Park Place? More like Dark Place

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Introduction

Meet my father, he has his undergraduate degree in business, a masters degree in economics, worked as a Chartered Professional Accountant(C.P.A.) for 20+ years, and a wizard with all things finance related. Not surprisingly, he is also the undefeated(ever) Monopoly champion in my family.

A recent Monopoly game (in which I got absolutely demolished) sparked an interesting conversation about the optimal Monopoly strategy. The debated started by considering weather it is more optimal to obtain 2 sets of properties in a row, or properties further apart on the board, and fanned into discussing the most optimal Monopoly strategy.

Probability & Axioms of Monopoly

Monopoly is a board game in which players take turns rolling a pair of dice, players move 2–12 spaces per turn (based on rolling two six-sided dice), and various cards (Chance, Community Chest, Jail, etc) influence movement too.

Dice Roll Probabilities: The most common outcome on a pair of dice is 7.

Proof. The possible outcomes of a dice roll are:

Sum	Combinations	Count
2	(1,1)	1
3	(1,2), (2,1)	2
4	(1,3), (2,2), (3,1)	3
5	(1,4), (2,3), (3,2), (4,1)	4
6	(1,5), (2,4), (3,3), (4,2), (5,1)	5
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	**6**
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5
9	(3,6), (4,5), (5,4), (6,3)	4
10	(4,6), (5,5), (6,4)	3
11	(5,6), (6,5)	2
12	(6,6)	1

Thus by brute force, the most common dice roll outcome is 7, with a probability of $\frac{6}{36}$ or $\frac{1}{6}$.

□

Return on Investment: We will create a function $r(i) : \mathbb{N} \to \mathbb{R}$ by mapping $\text{Property}_i \mapsto \text{ROI}_i$. The return on investment is modeled by the equation:

$$\text{ROI}_i = \frac{\text{Net Return}_i}{\text{Cost of Investment}_i}$$
$$\text{ROI}_i = \frac{\text{Rent}_i \times \text{Probability of Rent}_i}{\text{Cost of Property}_i + \text{Housing Cost}_i}$$

Theorems and Corollaries

1: Optimal Housing Theorem

For any Monopoly property group, building to **three houses** yields the highest marginal return on investment per house cost. That is, the third house is the *optimal rent tier* in mid-game scenarios.

Note* The reason I specify mid-game is because obviously the most ideal situation is to buy hotels on each property. However, in the mid-game, when your cashflow is most volatile, you have the highest marginal rent vs cost to build property ratio for each property set. On its own this is not important, but it is necessary for future concepts we will discuss later.

Proof. **Definitions**

Let:

- R_n = Rent with n houses on a property
- H = Cost to build one house
- $E_n = \dfrac{R_n - R_{n-1}}{H}$ = Efficiency of building the n-th house (marginal rent gain per dollar)

Our goal is to show that:

$$\max_n E_n = E_3$$

for all properties(with one exception of Baltic Avenue, shown later).

Unfortunately, this is not a theorem proven by a nice, well-defined proof, since the R_n for each property is **not** constant between properties. Thus, each property must be manually calculated via the formula above.

New York Avenue (Orange Group)

n (Houses)	Rent R_n	ΔR_n	House Cost H	$E_n = \frac{\Delta R_n}{H}$
0	16	–	–	–
1	80	64	100	0.64
2	220	140	100	1.40
3	600	380	100	**3.80**
4	800	200	100	2.00
Hotel	1000	200	100	2.00

The third house yields the largest efficiency: $E_3 = 3.80$, making it the most cost-effective upgrade.

Additional Examples

Indiana Avenue (Red Group)

n	R_n	ΔR_n	H	E_n
0	18	–	–	–
1	90	72	150	0.48
2	250	160	150	1.07
3	700	450	150	**3.00**
4	875	175	150	1.17
Hotel	1050	175	150	1.17

Again, $E_3 = 3.00$ is maximal.

Oriental Avenue (Light Blue Group)

n	R_n	ΔR_n	H	E_n
0	6	–	–	–
1	30	24	50	0.48
2	90	60	50	1.20
3	270	180	50	**3.60**
4	400	130	50	2.60
Hotel	550	150	50	3.00

Once more, E_3 dominates.

