Neutrinoless double beta decay and the theory behind the possible Majorana nature of the neutrino

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Introductory remarks

Determining whether neutrinos are their own antiparticles would be a monumental discovery in modern physics. It would mark the first time a Majorana* (the name given to particles that are their own antiparticle) fermion was discovered in nature. All fermions described within the current Standard Model of physics are Dirac fermions, meaning that their antiparticles are distinct from their particle pairs. There are, of course, particles in the Standard Model that are their own antiparticles, such as the photon, but this is a boson, not a fermion. Before delving deeper into this question it is important to discuss the history of neutrinos, why they were postulated, some of their properties and how the theory can lead into testable experiments.

The neutrino was first proposed in 1930, by Wolfgang Pauli, as an attempt to describe experimental results from nuclear beta decay. When a nucleus undergoes weak beta decay a neutron decays into a proton, an electron and an anti electron neutrino. The kinetic energy spectrum of electrons emitted from this decay was measured and it turned out to be continuous and broad. This was different to alpha and gamma decay where the energy spectrum of the emitted particles was narrowly defined. Alpha and gamma particles gave a well defined energy since they were the sole particles emitted during their respective decays and thus carried away the difference in energy between the initial and final states of the nucleus. In beta decay a continuous spectrum implied that more than just an electron was emitted since some of its energy could be lost. Pauli theorised that this was due to a small, almost immeasurably, particle he dubbed the neutron. It wasn't until later that the name was changed to neutrino.

*There is an interesting mystery about the sudden disappearance of Ettore Majorana, who theorised these particles, when he was only 32 years old. Some think he fled to Argentina and maybe joined the 3rd Reich in Germany, others think he may have committed suicide. There is no clear consensus on the topic but it is well worth a read of some hypotheses!

It was known at the time that nuclei of even mass number had a spin of 1 or zero and nuclei of odd mass number has spin of multiples of ½. Beta decay was seen to have no effect on either the mass number or spin of the nucleus. The electron was known to have a spin of ½ so this process appeared to violate the conservation of angular momentum! The neutrino therefore was proposed to have a spin of ½ to compensate for this. This particle was later identified to be the electron neutrino. Since neutrinos only interact with the weak force they have a left-handed helicity. It is now known through experiment that neutrinos come in three different flavours, the electron, muon and tau. There may even exist, as some Grand Unified Theories predict, massive, right handed flavours called sterile neutrinos, υ_s, that don't interact with the weak force. These sterile neutrinos could provide evidence for something called the seesaw mechanism that aims to describe why the light neutrinos are so light.

Dirac and Majorna fermions

Fermions come in two different types: Dirac and Majorana. The antiparticle counterpart of a Dirac fermion is distinct whereas the antiparticle of a Majorana fermion is itself. This is not the only difference between Dirac and Majorana fermions. To find out more let us start by studying the Dirac equation for a free neutrino with wavefunction ψ:

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0, \tag{1}$$

where γ^μ are the gamma matrices and m is the mass of the neutrino. This follows directly from the free Lagrangian that governs the particle

$$\mathscr{L} = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi. \tag{2}$$

Since the neutrino is a spin ½ particle it must obey the Majorana condition on its spinor field ψ. This condition is that of self charge conjugation such that the Lagrangian is invariant under a particle/antiparticle transformation. Self conjugation takes the form

$$\psi = \psi^c \equiv C\bar{\psi}^T \tag{3}$$

with C being the charge conjugation matrix. C is unitary and has the property $C\gamma_\mu C^{-1} = -\gamma^{\mu T}$ which can be rearranged and substituted into (2). One can see that the mass term, $m\bar{\psi}\psi$, becomes $-m\psi^T C^{-1}\psi$ which has no dependence on $\bar{\psi}$. This has a profound implication! If the field is assigned a U(1) gauge field, let's say electric charge, and the theory is invariant under any of those U(1) transformations then the mass term for that particle does not make sense in the Lagrangian. Another way of saying this is that if the particle we are describing has a charge and the theory is invariant under U(1) charge transformations then the Majorana condition (3) cannot be imposed since the mass term in (2) will no longer be valid. This is why quarks, electrons and muons are not Majorana particles but the neutrino might be as it is neutral.

