Homework #3

Hand calculations

The following problems should be worked out by hand. You can check your work with Matlab/Octave. Upload a pdf of the completed calculations into a github repository called '03_review_1-10'. Then, submit your repo link to (https://goo.gl/forms/xZ5AqRmSdugurwl33)[https://goo.gl/forms/xZ5AqRmSdugurwl33]

1. Use the Newton-Raphson method to approximate when f(x)=0. Start with an initial guess of $x_0 = 0$.

   $f(x) = e^{-x} - x^3$

   a. Compute the first 3 iterations and calculate the approximate error for each.

   b. Compare the exact derivative to the derivative used in the modified secant for $\delta x = 0.1$ and $\delta x = 0.001$ at x_0.

2. A simple computer is being assembled with 5-bits of storage for each integer.

   a. How many different integers can be stored with 5 bits?

   b. If we want the maximum number of positive and negative integers, what is the largest and smallest integer we can store with 5 bits?

3. Convert the following binary numbers to base-10 in two ways, 1- the exact conversion, and 2- the conversion if only 4 digits are saved after addition/subtraction

   a. 1.001

   b. 100.1

   c. 1.001 + 100.1

   d. 1000 - 0.0001

4. In Problem 3c-d what kind of error is introduced by limiting the number of digits stored?

5. Solve the following problems with matrix A:

   $$A = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 6 \\ 1 & 2 & 1 \end{bmatrix}$$

   a. Compute the LU-decomposition

   b. Solve for x if $Ax = b$ and $b = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

6. Solve the following problems with matrix A:

   $$A = \begin{bmatrix} 4 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 2 \end{bmatrix}$$

   a. Compute the Cholesky factorization of A

   $C_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} C_{ki}^2}$

   $C_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} C_{ki} C_{kj}}{C_{ii}}$.

   b. Find the determinant of A, |A|.

   c. Find the inverse of A, A^{-1}

7.Determine the lower (L) and upper (U) triangular matrices with LU-decomposition for the following matrices, A. Then, solve for x when Ax=b:

a. $A = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix} b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} b = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} b = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

d. $A = \begin{bmatrix} 1 & 3 & 1 \\ -4 & -9 & 2 \\ 0 & 3 & 5 \end{bmatrix} b = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$

e. $A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & -3 & 2 \\ 0 & 3 & 5 \end{bmatrix} b = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$

f. $A = \begin{bmatrix} 1 & 3 & -5 \\ 1 & 4 & -8 \\ -3 & -7 & 9 \end{bmatrix} b = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$

g. $A = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 5 & 2 \\ 1 & -1 & 2 \end{bmatrix} b = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$

9. Calculate the determinant of A from 1a-g.

10. Determine the Cholesky factorization, C, of the following matrices, where

$$C_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} C_{ki}^2}$$

$$C_{ij} = \frac{a_{ij} - \sum_{k=1}^{i-1} C_{ki}C_{kj}}{C_{ii}}.$$

a. A= $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$

b. A= $\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}$

c. A= $\begin{bmatrix} 10 & -10 & 20 \\ -10 & 20 & 10 \\ 20 & 10 & 30 \end{bmatrix}$

d. A= $\begin{bmatrix} 21 & -1 & 0 & 0 \\ -1 & 21 & -1 & 0 \\ 0 & -1 & 21 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

11. Verify that $C^T C = A$ for 3a-d