

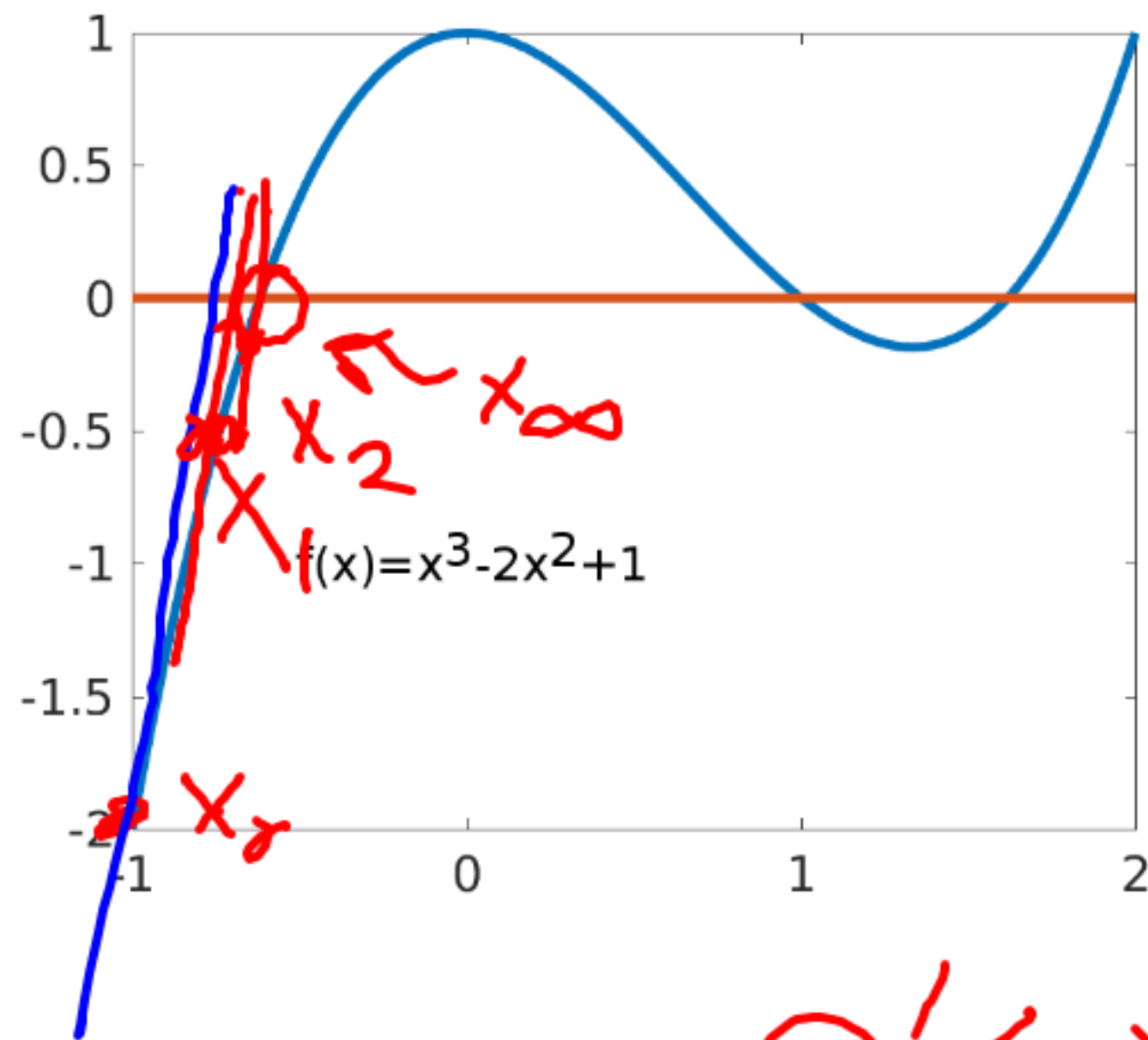
Root  
Finding

bisection

$$x_L = -1, x_U = 0$$

$$f(-1) \approx -2 \quad \textcircled{1} \quad x_r = -1/2$$

$$\textcircled{2} \quad x_L = -1, x_U = -1/2 \quad x_r = -3/4$$



$$x_0 = -1$$

NR method  
is open

$$f'(x) = 3x^2 - 4/x$$

$$x_0 = -1$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = -1 - \frac{-1 - 2 + 1}{-1 - 2} = -0.7143$$