We arbitrarily chose the U(1) transformations to be in charge space but lepton number is also a U(1) symmetry. Lepton number conservation violation has not yet been observed in any experiments and is thought to maybe be globally conserved. If it really is globally conserved then, like before, the mass term in the Lagrangian would no longer be valid and thus neutrinos would have to be Dirac particles. A process that is thought to exist that violates lepton number conservation is neutrinoless double beta decay, this will be explained in detail in a subsequent section. Finding this process may have even greater implications than one may think. If lepton number is not universally conserved then it may shed light on the matter antimatter imbalance in the universe. Non conservation of lepton number may just describe why the universe and most things in it (including you!) are made of matter rather than antimatter. A rather significant discovery is at stake for solving the puzzle of neutrinoless double beta decay so to quote Sherlock Holmes "The game, Mrs Hudson, is on!"

Another interesting property of Majorana fermions arises when expanding out ψ in free field via fourier transform:

$$\psi(x) = \int \frac{d^3p}{\sqrt{(2\pi)^3 2E_P}} \sum_s [a_s(\boldsymbol{p})u_s(\boldsymbol{p})e^{-ip\cdot x} + a_s{}^{\dagger}(\boldsymbol{p})v_s(\boldsymbol{p})e^{ip\cdot x}] \tag{4}$$

Second quantised notation reduces the complexity of ψ by assigning fermion annihilation and creation operators. $a_s(\boldsymbol{p})$ is equivalent to annihilating a fermion of momentum $\boldsymbol{p}$ and spin s and $a_s{}^{\dagger}(\boldsymbol{p})$ creates a fermion of momentum $\boldsymbol{p}$ and spin s. These operators only apply to Majorana particles and not for Dirac particles. If Dirac fermions are used then the creation operator $a_s{}^{\dagger}(\boldsymbol{p})$ is replaced by a $b_s{}^{\dagger}(\boldsymbol{p})$ that instead creates an antifermion of momentum $\boldsymbol{p}$ and spin s due to the distinction between Dirac particles and their anti particles. The summation is performed over the spin of the particles. $u_s(\boldsymbol{p})$ and $v_s(\boldsymbol{p})$ are spinor fields and they obey the anticommutation relation. It is possible, but I feel beyond the scope of this paper, to show that using the definitions of the spinor fields along with this relation and in conjunction with the condition in (3) that the vector current $\bar{\psi}\gamma_\mu\psi$ is zero. Physically this means that Majorana neutrinos have no magnetic moment. Any experiment that can conclusively measure a magnetic moment of a neutrino would therefore void the Majorana condition and thus show that neutrinos are in fact Dirac particles.

Double beta decay

In beta minus decay a neutron weakly interacts and changes into a proton while emitting an electron and an anti electron neutrino. It is just by convention that the neutrino emitted here an antineutrino since scientists who first encountered this problem assumed lepton number to be universally conserved which an antilepton would do here. Double beta decay is a process whereby two neutrons simultaneously change into protons emitting two electrons and two anti electron neutrinos. There may be another decay route for this process in which no anti electron neutrinos are emitted. I will go into detail later about how this may be physically possible. If this is a real process however, it will be the first experimentally observed violation of lepton number conservation, to wit, neutrinos are Majorana and therefore their own antiparticles!

Only certain nuclei can actually undergo double beta decay. The nucleus that decays, (A, Z) in Figure 1, must be less bound than the final nucleus (A, Z+2) but more bound than the intermediate nucleus (A, Z+1). In nature this corresponds to nuclei with even numbers of both protons and neutrons due to spin orbit coupling increasing stability. Regular double beta decay, unfortunately, is very rare. The half lives of isotopes whose decay route is only through double beta decay range from 10^{19} to 10^{25} years! Isotopes like this are considered stable for all intents and purposes. So to detect double beta decay one would need a very sensitive detector, a huge pile of the isotope and to wait an excruciating amount of time to make any significant measurements. We're not looking for double beta decay though, we're looking for neutrinoless double beta decay and this process is even more elusive, by a factor of around 100. Experiments that have already been conducted have managed to put a lower bound on the decay rates of neutrinoless double beta decay for a few isotopes. An example of this is that of ^{76}Ge whose half life has a lower bound of $T_{1/2}^{ov} > 0.8 \times 10^{25}$ years at the 90% confidence level.[3] [4]. This result corresponds to a constraint on the mass of the neutrino of $\langle m_v \rangle < (0.5 - 1.5)eV$ at the 90% confidence level. More on the how this rate can give a constraint on neutrino mass and the complexities of detectors will come later.

