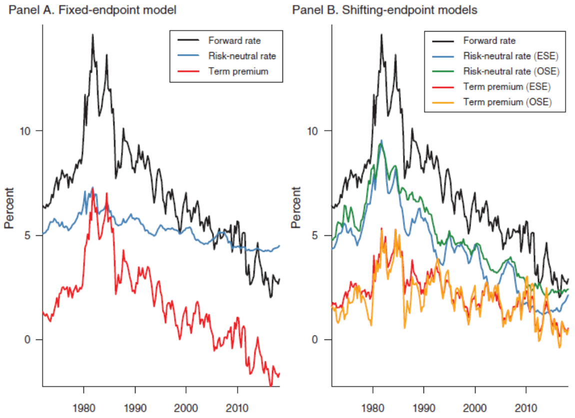

Figure 7. Expectations and Term Premium Components in Long-Term Interest Rates

Figure: Expectations and Term Premium Components in Long-Term Interest Rates

OOS

TABLE 5—ACCURACY OF OUT-OF-SAMPLE FORECASTS FOR THE TEN-YEAR YIELD

	Horizon in quarters				
	4	10	20	30	40
Panel A. Quarterly sample, 1976:III–2008:I (*127 quarters*)					
Random walk (RW)	1.33	1.85	2.52	2.60	2.88
Fixed endpoint (FE)	1.42	2.25	3.28	3.72	4.19
Observed shifting endpoint (OSE)	1.17	1.76	2.37	2.39	2.60
p-value: OSE $\geq$ RW	0.05	0.00	0.00	0.03	0.04
p-value: OSE $\geq$ FE	0.00	0.00	0.01	0.03	0.05
	Horizon in years				
	1	2	3	4	5
Panel B. Blue Chip sample, 1988:I–2011:IV (*48 Blue Chip surveys*)					
Blue Chip (BC)	1.06	1.39	1.59	1.79	1.99
Random walk (RW)	0.85	1.08	1.21	1.37	1.56
Fixed endpoint (FE)	1.53	2.08	2.52	2.96	3.34
Observed shifting endpoint (OSE)	0.87	0.95	1.04	1.18	1.37
p-value: OSE $\geq$ BC	0.10	0.08	0.15	0.18	0.20
p-value: OSE $\geq$ RW	0.58	0.05	0.01	0.04	0.08
p-value: OSE $\geq$ FE	0.00	0.00	0.00	0.00	0.00

Figure: Accuracy of Out-of-Sample Forecasts for the Ten-Year Yield

Conclusion

- Link macroeconomic and finance view of long-run trends
- Provide internally consistent formulation of that equilibrium trend and bond risk premia
- Potential future research:
    - A joint model with shifting endpoints of real yields, nominal yields, and inflation expectations
    - Shadow-rate paradigm
    - Stochastic volatility