$$x_2 = x_1 - \frac{f(x_1)^{3+4}}{f'(x_1)} = -0.6266$$

Mod. Secant

NR

$$x_r = x_1 - \frac{f(x_1)}{f'(x_1)}$$

MS

$$x_r = x_1 - \frac{f(x_1) \delta x}{f(x_1 + \delta x) - f(x_1)}$$

ok.  $f(x_1 + \delta x) - f(x_1)$



$$f'(x) = \lim_{\delta x \rightarrow 0} \frac{f(x + \delta x) - f(x)}{\delta x}$$

$$f'(x) \approx \frac{f(x + \delta x) - f(x)}{\delta x}$$

$$\textcircled{a} \quad x = -1 \quad ; \quad f'(-1) = 7$$

$$\delta x = 0.001$$

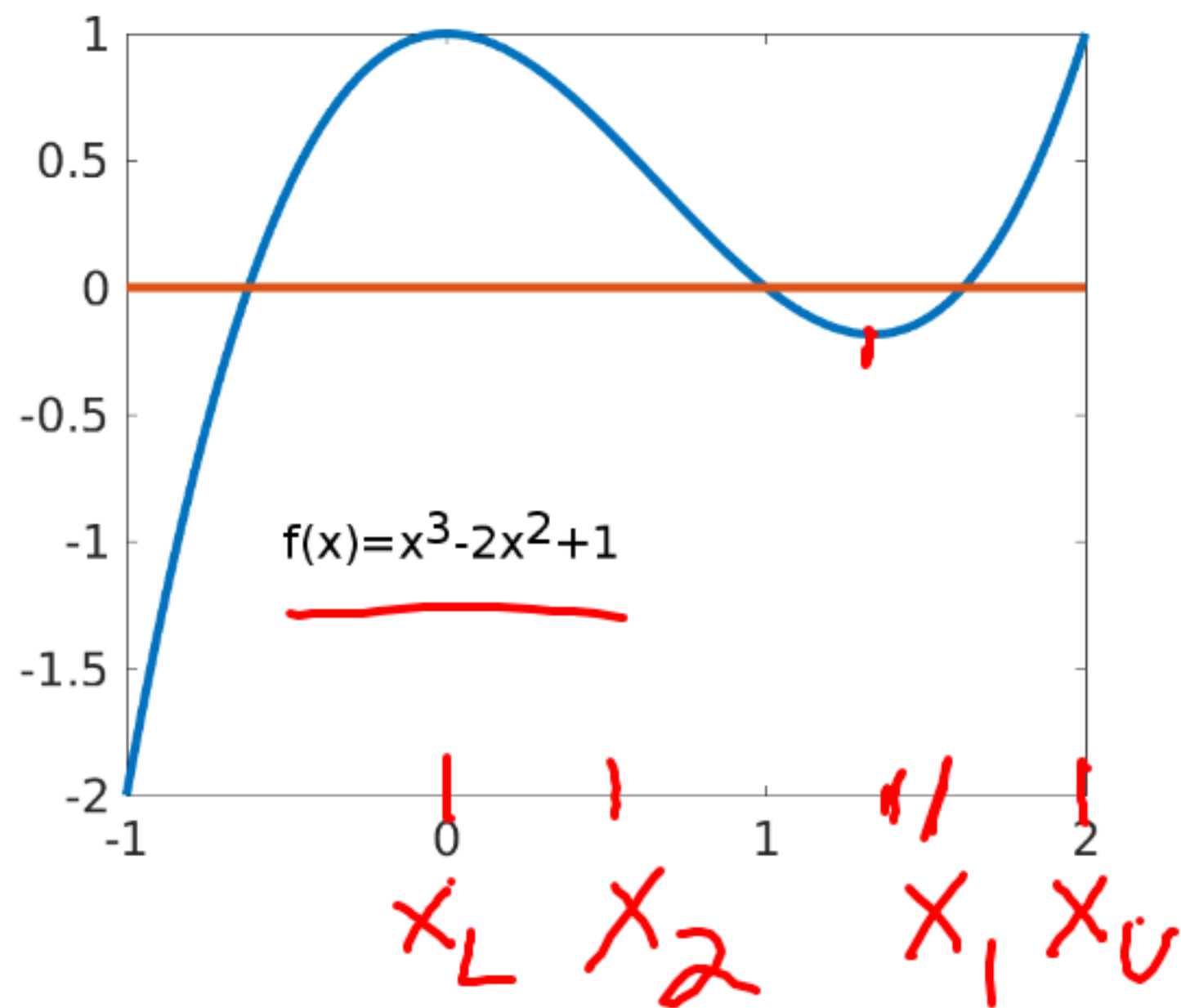
$$f'(x = -1) = \frac{f(-0.999) - f(-1)}{0.001}$$