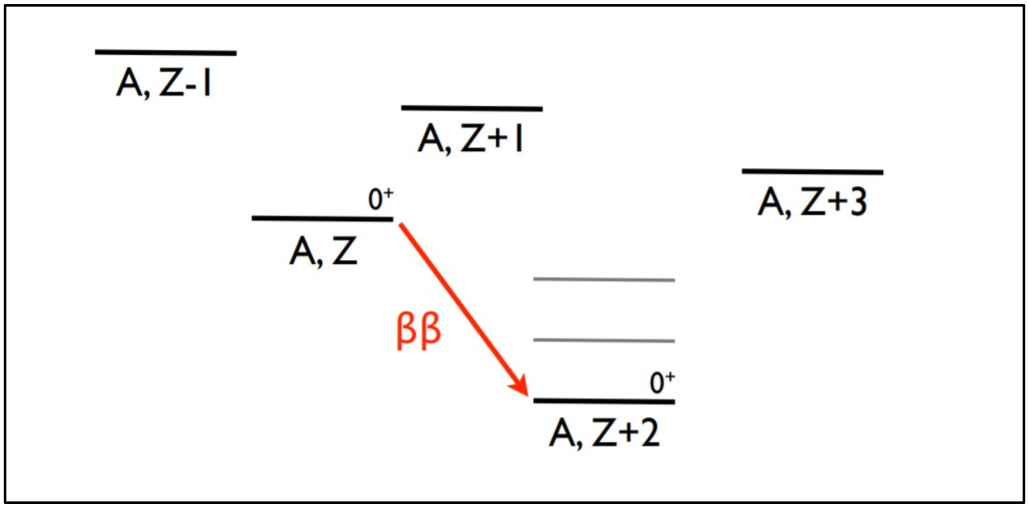

Figure 1: Here the nucleus that decays is denoted by (A, Z) and it follows the red ββ decay path. It releases two beta particles and two anti electron neutrinos while mutating into the (A, Z+2) nucleus. The intermediate nucleus (A, Z+1) is less stable than (A, Z) as indicated by its height and the final nucleus is more stable than (A, Z).

The process of neutrinoless double beta decay

Before diving directly into the process of neutrinoless double beta decay is important to understand the mechanisms behind regular beta decay and regular double beta decay. These two processes are completely valid in the standard model and they have been observed in nature countless times. To begin, beta decay occurs when a neutron transforms into a proton but more specifically one of the down quarks in the neutron emits a W^- boson through a weak process thereby changing it into an up quark. This W^- boson then propagates and decays into an electron and an anti electron neutrino (decay into a positron/neutrino pair is also allowed). Figure 2 shows a Feynman diagram of this process. For regular double beta decay this process occurs twice, simultaneously, as seen in figure 3.

Neutrinoless double beta decay is not allowed within the standard model, this is due to the fact that it does not conserve lepton number. Lepton number is increased (or decreased) globally by two in this process. A possible extension to the model that allows this decay to happen includes a mediating Majorana mass that acts virtually between two propagating W^- bosons causing only two electrons to be emitted. This process can be seen in figure 3.

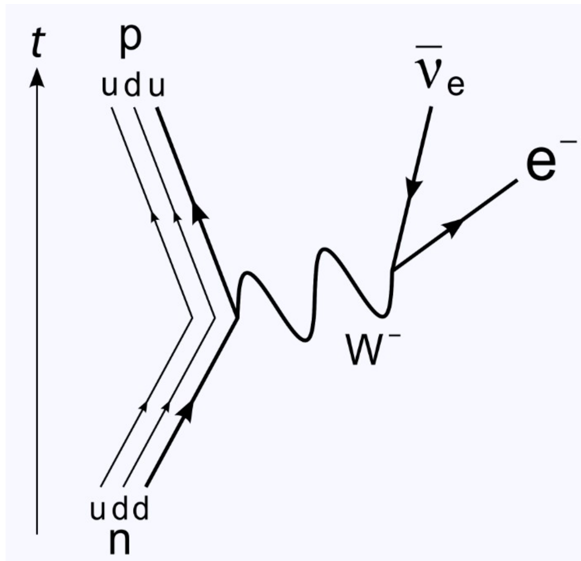

Figure 2: This figure shows the process of beta minus decay where a down quark emits a W^- boson that then decays into an electron (e^-) and an anti electron neutrino ($\overline{v}_e$). The fact that the neutrino here is an antineutrino is just convention.

