Solution to Form #1

In [1]:
```
[1,2,3]
```

```
ans =

   1   2   3
```

In [8]:
```
[1,2,3]*[1;2;3]=?
```

In [ ]:
```
```

The first source of error is roundoff error

Just storing a number in a computer requires rounding

In [11]:
```
fprintf('realmax = %1.20e\n',realmax)
fprintf('realmin = %1.20e\n',realmin)
fprintf('maximum relative error = %1.20e\n',eps)
```

```
realmax = 1.79769313486231570815e+308
realmin = 2.22507385850720138309e-308
maximum relative error = 2.22044604925031308085e-16
```

In [13]:
```
s=1;
for i=1:1000
    s=s+eps/10;
end
s==1
```

```
ans =  1
```

Freefall Model (revisited)

Octave solution (will run same on Matlab)

Set default values in Octave for linewidth and text size

In [14]:
```
%plot --format svg
```

In [15]:
```
set (0, "defaultaxesfontname", "Helvetica")
set (0, "defaultaxesfontsize", 18)
set (0, "defaulttextfontname", "Helvetica")
set (0, "defaulttextfontsize", 18)
set (0, "defaultlinelinewidth", 4)
```

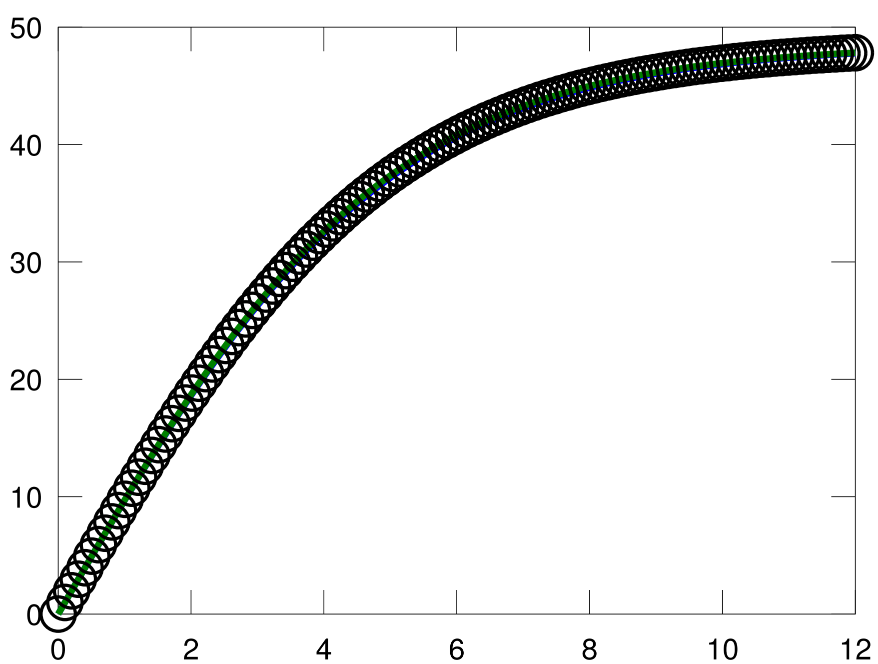
Define time from 0 to 12 seconds with N timesteps function defined as `freefall`

In [16]:
```
function [v_analytical,v_terminal,t]=freefall(N)
    t=linspace(0,12,N)';
    c=0.25; m=60; g=9.81; v_terminal=sqrt(m*g/c);

    v_analytical = v_terminal*tanh(g*t/v_terminal);
    v_numerical=zeros(length(t),1);
    delta_time =diff(t);
    for i=1:length(t)-1
        v_numerical(i+1)=v_numerical(i)+(g-c/m*v_numerical(i)^2)*delta_time
(i);
    end
    % Print values near 0,2,4,6,8,10,12 seconds
    indices = round(linspace(1,length(t),7));
    fprintf('time (s)|vel analytical (m/s)|vel numerical (m/s)\n')
    fprintf('-----------------------------------------------\n')
    M=[t(indices),v_analytical(indices),v_numerical(indices)];
    fprintf('%7.1f | %18.2f | %15.2f\n',M(:,1:3)');
    plot(t,v_analytical,'-',t,v_numerical,'o-')
end
```

In [19]:
```
[v_analytical,v_terminal,t]=freefall(120);
```

```
time (s)|vel analytical (m/s)|vel numerical (m/s)
-----------------------------------------------
    0.0 |               0.00 |            0.00
    2.0 |              18.76 |           18.82
    4.0 |              32.64 |           32.80
    6.1 |              40.79 |           40.97
    8.0 |              44.80 |           44.94
   10.0 |              46.84 |           46.93
   12.0 |              47.77 |           47.82
```



Types of error

Freefall is example of "truncation error"

Truncation error results from approximating exact mathematical procedure

We approximated the derivative as $\delta v/\delta t \approx \Delta v/\Delta t$

Can reduce error by decreasing step size -> Δt=`delta_time`

Another example of truncation error is a Taylor series (or Maclaurin if centered at a=0)

Taylor series: $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$

We can approximate the next value in a function by adding Taylor series terms:

Approximation	formula
0^{th}-order	$f(x_{i+1}) = f(x_i) + R_1$
1^{st}-order	$f(x_{i+1}) = f(x_i) + f'(x_i)h + R_2$
2^{nd}-order	$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + R_3$
n^{th}-order	$f(x_{i+1}) = f(x_i) + f'(x_i)h + \frac{f''(x_i)}{2!}h^2 + \ldots \frac{f^{(n)}}{n!}h^n + R_n$

Where $R_n = \frac{f^{(n+1)}(\xi)}{(n+1)!}h^{n+1}$ is the error associated with truncating the approximation at order n.

The n^{th}-order approximation estimates that the unknown function, $f(x)$, is equal to an n^{th}-order polynomial.

In the Freefall example, we estimated the function with a 1^{st}-order approximation, so

$v(t_{i+1}) = v(t_i) + v'(t_i)(t_{i+1} - t_i) + R_1$

$v'(t_i) = \frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} - \frac{R_1}{t_{i+1} - t_i}$

$\frac{R_1}{t_{i+1} - t_i} = \frac{v''(\xi)}{2!}(t_{i+1} - t_i)$

or the truncation error for a first-order Taylor series approximation is

$\frac{R_1}{t_{i+1} - t_i} = O(\Delta t)$

1. digital representation of a number is rarely exact
2. arithmetic (+,-,/,*) causes roundoff error

In [20]:
```
fprintf('%1.20f\n',double(pi))
fprintf('%1.20f\n',single(pi))
```

```
3.14159265358979311600
3.14159274101257324219
```

In [ ]: