Homework #5

due 3/28/17 by 11:59pm

Include all work as either an m-file script, m-file function, or example code included with "' and document your code in the README.md file

1. Create a new github repository called 'linear_algebra'.
   a. Add rcc02007 and pez16103 as collaborators.
   b. Clone the repository to your computer.
2. Create an LU-decomposition function called `lu_tridiag.m` that takes 3 vectors as inputs and calculates the LU-decomposition of a tridiagonal matrix. The output should be 3 vectors, the diagonal of the Upper matrix, and the two off-diagonal vectors of the Lower and Upper matrices.

   ```
   [ud,uo,lo]=lu_tridiag(e,f,g);
   ```
3. Use the output from `lu_tridiag.m` to create a forward substitution and back-substitution function called `solve_tridiag.m` that provides the solution of Ax=b given the vectors from the output of [ud,uo,lo]=lu_tridiag(e,f,g). *Note: do not use the backslash solver* `\`*, create an algebraic solution*

   ```
   x=solve_tridiag(ud,uo,lo,b);
   ```
4. Test your function on the matrices A3, A4, ..., A10 generated with `test_arrays.m` Solving for `b=ones(N,1);` where N is the size of A. In your `README.md` file, compare the norm of the error between your result and the result of AN.

```
| size of A | norm(error) |
|-----------|-------------|
|  3 | ... |
| 4 | ... |
```

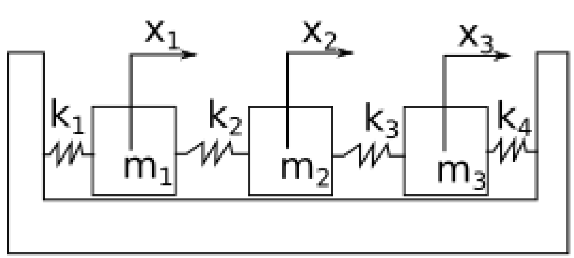

Figure 1: Spring-mass system for analysis

5. In the system shown above, determine the three differential equations for the position of masses 1, 2, and 3. Solve for the vibrational modes of the spring-mass system if k1=10 N/m, k2=k3=20 N/m, and k4=10 N/m. The masses are m1=1 kg, m2=2 kg and m3=4 kg. Determine the eigenvalues and natural frequencies.
6. The curvature of a slender column subject to an axial load P (Fig. P13.10) can be modeled by

$\frac{d^2y}{dx^2} + p^2 y = 0$

where $p^2 = \frac{P}{EI}$

where E = the modulus of elasticity, and I = the moment of inertia of the cross section about its neutral axis.

This model can be converted into an eigenvalue problem by substituting a centered finite-difference approximation for the second derivative to give $\frac{y_{i+1}-2y_i+y_{i-1}}{\Delta x^2} + p^2 y_i$

where i = a node located at a position along the rod's interior, and Δx = the spacing between nodes. This equation can be expressed as $y_{i-1} - (2 - \Delta x^2 p^2) y_i y_{i+1} = 0$ Writing this equation for a series of interior nodes along the axis of the column yields a homogeneous system of equations. (See 13.10 for 4 interior-node example)

Determine the eigenvalues for a 5-segment (4-interior nodes), 6-segment (5-interior nodes), and 10-segment (9-interior nodes). Using the modulus and moment of inertia of a pole for pole-vaulting (http://people.bath.ac.uk/taf21/sports_whole.htm) E=76E9 Pa, I=4E-8 m^4, and L= 5m.

Include a table in the `README.md` that shows the following results: What are the largest and smallest eigenvalues for the beam? How many eigenvalues are there?

```
| # of segments | largest | smallest | # of eigenvalues |
| --- | --- | --- | --- |
| 5 | ... | ... | ... |
| 6 | ... | ... | ... |
| 10 | ... | ... | ... |
```

If the segment length (Δx) approaches 0, how many eigenvalues would there be?