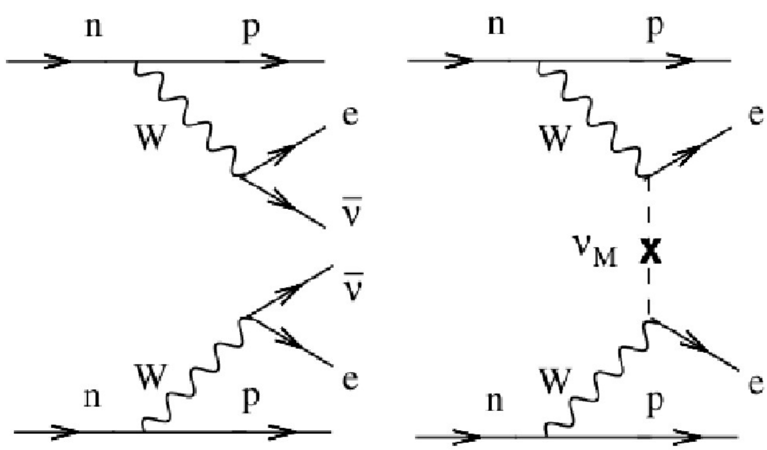

Figure 3: On the left the process of regular double beta decay with emissions of neutrinos is shown and to the right, the neutrinoless version can be seen. The ν_M particle is a virtual Majorana mass that represents the two interacting neutrinos.

Using what is known as the random phase approximation, which I deem far beyond the mathematical scope of this paper, one can calculate the rate of regular double beta decay to be

$$(T_{1/2}^{2v})^{-1} = G_{2v}(Q_{\beta\beta}, Z)|M_{2v}|^2. \tag{5}$$

Here $G_{2v}(Q_{\beta\beta}, Z)$ denotes the four-particle phase space factor and $|M_{2v}|^2$ is the squared nuclear matrix element. This rate does not have any preference over the Dirac or Majorana nature of the neutrinos involved and nor does it have any strong, explicit dependence on the mass of the neutrinos. Using the same approach for neutrinoless double beta decay the rate is

$$(T_{1/2}^{0v})^{-1} = G_{0v}(Q_{\beta\beta}, Z)|M_{0v}|^2\langle m_{\beta\beta}\rangle^2. \tag{6}$$

Where $G_{ov}(Q_{\beta\beta}, Z)$ is the phase space factor for the two electron emission that is a function of $Q_{\beta\beta}$ the energy of the decay and Z the proton number of the parent isotope. $|M_{0v}|^2$ is the squared nuclear matrix element for the decay and $\langle m_{\beta\beta}\rangle$ is the effective Majorana mass of an electron neutrino, explicitly

$$\langle m_{\beta\beta}\rangle \equiv |\sum_k m_k U_{ek}^2|. \tag{7}$$

m_k represents all three masses of each light neutrino, electron, muon and tau. The distinction of light neutrinos is made here since a proposed deeper mechanism of this problem contains heavier, sterile, right handed neutrinos. U_{ek} are the elements of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, a matrix that contains information about how different flavours of neutrinos mix and how that relates to their masses. Recent results from Super Kamiokande and Sudbry Neutrino Observatory [6] have proven that neutrinos both oscillate and have mass. These experiments have allowed the U_{ek} values to be constrained to a good degree. For an experiment to actually be able to determine the effective mass it is useful to define the nuclear structure form factor

$$F_N \equiv G_{0v}(Q_{\beta\beta}, Z)|M_{0v}|^2 {m_e}^2. \tag{8}$$

The value m_e is the mass of the electron. This allows the effective mass to be re-written in terms of F_N such that

$$\langle m_{\beta\beta}\rangle \equiv m_e\left[F_N T_{1/2}^{0v}\right]^{-1/2}. \tag{9}$$

To be sensitive to a certain value of $\langle m_{\beta\beta}\rangle$ an experiment must be able to measure a given half-life. This half-life can be estimated by using calculated values of F_N. For example, calculated F_N values fall in the 10^{-13} and 10^{-14} years^{-1} and if we want to design an experiment that is sensitive to masses $\langle m_{\beta\beta}\rangle$ of 0.1eV, the current highest standard, then the detector must be able to observe half lives of between 10^{26} and 10^{27} years. 10^{27} years is around 100 Quadrillion times the age of the *Universe*! To combat this, experiments must be run with many kilograms of isotopes and for very long periods of time. The GERDA collaboration in Italy estimates that their source of 38kg of ^{76}Ge will produce less than ONE observable neutrinoless double beta decay per kilogram per YEAR. This makes it abundantly clear that any background radiation in the event range must be limited as much as possible so that we can false positives are reduced, we want to be sure that this is the reaction that is detected. For ^{76}Ge the characteristic footprint of a decay would be a sharp peak in the detected energy spectrum at 2039keV.

The GERDA experiment

Observing neutrinoless double beta decay is an expensive and arduous process. Materials that can exhibit this decay type occur rarely in nature and the processes used to isolate these isotopes are complicated and costly. The Germanium Detector Array (GERDA) is an experiment that uses high purity germanium detectors containing large amounts of ^{76}Ge as a source of decay. GERDA uses hundreds of these HPGe's to both produce and detect neutrinoless double beta decay. A neutrinoless double beta decay would leave a footprint within the detector that can be traced and identified.