$$f'(x) \approx 7$$

$$x \rightarrow -1$$


$$\Delta x = 0.1$$

$$\frac{f(-0.9) - f(-1)}{0.1} \approx 7.51$$



# Golden Ratio

$$\phi = 1.61803$$



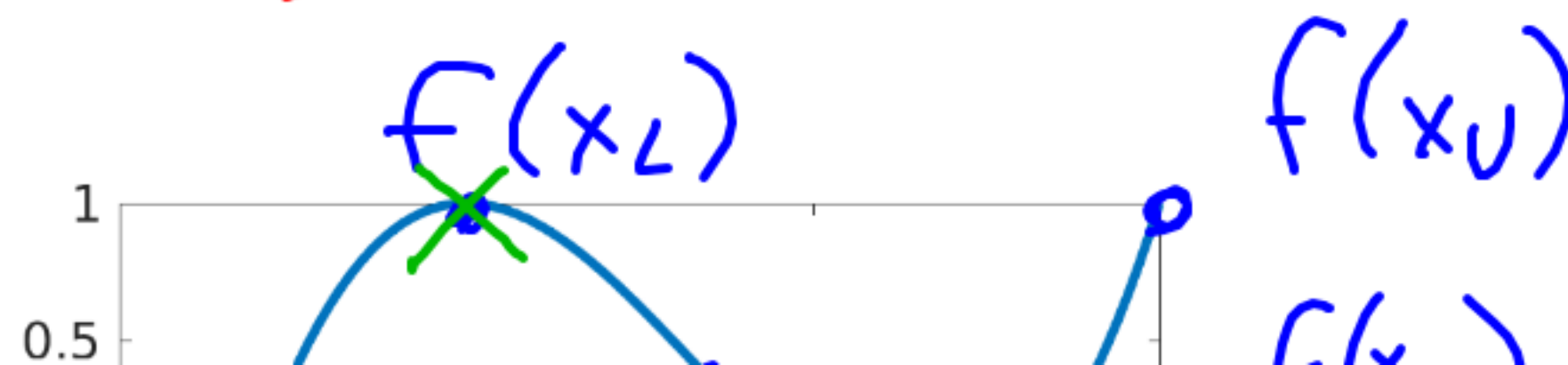
$$\frac{l_1}{l_2} = \frac{l_1 + l_2}{l_1}$$