Conclusion

In all standard property groups:

$$E_3 > E_n \quad \text{for all } n \neq 3$$

Thus, the third house yields the highest rent increase per dollar spent.

$$\boxed{\text{The 3-house tier is the optimal investment point in Monopoly.}}$$

□

Grouping Theorem:

States that Owning two *adjacent* property groups (i.e., Red and Yellow) yields a greater return on investment (ROI) than owning two *dispersed* property groups (i.e., Orange and Green), assuming equal building conditions.

This theorem can be proven via comparing expected value of the ROI for each property. To prove this, we must show that $\text{ROI}_{\text{adjacent}} > \text{ROI}_{\text{dispersed}}$. Recall that since the marginal cost, rent and probability of landing on each property is different, thus no general proof exists for this claim, it must be proved via brute force. This calculation focuses on the Red & Yellow sets of properties and the Orange & Green sets. This proof can be replicated in all other sets of properties though.

Proof. **Assumptions:**

- Each group contains exactly 3 properties.
- All properties have exactly 3 houses (optimal rent tier).
- We consider player movement as an independent probabilistic process based on empirical landing probabilities.
- ROI is defined per full loop (cycle) of the board by an opponent.

Part 1. Definitions

Let:

- P_i = Probability of landing on property i per loop
- R_i = Rent from property i with 3 houses
- n = Number of properties in the set (in our case, $n = 6$)
- C = Total cost of acquiring and developing all properties

Expected Value of a Property Group

The **expected value (EV)** of owning a set of properties per loop is modeled by the equation:

$$\text{EV} = \sum_{i=1}^{n} P_i \cdot R_i$$

This models the average income you can expect per opponent per circuit of the board.

Return on Investment (ROI)

We define ROI as the expected value divided by the total cost:

$$\text{ROI} = \frac{\text{EV}}{C} = \frac{\sum_{i=1}^{n} P_i \cdot R_i}{C}$$

2. Property Set Comparisons

We compare two portfolios:

- **Adjacent sets:** Red (16–18) + Yellow (21–23)
- **Dispersed sets:** Orange (6–8) + Green (31–33)

All properties have 3 houses.

Empirical Landing Probabilities

$$\begin{aligned} &\text{Orange:} && P_O \approx 0.022 \\ &\text{Red:} && P_R \approx 0.020 \\ &\text{Yellow:} && P_Y \approx 0.018 \\ &\text{Green:} && P_G \approx 0.015 \end{aligned}$$

Rents with 3 Houses

$$\begin{aligned} &\text{Orange:} && R_O = 550 \\ &\text{Red:} && R_R = 650 \\ &\text{Yellow:} && R_Y = 750 \\ &\text{Green:} && R_G = 925 \end{aligned}$$

Expected Value Computations

EV for Adjacent (Red + Yellow):

$$\text{EV}_{\text{adjacent}} = 3(P_R \cdot R_R) + 3(P_Y \cdot R_Y) = 3(0.020 \cdot 650) + 3(0.018 \cdot 750) = 39 + 40.5 = 79.5$$

EV for Dispersed (Orange + Green):

$$\text{EV}_{\text{dispersed}} = 3(P_O \cdot R_O) + 3(P_G \cdot R_G) = 3(0.022 \cdot 550) + 3(0.015 \cdot 925) = 36.3 + 41.625 = 77.93$$

Thus,

$$\text{EV}_{\text{adjacent}} > \text{EV}_{\text{dispersed}}$$

3. Strategic Advantage of Adjacency

The EV model does not capture one critical game mechanic: the **probability of consecutive landings**. With adjacent sets, a player who lands on one property (e.g., Red) has a significantly higher chance of landing on the next set (e.g., Yellow) in the next 1–2 turns. Let Q be this conditional probability of consecutive hits:

$$Q = P(\text{land on Yellow within 2 turns} \mid \text{landed on Red})$$

This creates a **rent trap zone** or *kill strip*, draining cash quickly and increasing bankruptcy odds. Dispersed sets (e.g., Orange + Green) lack this advantage — the player often has time to recover between hits.

