

Ben: Centroids

Ben: SVD bases

~~Tangent distance~~

~~Tensor SVD~~

Omar: eigenfaces (eigensigns)

Smoothing

Neural Networks?

Project Submission Guidelines:

- Due by May 9, 2023.
- The format of the project report will have the following
 - Title, Author etc.
 - Brief Abstract.
 - Brief Introduction.
 - What you have accomplished.
 - Observation.
 - Conclusion.
 - References.
 - Codes
 - Appendix if needed.
- Submit/upload your project report here.

You are encouraged to use a Jupyter notebook markdown for the report.

American Sign Language Image Translator

By Benjamin Singleton and Omar Abdelmotaleb

Abstract

In this project, we seek to utilize multiple image classification methods for classifying 28 x 28 resolution images of hand signs to label them with a respective letter. These hand signs are meant to represent a letter in the English language through translation of American Sign Language. The first method we explore is eigensigns, which is our take on the eigenface method using eigen-decomposition on face images but instead on hand sign images. Then, we look at using image

centroids, implementing multiple distance calculation methods to find which hand sign is the image closest to, with the hand signs in comparison are the mean images of each label. Next, we use SVD bases, taking the Singular Value Decomposition of the hand signs as well as calculating the residuals to classify which one the hand sign represents the closest. Finally, smoothing is implemented as our last method in an attempt to optimize the performance of our previous models. Overall, we find that some methods are less effective than others in their accuracy of classifying a hand sign to be the correct English letter. Eigensigns and SVD bases yields an accuracy above 80%, while image centroids and our attempted optimization with smoothing yielded no more than 50%.

Introduction

American Sign Language is a popular means of communication for those who may be deaf or hard of hearing. Utilizing hand signs effectively demonstrates letters in the English language which can be used to construct words and sentences. Translating English words to hand signs is a straightforward task which has been accomplished in numerous ways. However, it's not frequently done the other way around. Translating American Sign Language to English is a more challenging task because it requires methods involving image classification. More importantly, attaining a high accuracy is not as simple and straightforward due to the flaws in existing image classification as well as potential clarity issues with the images themselves. Translating hand sign images to English serves a purpose in allowing those who are not familiar with ASL to understand or learn from hand signs. This can also help in recording American Sign Language in English for those who want to transcribe it.

What you have accomplished

Our dataset [Sign Language MNIST](#) was retrieved from Kaggle and read using a pandas dataframe. The training dataset is comprised of over 27000 rows and the testing dataset of over 7000 rows. The first header both is the label, which is a numerical value corresponding to a letter in the English alphabet. The remaining 784 columns represent an individual pixel (i.e. pixel1, pixel2, ..., pixel784) of which is a 1-dimensional representation of the 28 x 28 image, able to be displayed once reshaped. Each row will represent an individual image.

Neural Network

Centroids

SVD Bases

Eigensigns

Smoothing

```
In [ ]: import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

#from google.colab import files
#from google.colab import drive
from tqdm import tqdm
```

```
import io
from sys import maxsize
from sklearn.utils.extmath import randomized_svd
from scipy.ndimage import gaussian_filter
from scipy.ndimage import gaussian_filter1d
import multiprocessing as mp

from sklearn.svm import SVC
from sklearn.model_selection import train_test_split
from sklearn.decomposition import PCA
from sklearn.metrics import confusion_matrix, classification_report
from sklearn.neural_network import MLPClassifier
from sklearn.preprocessing import MinMaxScaler
import warnings
warnings.filterwarnings("ignore")

# import tensorflow as tf
```

```
In [ ]: # Assessment metrics
from sklearn.metrics import classification_report
from sklearn.metrics import accuracy_score
from sklearn.metrics import precision_score
```

```
In [ ]: import math
```

```
In [ ]: from PIL import Image
```

```
In [ ]: sign_map = {
    0: "A",
    1: "B",
    2: "C",
    3: "D",
    4: "E",
    5: "F",
    6: "G",
    7: "H",
    8: "I",
    9: "J",
    10: "K",
    11: "L",
    12: "M",
    13: "N",
    14: "O",
    15: "P",
    16: "Q",
    17: "R",
    18: "S",
    19: "T",
    20: "U",
    21: "V",
    22: "W",
    23: "X",
    24: "Y",
}
```

```
    25: "Z"  
}
```

```
In [ ]: reduced_sign_map = {  
    0: "A",  
    1: "B",  
    2: "C",  
    3: "D",  
    4: "E",  
    5: "F",  
    6: "G",  
    7: "H",  
    8: "I",  
    9: "K",  
   10: "L",  
   11: "M",  
   12: "N",  
   13: "O",  
   14: "P",  
   15: "Q",  
   16: "R",  
   17: "S",  
   18: "T",  
   19: "U",  
   20: "V",  
   21: "W",  
   22: "X",  
   23: "Y"  
}
```

```
In [ ]: #df_train = pd.read_csv('/content/drive/MyDrive/sign data/sign_mnist_train/sign_mnist_t
```

```
In [ ]: #df_test = pd.read_csv('/content/drive/MyDrive/sign data/sign_mnist_test/sign_mnist_tes
```

```
In [ ]: df_train = pd.read_csv('sign_mnist_train.csv')
```

```
In [ ]: df_test = pd.read_csv('sign_mnist_test.csv')
```

```
In [ ]: df_train.head(10)
```

```
Out[ ]:
```

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pixel777
0	3	107	118	127	134	139	143	146	150	153	...	207	207	207
1	6	155	157	156	156	156	157	156	158	158	...	69	149	149
2	2	187	188	188	187	187	186	187	188	187	...	202	201	201
3	2	211	211	212	212	211	210	211	210	210	...	235	234	234
4	13	164	167	170	172	176	179	180	184	185	...	92	105	105
5	16	161	168	172	173	178	184	189	193	196	...	76	74	74

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pixel777
6	8	134	134	135	135	136	137	137	138	138	...	109	102	
7	22	114	42	74	99	104	109	117	127	142	...	214	218	
8	3	169	174	176	180	183	185	187	188	190	...	119	118	
9	3	189	189	189	190	190	191	190	190	190	...	13	53	

10 rows × 785 columns

```
In [ ]: df_test.head(10)
```

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pixel777
0	6	149	149	150	150	150	151	151	150	151	...	138	148	
1	5	126	128	131	132	133	134	135	135	136	...	47	104	
2	10	85	88	92	96	105	123	135	143	147	...	68	166	
3	0	203	205	207	206	207	209	210	209	210	...	154	248	
4	3	188	191	193	195	199	201	202	203	203	...	26	40	
5	21	72	79	87	101	115	124	131	135	139	...	187	189	
6	10	93	100	112	118	123	127	131	133	136	...	173	175	
7	14	177	177	177	177	177	178	179	179	178	...	232	223	
8	3	191	194	196	198	201	203	204	205	205	...	43	57	
9	7	171	172	172	173	173	173	173	173	172	...	199	199	

10 rows × 785 columns

```
In [ ]: def getImageFromTrain(index):
        row = df_train.loc[index].tolist()
        sign = row[0]
        row = row[1:]
        width, height = 28, 28
        img = Image.new("L", (width, height))
        img.putdata(row)
        newsize = (300, 300)
        img = img.resize(newsize)
        img.show()
        print("Sign: ", sign_map[sign])
```

```
In [ ]: def getImageFromTest(index):
        row = df_test.loc[index].tolist()
        sign = row[0]
        row = row[1:]
        width, height = 28, 28
        img = Image.new("L", (width, height))
```

```
img.putdata(row)
newsize = (300, 300)
img = img.resize(newsize)
img.show()
print("Sign: ", sign_map[sign])
```

```
In [ ]: def getImage(row):
        print(row)
        sign = row['label']
        print(sign)
        row = row[1:]
        width, height = 28, 28
        img = Image.new("L", (width, height))
        img.putdata(row)
        newsize = (300, 300)
        img = img.resize(newsize)
        img.show()
        print("Sign: ", sign_map[sign])
```

```
In [ ]: getImage(df_train.loc[2])
```

```
label      2
pixel1     187
pixel2     188
pixel3     188
pixel4     187
...
pixel780   199
pixel781   198
pixel782   195
pixel783   194
pixel784   195
Name: 2, Length: 785, dtype: int64
2
Sign:  C
```

Method 0: Eigensigns

```
In [ ]: sample_size = 300
        train = df_train.values[:,1:]
        train = train[:sample_size,]
        test = df_test.values[:,1:]
        test = test[:200,]
        # Label at index 0
        # 784 pixels = 28 x 28
        # print(first.reshape(28,28))
        first = train[6]
        # plt.imshow(first.reshape(28,28), cmap="gray")
        labels = df_train.values[:,0]
        print(np.sort(labels))
        # sign_train_mean = train.mean(axis=0)
        # sign_test_mean = test.mean(axis=0)
        # plt.imshow(sign_test_mean.reshape(28,28), cmap="gray")
```

```
[ 0  0  0 ... 24 24 24]
```

```
In [ ]: train_mean = train.mean(axis=0)
        train_pca = np.subtract(train, train_mean)

        example = train_pca[6]
        r = np.asarray(example).reshape(28,28)
        # plt.imshow(r, cmap="gray")
```

```
In [ ]: train_pca_t = np.transpose(train_pca)
        #  $Y * Y_t / size\_train = the\ covariance\ matrix$ 
        yy_t = np.dot(train_pca, train_pca_t)
        n_train, _ = train.shape
        cov = np.divide(yy_t, n_train)
```

```
In [ ]: # @tf.function
        # def oper(m):
        #     tensor = tf.convert_to_tensor(m)
        #     eigenvalues, eigenvectors = tf.linalg.eig(tensor)
        #     return eigenvalues, eigenvectors

        # eigenvalues, eigenvectors = oper(cov)
```

```
In [ ]: # eigenvalues = eigenvalues.numpy()
        # eigenvectors = eigenvectors.numpy()
```

```
In [ ]: eigenvalues, eigenvectors = np.linalg.eig(cov)
```

```
In [ ]: # Top K = 150 eigensigns computed
        K = 150

        eigenvalues_index_sorted = np.argsort(eigenvalues)[::-1]
        eigenvalues_sorted = eigenvalues[eigenvalues_index_sorted][0:K]
        # eigenvectors_sorted = eigenvectors[:, eigenvalues_index_sorted]
        eigenvectors_sorted = eigenvectors[:,eigenvalues_index_sorted][:,0:K]
```

```
In [ ]: eigensigns = np.dot(train_pca_t, eigenvectors_sorted)
```

```
In [ ]: # labels_a_list = np.array([0,1,2,3,4,5,6,7,8,10,11,12,13,14,15,16,17,18,19,20,21,22,23]
        # distances = [np.linalg.norm(eigensigns - labels_a) for labels_a in labels_a_list]
        # closest_label_index = np.argmin(distances)
        # closest_label = labels[closest_label_index]
```

```
In [ ]: eigensigns_normal = eigensigns / np.linalg.norm(eigensigns, axis=0)
```

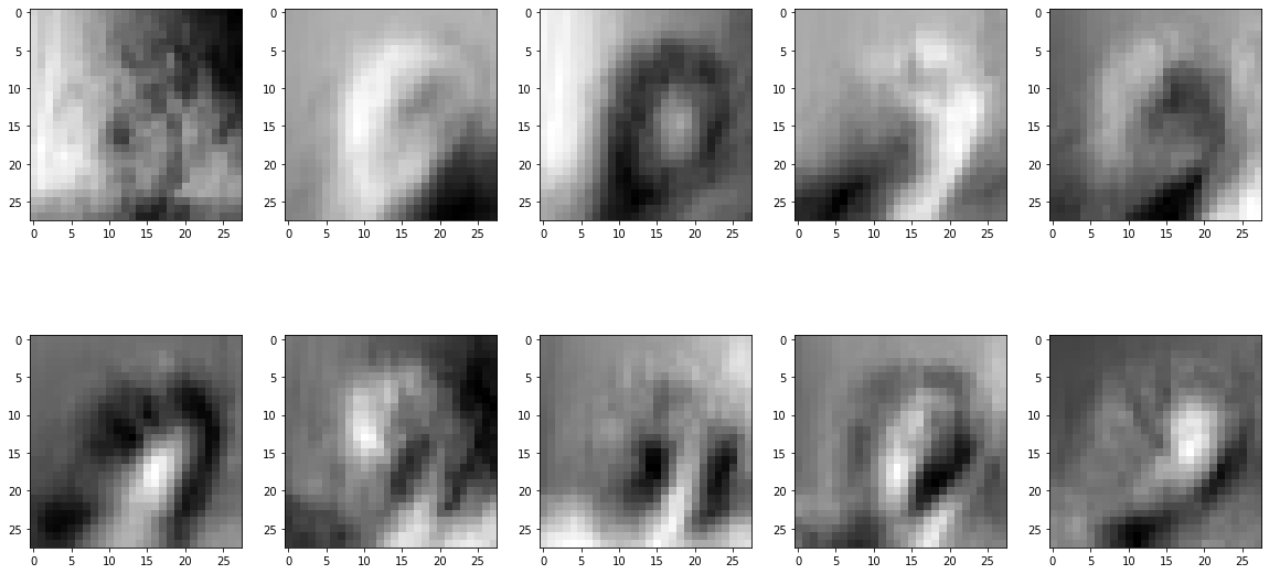
```
In [ ]: fig = plt.figure(figsize=(20, 10))
        for i in range(10):
            v = eigensigns_normal[:, i]
            r = np.asarray(v).reshape(28, 28)
```

```

fig.add_subplot(2, 5, i + 1)
plt.imshow(r, cmap='gray')

plt.savefig(f"eigensign_figures/K={K}_eigensign.pdf")

```



```

In [ ]: eigensigns_normal_k_t = np.transpose(eigensigns_normal)

# test_t      = test          - train_mean
# test_t_e    = test_t        @ eigensigns_normal
# test_t_e_et = test_t_e      @ eigensigns_normal_k_t
# test_final  = test_t_e_et   + train_mean

test_final = (((test - train_mean) @ eigensigns_normal) @ eigensigns_normal_k_t) + train_mean

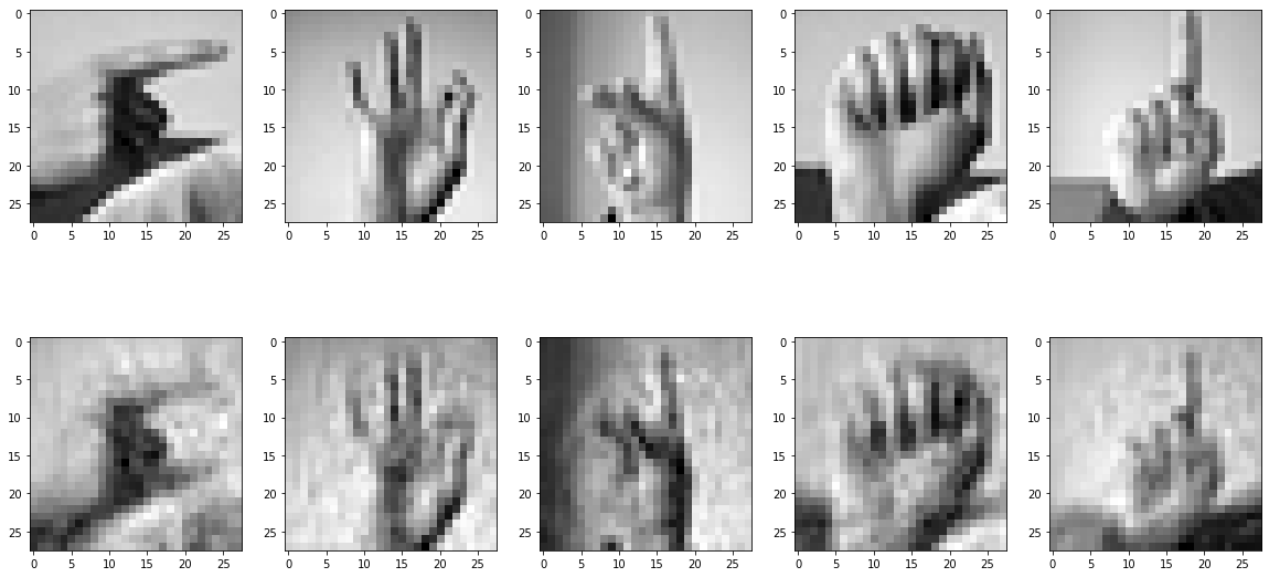
fig_r = plt.figure(figsize=(20, 10))
i = 1

for v in test:
    r = np.asarray(v).reshape(28, 28)
    fig_r.add_subplot(2, 5, i)
    i += 1
    plt.imshow(r, cmap="gray")
    if i > 5:
        break

for v in test_final:
    r = np.asarray(v).reshape(28, 28)
    fig_r.add_subplot(2, 5, i)
    i += 1
    plt.imshow(r, cmap="gray")
    if i > 10:
        break

plt.savefig(f"eigensign_figures/K={K}_test_vs_reconstructed.pdf")

