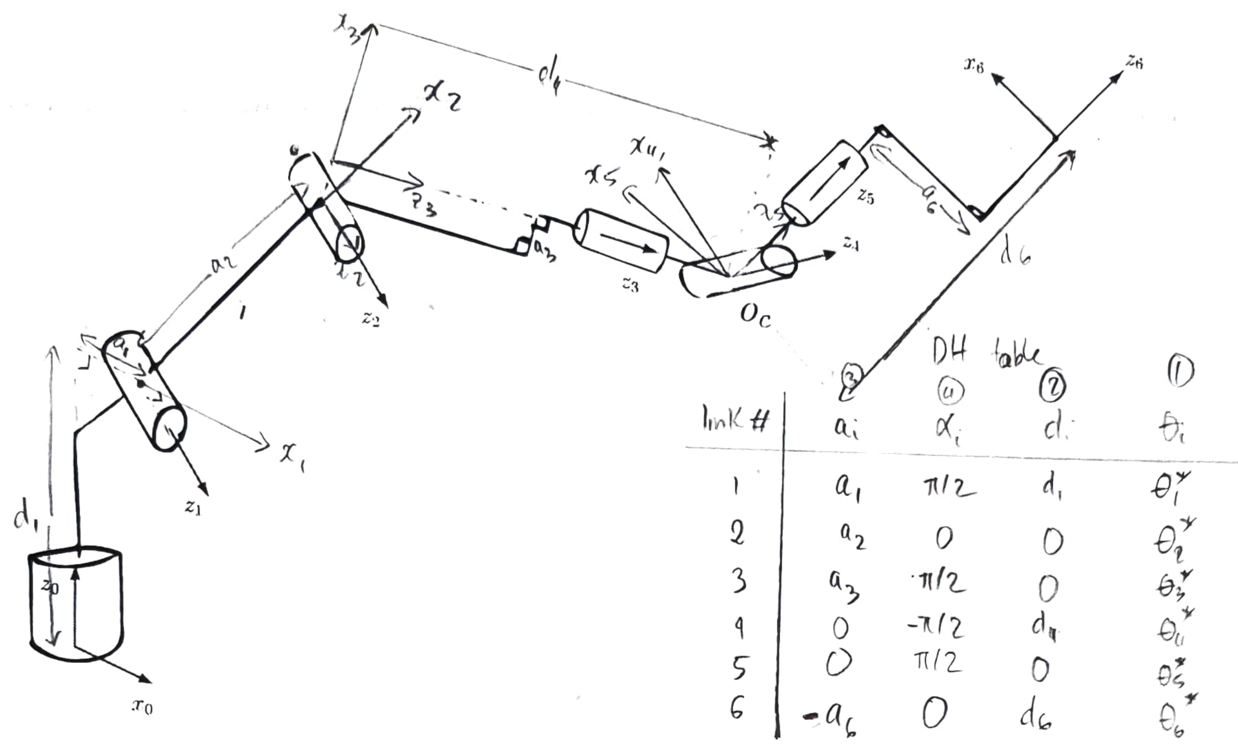
Prelab, part 1



DH table

link #	(3) a_i	(4) α_i	(2) d_i	(1) θ_i
1	a_1	$\pi/2$	d_1	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	$\pi/2$	0	θ_3^*
4	0	$-\pi/2$	d_4	θ_4^*
5	0	$\pi/2$	0	θ_5^*
6	$-a_6$	0	d_6	θ_6^*

Figure 3: Schematic diagram of KUKA robot arm not at rest.

up solution. Write your derivations neatly on paper. As in lab 1, one can solve the inverse kinematics problem by the technique of kinematic decoupling in which the problem is divided in two parts: inverse position and inverse orientation.

- The position of the wrist centre o_c is shown in Figure 2. First find $(\theta_1, \theta_2, \theta_3)$ such that $o_c^0(\theta_1, \theta_2, \theta_3) = \begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = o_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$.
- Then solve the equation
$$R_6^3(\theta_4, \theta_5, \theta_6) = (R_3^0)^\top R_d.$$
for $(\theta_4, \theta_5, \theta_6)$.

3. Modify your inverse kinematics function from lab 1 to incorporate the changes of this setup. Specifically, write Matlab functions `mykuka.m`, `forward_kuka.m`, and `inverse_kuka.m` as follows.

   `myrobot = mykuka(DH)` defines the robot structure of the KUKA robot with the 6×4 DH table you found earlier.

   `H = forward_kuka(q,myrobot)` returns the homogeneous transformation matrix `H` of the end effector, where q is the 6×1 vector of joint angles, and `myrobot` is the robot structure defined above.

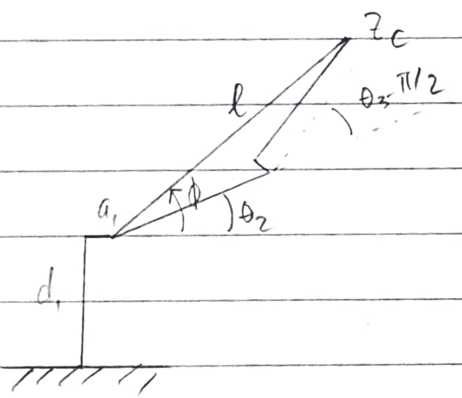
   `q = inverse_kuka(H,myrobot)` returns the 6×1 vector of joint angles $q = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)$, where `H` is the 4×4 homogeneous transformation matrix $H = \begin{bmatrix} R_d & o_d^0 \\ 0 & 1 \end{bmatrix}$.