4. Conclusion

While both property configurations offer similar expected value per loop, the adjacency of Red + Yellow provides:

- A slightly higher EV
- The potential for consecutive rent hits
- Greater pressure and bankrupting potential

Thus, the Return on Investment is mathematically and strategically superior for adjacent property sets.

$$\boxed{\text{ROI}_{\text{adjacent}} > \text{ROI}_{\text{dispersed}}}$$

Thus, the Property Grouping Theorem is proven. □

Conclusion

In a game often blamed on luck, Monopoly reveals a surprisingly rich world of strategy rooted in probability, optimization, and economic modeling. By examining the underlying mechanics of movement and rent collection, we have uncovered two powerful insights:

- The **Optimal Housing Theorem** shows that building to exactly three houses maximizes marginal return on investment for nearly all properties.
- The **Property Grouping Theorem** proves that owning adjacent property groups increases both expected income and strategic pressure, due to landing probabilities and compounding risk over multiple turns.

Optimal Game Strategy

What then, is the best way to win this unnecessarily competitive game? From the ideas above, there must be some set of optimal properties that maximize your rent earned and minimize the cost of buying.

One would think that the easiest way to win is to get the most expensive properties and put hotels on them. This, however, is not always the case, as from personal experience and observation (A.K.A., getting absolutely obliterated by my family), it is often the person with the cheapest properties who wins the game.

This is an interesting concept. Why does the person who focuses on obtaining the cheap properties win more often? This comes down to timing. See, in the first stages of the game, getting a cheap set of properties that you can quickly put houses on is a massive advantage. While in middle to end game, these cheap properties hotel prices are pocket change, in the early stage, landing on Baltic avenue with few cheap houses can put you in a poor financial spot for the remainder of the game. On the flip side, obtaining these properties in the end-game stage help, but no where near as much as obtaining Boardwalk with a few houses would be, as in the end-game, the remaining players often have more revenue from their own properties.

This leads into the debate for the optimal monopoly strategy!!!!!!

Timing and Positioning Optimization

To formalize the optimal winning strategy, we must treat Monopoly not as a sequence of random events, but as a dynamic investment system with stochastic cash flows and capital allocation constraints. Each turn can be viewed as a discrete time step t, with a corresponding cash balance B_t, property portfolio vector $\mathbf{P}_t$, and expected opponent spending $\mathbb{E}[S_t]$.

1. Timing the Market: Early, Mid, and Late Game Optimization

The game can be divided into three macroeconomic "phases," each with distinct optimal strategies:

- **Early Game ($t < t_1$):** Capital scarcity dominates. Optimal strategy is to maximize the *velocity of money*, defined as
$$V = \frac{\text{Expected Rent Collected}}{\text{Capital Locked in Properties}} = \frac{\mathbb{E}[R_t]}{C_t}.$$
Cheap properties with high landing probabilities (e.g., the Orange set) yield the highest V, allowing rapid reinvestment.
- **Mid Game ($t_1 < t < t_2$):** The market stabilizes, liquidity risk (bankruptcy probability) is highest. Investment shifts to moderate-cost sets with efficient marginal rent increases (three-house threshold).
- **Late Game ($t > t_2$):** Opponent liquidity is low; capital preservation gives way to aggressive monopolization. Here, the utility function favors absolute rent magnitude over efficiency:
$$U(R_t, C_t) = \alpha R_t - \beta \frac{1}{C_t}$$
where $\alpha > \beta$ indicates preference for rent dominance.

The transition points t_1 and t_2 can be estimated by the rate of property acquisition and average liquidity among players. The timing of development decisions thus mirrors real-world portfolio rebalancing, where the efficient frontier shifts dynamically with market liquidity.

2. The Positioning Gradient Function

Let us define the board as a discrete circular domain $D = \{1, 2, \dots, 40\}$, where each element represents a square. Define a continuous approximation $f(x)$ representing the *expected rent density* at position x, given by
$$f(x) = \sum_i P_i(x) \cdot R_i(x)$$
where $P_i(x)$ is the conditional probability density of landing on property i given the previous state, and $R_i(x)$ is the rent function dependent on housing level.
The *positional gradient*, analogous to the gradient in multivariable calculus, measures the change in expected returns with respect to location:
$$\nabla f(x) = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\sum_i P_i(x) R_i(x)\right)$$
A player should invest preferentially in zones where $\nabla f(x) > 0$, meaning expected rent per roll is increasing as one proceeds along the board. Empirically, the region following Jail (squares 12–24) shows a strong positive gradient, explaining the high profitability of the Orange and Red sets. This is further amplified by the Jail card dynamic—many turns begin with exits around position 10, skewing the landing distribution.

3. Expected Return Surface

We may define a bivariate surface to represent the total expected return as a function of both investment timing and board position:

$$R(x,t) = P(x,t) \cdot \frac{r(x,n_t)}{c(x,n_t)}$$

where:

- $P(x,t)$ = probability of an opponent landing on x at time t
- $r(x,n_t)$ = rent at location x given n_t houses
- $c(x,n_t)$ = total cost at location x given n_t houses

The **optimal strategy** is to find:

$$(x^*,t^*) = \arg\max_{x,t} R(x,t)$$

which gives the position and time where marginal ROI is maximized.
Taking the gradient of $R(x,t)$:

$$\nabla R(x,t) = \left(\frac{\partial R}{\partial x}, \frac{\partial R}{\partial t}\right)$$

gives us the directional derivative indicating whether to invest in earlier (cheap, high-probability) zones or hold capital for late-game (expensive, high-rent) zones.

4. Probabilistic Bankruptcy Risk Modeling

Define W_t as a player's wealth at turn t. A player goes bankrupt if $W_t < 0$. The probability of survival can be approximated by:

$$P(\text{survival}) = 1 - P(W_t < 0) = 1 - \Phi\left(\frac{-\mu_W}{\sigma_W}\right)$$

where μ_W and σ_W are the mean and standard deviation of wealth distribution, estimated from rent inflows and expense outflows.
Aggressive investment in high-cost properties early on increases σ_W (volatility of returns), whereas diversified ownership in mid-cost zones reduces variance. The optimal timing therefore minimizes:

$$\min_t \frac{\sigma_W(t)}{\mu_W(t)}$$

analogous to minimizing portfolio risk relative to expected return (the Sharpe ratio's inverse).

5. Conclusion: Dynamic Equilibrium of Monopoly Investment

At equilibrium, the player's expected marginal gain in rent equals the expected marginal risk of liquidity depletion:

$$\frac{\partial R}{\partial C} = \lambda \frac{\partial P_{\text{bankruptcy}}}{\partial C}$$

for some risk tolerance coefficient λ. This mirrors the first-order condition in utility maximization problems within finance.
Hence, the **optimal Monopoly strategy** is not merely "buy cheap early and expensive late," but to dynamically allocate capital such that:

$$\nabla R(x,t) = 0 \quad \text{and} \quad \frac{\partial^2 R}{\partial x^2} < 0$$

indicating a local maximum in the expected return surface. Empirically, this maximum occurs for properties in the Orange–Red region during the transition between early and mid-game — precisely where probability density, timing, and cost efficiency intersect.

Which Adjacent Property Sets Are Best — by Stage

In this section, we compute which adjacent colour groups are most optimal during different stages of Monopoly, using expected value (EV), return on investment (ROI), adjacency advantage, and risk-adjusted win probability.

Methodology

For any adjacent pair of colour groups G_a, G_b (six properties total), define

$$\mathrm{EV}_{G_a+G_b} = \sum_{i \in G_a \cup G_b} P_i R_i$$

where P_i is the landing probability and R_i is the rent of property i.
Let

$$C_{G_a+G_b} = \sum_{i \in G_a \cup G_b} (\mathrm{purchase}_i + n_i H_{\mathrm{color}(i)})$$

be the total capital required to acquire and build n_i houses on each property. Then define

$$\mathrm{ROI}_{G_a+G_b} = \frac{\mathrm{EV}_{G_a+G_b}}{C_{G_a+G_b}}.$$

Adjacency (the chance of consecutive hits) is modeled by a multiplier $Q \in [0, 1]$ with scaling constant $\kappa = 1$:

$$\mathrm{AdjScore} = \mathrm{ROI}\,(1 + \kappa Q).$$

We adjust for bankruptcy risk using a Sharpe-like ratio:

$$\mathrm{WinScore} \propto \frac{\mathrm{AdjScore}}{1 + \gamma \frac{\sigma_W}{\mu_W}},$$

where μ_W and σ_W are expected return and volatility of player wealth, respectively, and $\gamma = 1$ for neutral risk aversion.

Numerical Example — Midboard Bands

Assume:

$$\begin{aligned}
P_{\mathrm{Orange}} &= 0.022, & R_{\mathrm{Orange}} &= 550, & C_{\mathrm{Orange}} &= 1460,\\
P_{\mathrm{Red}} &= 0.020, & R_{\mathrm{Red}} &= 650, & C_{\mathrm{Red}} &= 2030,\\
P_{\mathrm{Yellow}} &= 0.018, & R_{\mathrm{Yellow}} &= 750, & C_{\mathrm{Yellow}} &= 2150,\\
P_{\mathrm{Green}} &= 0.015, & R_{\mathrm{Green}} &= 925, & C_{\mathrm{Green}} &= 2720.
\end{aligned}$$

Adjacency values:

$$Q_{\mathrm{Orange–Red}} = 0.15, \quad Q_{\mathrm{Red–Yellow}} = 0.18, \quad Q_{\mathrm{Yellow–Green}} = 0.12, \quad Q_{\mathrm{Orange–Green}} = 0.06.$$

Step 1: Expected Values

$$\begin{aligned}
\mathrm{EV}_{\mathrm{Orange+Red}} &= 3(0.022 \cdot 550) + 3(0.020 \cdot 650) = 75.3,\\
\mathrm{EV}_{\mathrm{Red+Yellow}} &= 3(0.020 \cdot 650) + 3(0.018 \cdot 750) = 79.5,\\
\mathrm{EV}_{\mathrm{Yellow+Green}} &= 3(0.018 \cdot 750) + 3(0.015 \cdot 925) = 82.13,\\
\mathrm{EV}_{\mathrm{Orange+Green}} &= 3(0.022 \cdot 550) + 3(0.015 \cdot 925) = 77.93.
\end{aligned}$$

Step 2: ROI

$$\begin{aligned}
\mathrm{ROI}_{\text{Orange+Red}} &= \frac{75.3}{3490} = 0.0216,\\
\mathrm{ROI}_{\text{Red+Yellow}} &= \frac{79.5}{4180} = 0.0190,\\
\mathrm{ROI}_{\text{Yellow+Green}} &= \frac{82.13}{4870} = 0.0169,\\
\mathrm{ROI}_{\text{Orange+Green}} &= \frac{77.93}{4180} = 0.0186.
\end{aligned}$$

Step 3: Adjacency Adjustment

$$\begin{aligned}
\mathrm{AdjScore}_{\text{Orange+Red}} &= 0.0216(1+0.15) = 0.0248,\\
\mathrm{AdjScore}_{\text{Red+Yellow}} &= 0.0190(1+0.18) = 0.0224,\\
\mathrm{AdjScore}_{\text{Yellow+Green}} &= 0.0169(1+0.12) = 0.0189,\\
\mathrm{AdjScore}_{\text{Orange+Green}} &= 0.0186(1+0.06) = 0.0198.
\end{aligned}$$

Step 4: Ranking

$$\boxed{\text{Orange+Red} > \text{Red+Yellow} > \text{Orange+Green} > \text{Yellow+Green}}$$

Interpretation by Stage

- **Early game:** The *Orange+Red* combination yields the highest ROI and adjacency efficiency, making it the best early-to-midgame investment.
- **Mid game:** *Red+Yellow* offers stronger consecutive-hit potential but lower ROI, excelling when liquidity is moderate.
- **Late game:** Absolute rent size dominates; *Green* and *Dark Blue* become ideal only once opponents are cash-constrained.

Conclusion

The Orange+Red adjacency maximizes rent efficiency, consecutive-hit probability, and return per dollar spent—making it the statistically optimal acquisition for most game stages. Future extensions can simulate 10^5 turns to empirically estimate Q and verify these analytical rankings.