$$\Delta x = 2 - 0$$

$$x_1 = x_L + (\phi - 1) \Delta x$$

$$x_2 = x_U - (\phi - 1) \Delta x$$

$$x_1 = 1.236$$



$$x_2 = 0.7639$$

$$f(x_1) = -0.1672$$

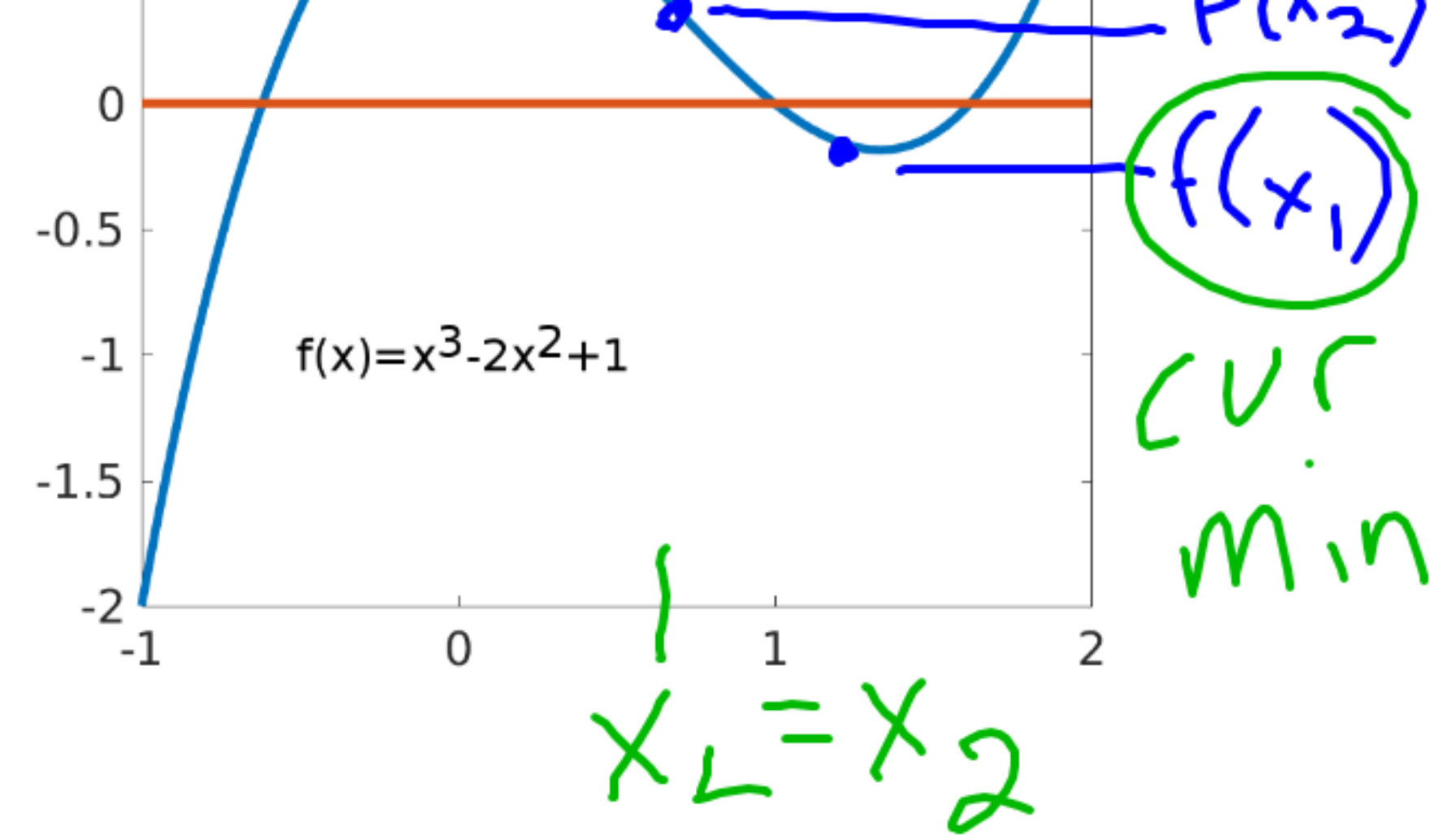
$$f(x_2) = 0.2787$$

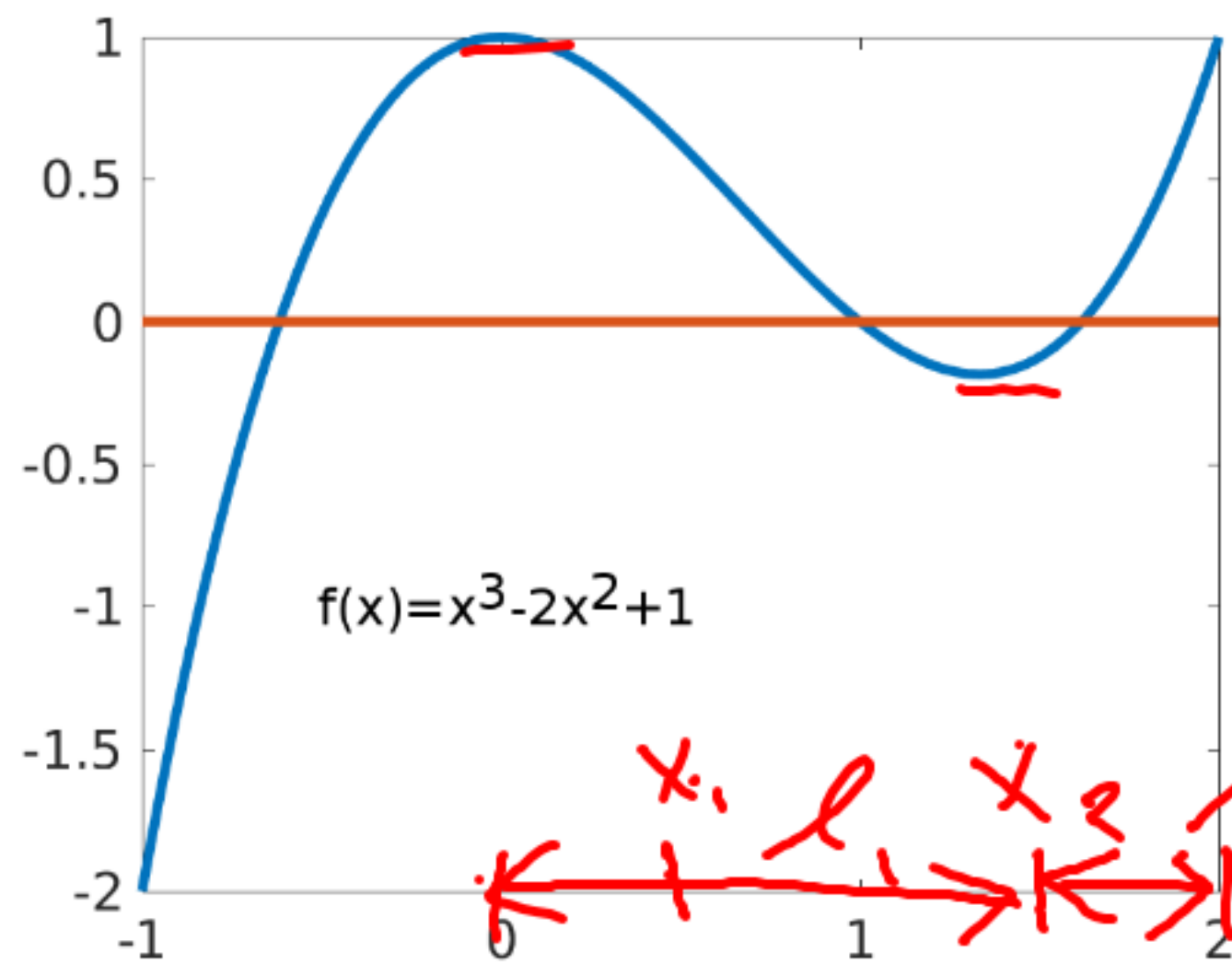
next iteration

$$x_L = x_2 = 0.7639$$

$$x_u = x_u = 2$$

$$x_{opt} = x_1 = 1.236$$





golden ratio

$$x_L = 0 \quad x_U = 2$$

$$\frac{l_1 + l_2}{l_1} = \frac{l_1}{l_2}$$

$$1 + \frac{1}{\phi} = \phi = \frac{l_1}{l_2}$$



$$\phi^2 - \phi - 1 = 0$$

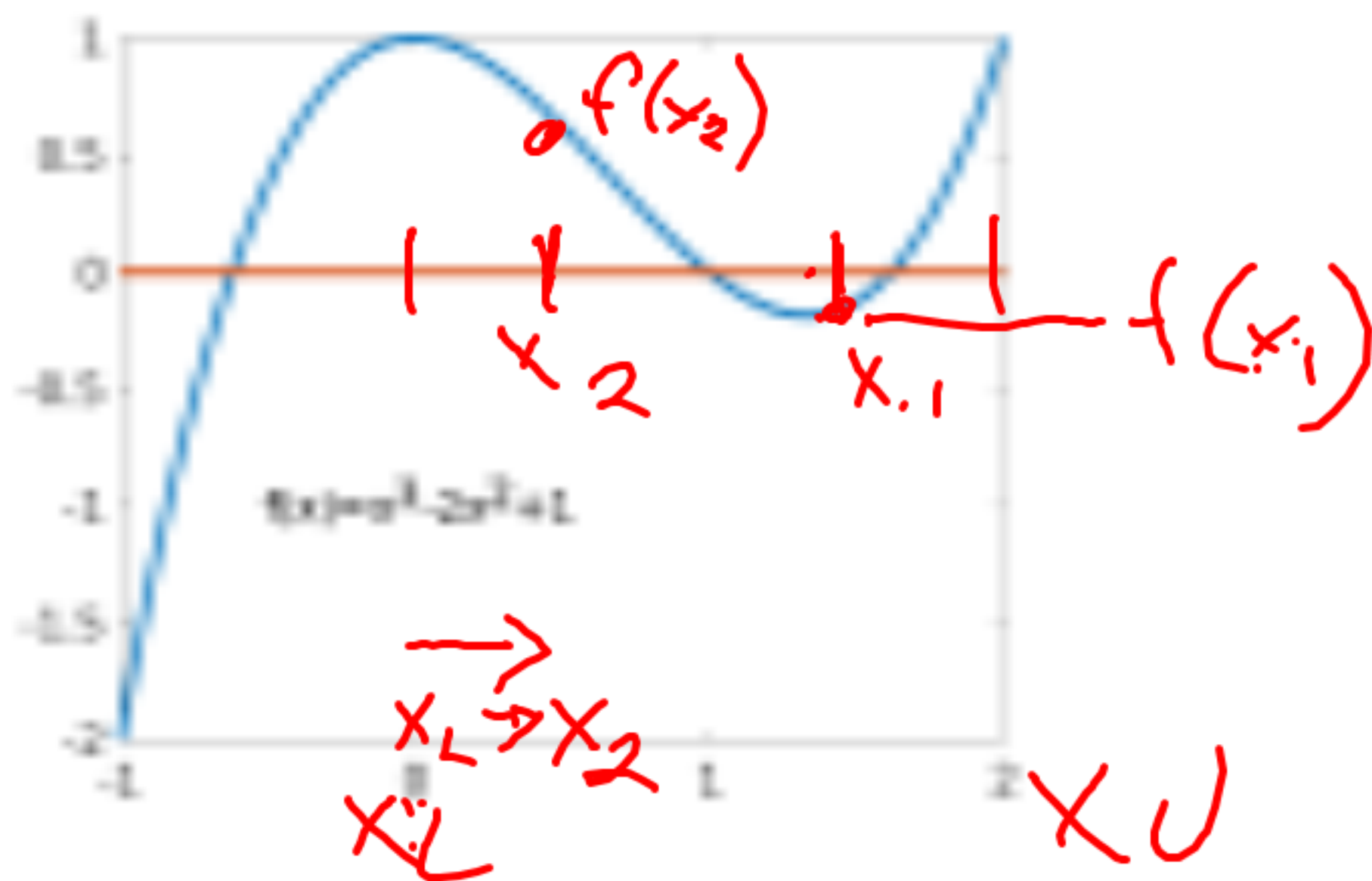