4. Test your software: you should get

```
>> kuka=mykuka(DH);
>> forward_kuka([pi/5 pi/3 -pi/4 pi/4 pi/3 pi/4]',kuka)

ans =
```

let $\begin{bmatrix} x_c \\ y_c \\ z_c \end{bmatrix} = O_d^0 - R_d \begin{bmatrix} -a_6 \\ 0 \\ d_6 \end{bmatrix}$

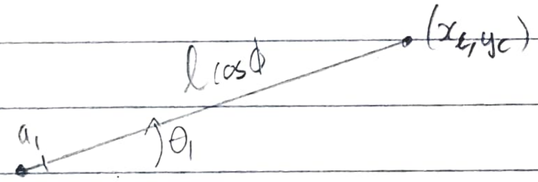


$z_c - d_1 = l \sin\phi$

$l\cos\phi = \sqrt{x_c^2 + y_c^2} - a_1$

$(a_1 + l\cos\phi)\cos\theta_1 = x_c$

$(a_1 + l\sin\phi)\sin\theta_1 = y_c$

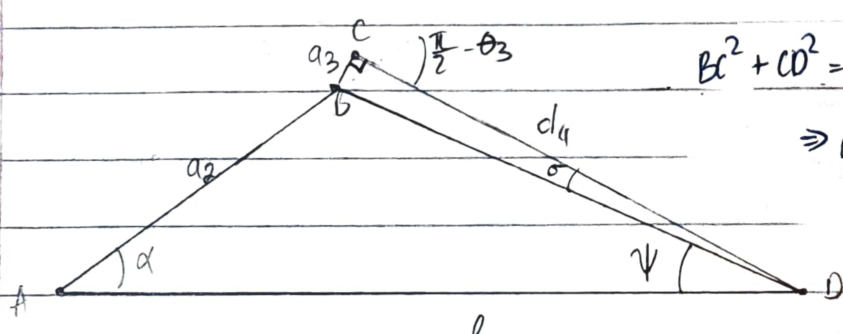


$\Rightarrow \boxed{\theta_1 = \operatorname{atan2}(y_c, x_c)}$

$\tan\phi = \dfrac{z_c - d_1}{\sqrt{x_c^2 + y_c^2} - a_1} \Rightarrow \phi = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2} - a_1\right)$

$\Rightarrow \quad \Rightarrow l = \dfrac{z_c - d_1}{\sin\phi}$

Given that l and ϕ is known, we have the following elbow up configuration:



$BC^2 + CD^2 = AB^2 + l^2 - 2AB \cdot l \cos\alpha$

$\Rightarrow \alpha = \cos^{-1}\left(\dfrac{BC^2 + CD^2 - AB^2 - l^2}{-2AB \cdot l}\right)$

$= \cos^{-1}\left(\dfrac{a_3^2 + d_4^2 - a_2^2 - l^2}{-2a_2 \cdot l}\right)$

$$\boxed{\theta_2 = \phi + \alpha = \phi + \cos^{-1}\left(\frac{a_3^2 + d_4^2 - a_2^2 - l^2}{-2a_2 l}\right) \quad \text{where } \begin{cases} \phi = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2 + y_c^2} - a_1\right) \\ l = \dfrac{z_c - d_1}{\sin\phi} \end{cases}}$$

$$a_2^2 = \ell^2 + a_3^2 + d_4^2 - 2\ell\sqrt{a_3^2+d_4^2}\cos\psi$$

$$\Rightarrow \psi = \cos^{-1}\left(\frac{a_2^2 - \ell^2 - a_3^2 - d_4^2}{-2\ell\sqrt{a_3^2+d_4^2}}\right)$$

$$\sigma = \operatorname{atan2}(a_3, d_4)$$

$$\frac{\pi}{2} - \theta_3 = \alpha + \sigma + \psi$$

$$\boxed{\begin{aligned} \Rightarrow \theta_3 &= \frac{\pi}{2} - \left[\cos^{-1}\left(\frac{a_3^2+d_4^2-a_2^2-\ell^2}{-2a_2\ell}\right) + \operatorname{atan2}(a_3, d_4) + \cos^{-1}\left(\frac{a_2^2-\ell^2-a_3^2-d_4^2}{-2\ell\sqrt{a_3^2+d_4^2}}\right)\right] \\ \text{where } &\begin{cases} \phi = \operatorname{atan2}\left(z_c - d_1, \sqrt{x_c^2+y_c^2} - a_1\right) \\ \ell = \dfrac{z_c - d_1}{\sin\phi} \end{cases} \end{aligned}}$$

from $\theta_1, \theta_2, \theta_3$ above, we can calculate R_3^0 (by finding $H_3^0 = H_1^0 H_2^1 H_3^2$, and then extract R_3^0)

$$R_6^3 = (R_3^0)^T R_d = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

b/c the wrist has ZYZ configuration, the solution can be found analytically to be:

$$\boxed{\begin{aligned} \theta_4 &= \operatorname{atan2}(a_{23}, a_{13}) \\ \theta_5 &= \operatorname{atan2}\left(\sqrt{1-a_{33}^2}, a_{33}\right) \\ \theta_6 &= \operatorname{atan2}(a_{32}, -a_{31}) \end{aligned}}$$