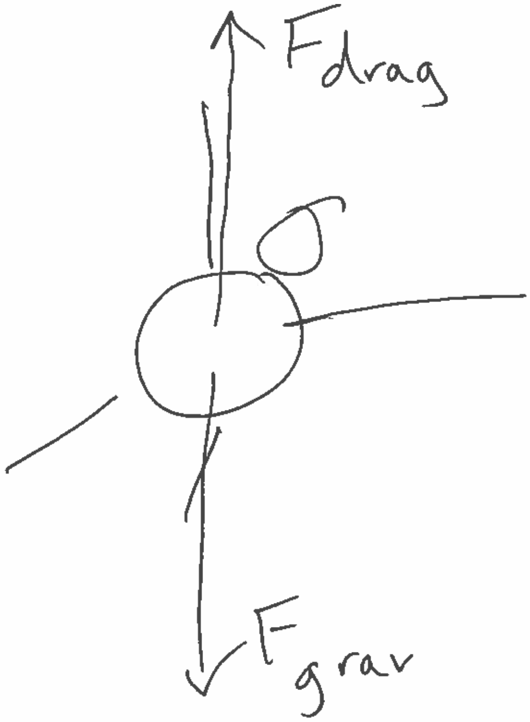

$$\Sigma \bar{F} = \bar{F}_D + \bar{F}_g$$

$$\bar{F}_g = mg\hat{i}$$

drag coeff.

$$\bar{F}_D = -C_D v^2 \hat{e}_v$$

$$\hat{e}_v = \frac{\bar{v}}{|\bar{v}|}$$

$$\Sigma \bar{F} = m\bar{a}$$

$$\underbrace{\bar{a}}_{\text{ind. var}} = \underbrace{\frac{\bar{F}}{m}}_{\text{forcing fctn}}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{1}{m}\left(mg - c_d v^2\right)$$

$$\frac{d\bar{v}}{dt} = g - \frac{c_d}{m} v^2$$

@ terminal velocity

$$\frac{d\bar{v}}{dt} = 0 = g - \frac{c_d}{m} v^2$$

$$v_{terminal} = \sqrt{\frac{mg}{c_d}} \quad @\ t = \infty$$

$$\frac{dv}{dt} = g - \frac{c_d}{m} v^2$$

$$\underbrace{\frac{dv}{1 - \frac{c_d}{mg} v^2}}_{f(v)} = \underbrace{g\,dt}_{g(t)}$$

$$\int \frac{dv}{1 - \frac{v^2}{v_{term}^2}} = \int g\,dt$$

note:

$$\int \frac{dz}{1 - z^2} = \operatorname{arctanh}(z)$$

$$z = \frac{v}{v_{term}}, \quad dz = \frac{dv}{v_{term}}$$

$$\operatorname{arctanh}(z) = gt + C$$

$$\frac{v}{v_{term}} = \tanh\left(\frac{gt}{v_{term}}\right) + C'$$

$$v(t) = v_{term} \tanh\left(\frac{gt}{v_{term}}\right) + C''$$

$$v(0) = 0 = C''$$

Analytical Result

$$\underbrace{v(t)}_{\text{dep var}} = \sqrt{\underbrace{\frac{mg}{c_d}}_{\text{system props}}}\tanh\left(\sqrt{\frac{g c_d}{m}}\cdot t\right)$$

g : forcing fctn

Numerical Approach

$$\frac{dv}{dt} = g - \frac{c_d}{m}v^2$$

$$\frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i}$$

$$\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c_d}{m}v^2(t_i)$$

$$\underbrace{v(t_{i+1})}_{\text{dep. var. @ next time}} = \underbrace{v(t_i)}_{\text{dep. var @ current time}} + \left(g - \frac{c_d}{m}v^2(t_i)\right)\underbrace{[t_{i+1} - t_i]}_{\text{ind. var.'s}}$$

dep. var @ $t = t + \Delta t$ + slope $\times \Delta t$

$\rightarrow$ Euler's method