$$\phi = 1.61803$$

$$x_L = 0, x_U = 2$$

$$x_1 = x_L + d$$

$$x_2 = x_U - d$$

$$d = (\phi - 1)(x_U - x_L)$$



$$f(x_1) = f(1.236) = -0.1672 \text{ (?) min}$$

$$f(x_2) = f(0.7639) = 0.2787 \text{ (X) min}$$

$$x_{opt} = x_1 = -0.1672$$

$$x_L = 0.2787, x_U = 2$$

$$x_{opt} = -0.1672$$

$$Ax = y$$

$$4x_1 + 6x_2 + 2x_3 = 1$$

$$0x_1 + 2x_2 + 6x_3 = 2$$

$$1x_1 + 2x_2 + 1x_3 = 1$$

$$\begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 6 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ 1 \end{bmatrix}$$

$$\left[ \begin{array}{ccc|c} 4 & 6 & 2 & 1 \\ 0 & 2 & 6 & 2 \\ 1 & 2 & 1 & 1 \end{array} \right]$$

$$-\frac{\text{row 1}}{4} + \text{row 3}$$

$$\left[ \begin{array}{ccc|c} 4 & 6 & 2 & 1 \\ 0 & 2 & 6 & 2 \\ 0 & \frac{1}{2} & \frac{1}{2} & \frac{3}{4} \end{array} \right]$$

$$-\text{row 2}$$

2



$$\begin{array}{c} \hline 4. \\ \hline \end{array} + \text{row} \rightarrow$$

$$\left[ \begin{array}{ccc|c} 2 & 6 & 2 & 1 \\ 0 & 2 & 6 & 2 \\ 0 & 0 & -1 & 1/2 \end{array} \right]$$

$$x_1 = 1 - 6x_2 - 2x_3 = -2.25$$

$$x_2 = 2 - 6x_3 = 1.75$$

$$x_3 = -1/4$$