CS171 PS1

• Name: Tsung-Ying Chen

• SID: 861310198

• Date: 10/14/2017

Consider the following 1-dimensional regression dataset

x	y
0	1
2	-3
2	-2
3	-3
-1	-1
1	-1

Fit a third-degree polynomial to this data using least squares regression. You may use Matlab to do the calculations (matrix inversions, for instance), but show all of your steps. Write the resulting f(x) as a third degree polynomial in x.

$Let\ f(x) = ax^3 + bx^2 + cx + d$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} = \begin{bmatrix} {x_1}^3 & {x_1}^2 & x_1 & 1 \\ {x_2}^3 & {x_2}^2 & x_2 & 1 \\ {x_3}^3 & {x_3}^2 & x_3 & 1 \\ {x_4}^3 & {x_4}^2 & x_4 & 1 \\ {x_5}^3 & {x_5}^2 & x_5 & 1 \\ {x_6}^3 & {x_6}^2 & x_6 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -3 \\ -2 \\ -3 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 8 & 4 & 2 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \Leftrightarrow Y = Xw$$

$Use\ w = (X^T X)^{-1} X^T Y$

$$= \left(\begin{bmatrix} 0 & 8 & 8 & 27 & -1 & 1 \\ 0 & 4 & 4 & 9 & 1 & 1 \\ 0 & 2 & 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \\ 8 & 4 & 2 & 1 \\ 8 & 4 & 2 & 1 \\ 27 & 9 & 3 & 1 \\ -1 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 0 & 8 & 8 & 27 & -1 & 1 \\ 0 & 4 & 4 & 9 & 1 & 1 \\ 0 & 2 & 2 & 3 & -1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -2 \\ -3 \\ -1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7419 \\ -0.2339 \\ -1.5161 \\ 0.3962 \end{bmatrix}$$ (solved by Matlab)

Use Matlab to plot the resulting function and data on the same plot (the data as points, the function as a smooth curve), on the range of $x \in [-1, 4]$.

Now perform the same two steps again (calculate the third-degree polynomial fit and plot the resulting function with the data) for ridge regression with $\lambda = 5$.