Due to the extraordinarily low count rate discussed in an earlier section, one count per kilogram per year, huge amounts of shielding is needed around the detectors. Neutrinos from the Sun, Earth and cosmic rays can interfere with measurements and all add to background noise. Nearby materials have to be analysed or selected carefully, they need to be accounted for during calibrations. The Germanium detectors are submerged in a sixty-four cubic meter tank of liquid argon with copper shielding on the outside. This whole contraption is then held within a 590 cubic meter tank of ultra-pure water. To top it all off, the entire laboratory is located underneath the largest mountain in the Apennine range, Gran Sasso, which provides an average rock cover of 1400m, equivalent to another 3000m of water shielding. The inner set up can be seen in figure 4. Despite all this and running the detectors for multiple years no trace of neutrinoless double beta decay has come up. The latest results from GERDA put $T_{1/2}^{0v}$ to be >$0.8x10^{25}$ years

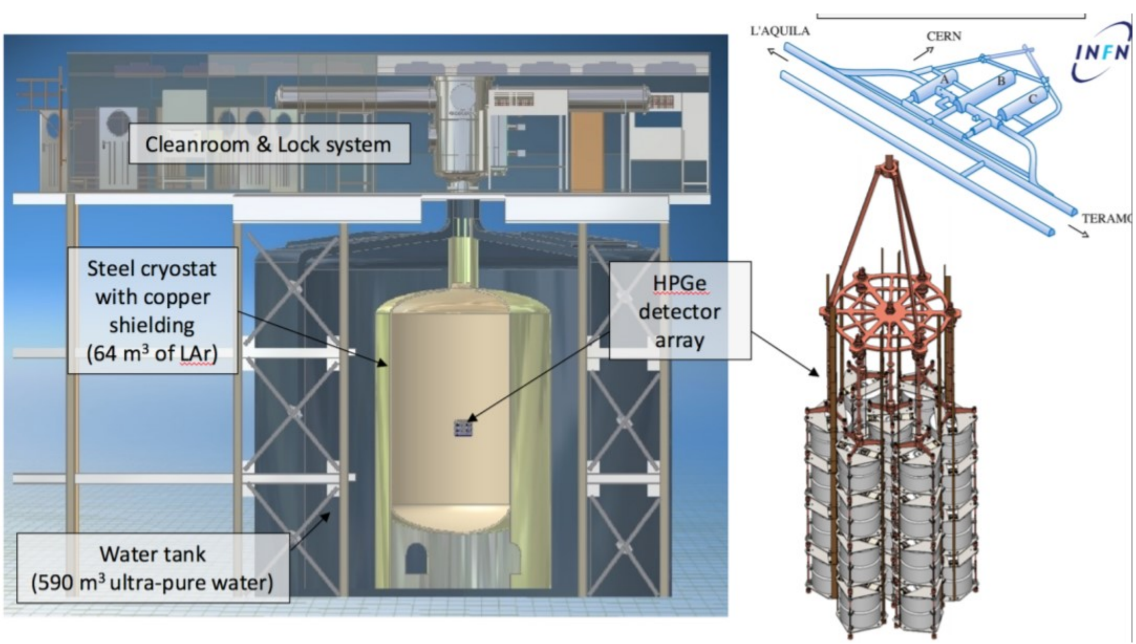

Figure 4: Here the interior set up of the GERDA detector is shown. The laboratory layout is visible in the top right corner; it is situated under the Mountain, Gran Sasso, in Italy.

Germanium, unfortunately doesn't come cheap, the next upgrade to GERDA wants to add another 200kg of Germanium to the current stockpile. Enrichment costs around $70 a gram and diode production (for use in detectors) is also extremely costly. The entire upgrade is estimated to cost more than 20 Million dollars and will take 4 years to implement. If current and proposed detectors continue to find no trace of neutrinoless decays then future detectors could be as costly as large scale accelerator projects and if more and more negative results come in we may have to rule out neutrinoless double beta decay altogether and search for another way to prove if neutrinos are their own antiparticles or not.

Conclusion

Neutrinos were initially postulated to solve an angular momentum conservation problem but they have shown that they may hold the secrets to some fundamental questions. By studying properties of neutrinos that are currently known, scientists have come up with ways of detecting a process whose half life is many magnitudes greater than the age of the Universe. Finding evidence that this process exists is very costly and will take a huge amount of time but it will be of great significance if and when it does get detected.

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