```

In []:

```
test_image_A = Image.open("test_images/A.jpg")
test_image_B = Image.open("test_images/B.jpg")
test_image_L = Image.open("test_images/L.jpg")

test_image_A = np.array(test_image_A.convert("L"))
test_image_B = np.array(test_image_B.convert("L"))
test_image_L = np.array(test_image_L.convert("L"))

test_image_A_1D = test_image_A.ravel()
test_image_B_1D = test_image_B.ravel()
test_image_L_1D = test_image_L.ravel()

def ops(test_img):
    return (((test_img - train_mean) @ eigensigns_normal) @ eigensigns_normal_k_t) + tr

test_image_A_final = ops(test_image_A_1D)
test_image_B_final = ops(test_image_B_1D)
test_image_L_final = ops(test_image_L_1D)

distances = np.sqrt(np.sum((df_test.values[:,1:] - test_image_A_final)**2, axis=1))

# Find the index of the row with the smallest distance
min_idx = np.argmin(distances)

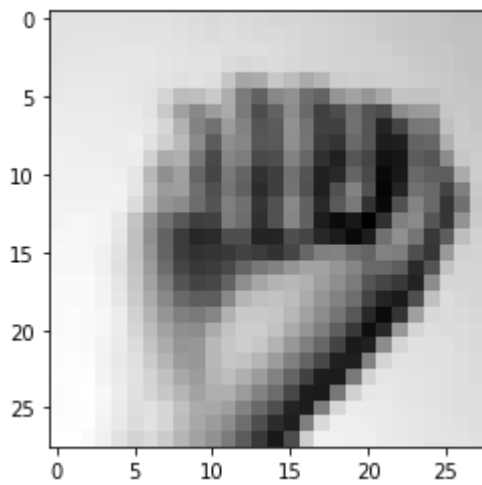
closest_image = df_test.values[:,1:][min_idx].reshape((28, 28))

print("Label: ", df_test.values[:,0][min_idx])
# plt.imshow(test_image_A_final.reshape(28,28), cmap="gray")
# plt.imshow(test_image_B_final.reshape(28,28), cmap="gray")
# plt.imshow(test_image_L_final.reshape(28,28), cmap="gray")
plt.imshow(closest_image, cmap="gray")

# for t in test_final:
#     print(t.shape)
#     break
```

Label: 0

Out[]: <matplotlib.image.AxesImage at 0x1952d3eb9e8>



Using existing packages to accomplish classification

```
In [ ]: eigensigns_X = df_train.drop("label", axis=1).values
        eigensigns_y = df_train["label"].values
```

```
In [ ]: # X_train, X_test, y_train, y_test = train_test_split(eigensigns_X, eigensigns_y)
        X_train = df_train.drop('label', axis=1).values
        y_train = df_train['label'].values
        X_test = df_test.drop('label', axis=1).values
        y_test = df_test['label'].values
        eigensigns_pca = PCA(n_components=50).fit(X_train) # 672 in total
```

```
In [ ]: X_train_pca = eigensigns_pca.transform(X_train)
```

```
In [ ]: classifier = SVC().fit(X_train_pca, y_train)
```

```
In [ ]: X_test_pca = eigensigns_pca.transform(X_test)
        predictions = classifier.predict(X_test_pca)
```

```
In [ ]: print(classification_report(y_test, predictions))
```

	precision	recall	f1-score	support
0	0.94	1.00	0.97	331
1	1.00	1.00	1.00	432
2	0.87	0.99	0.93	310
3	0.91	1.00	0.95	245
4	0.96	1.00	0.98	498
5	0.89	0.90	0.89	247
6	0.97	0.90	0.93	348
7	0.97	0.97	0.97	436
8	0.78	0.91	0.84	288
10	0.84	0.62	0.71	331
11	0.89	1.00	0.94	209
12	0.90	0.77	0.83	394
13	0.90	0.75	0.82	291
14	1.00	0.93	0.96	246

15	1.00	1.00	1.00	347
16	0.98	1.00	0.99	164
17	0.40	0.68	0.51	144
18	0.77	0.78	0.78	246
19	0.87	0.69	0.77	248
20	0.72	0.74	0.73	266
21	0.82	0.65	0.73	346
22	0.58	0.88	0.70	206
23	0.83	0.83	0.83	267
24	0.91	0.76	0.83	332
accuracy			0.87	7172
macro avg	0.86	0.86	0.86	7172
weighted avg	0.88	0.87	0.87	7172

Method 1: Image Centroids

```
In [ ]: df_train_A = df_train.loc[df_train['label'] == 0]
```

```
In [ ]: df_train.loc[2]
```

```
Out[ ]: label      2
pixel1      187
pixel2      188
pixel3      188
pixel4      187
...
pixel780    199
pixel781    198
pixel782    195
pixel783    194
pixel784    195
Name: 2, Length: 785, dtype: int64
```

```
In [ ]: df_train_A.mean().astype(int)
```

```
Out[ ]: label      0
pixel1      164
pixel2      165
pixel3      162
pixel4      161
...
pixel780    184
pixel781    182
pixel782    182
pixel783    178
pixel784    174
Length: 785, dtype: int32
```

The getImage started only working for means when you also convert to type int. I swear it wasn't like this before and I don't know what changed

```
In [ ]: getImage(df_train_A.mean().astype(int))
```

```
label      0
pixel1     164
```

```
pixel2      165
pixel3      162
pixel4      161
...
pixel780    184
pixel781    182
pixel782    182
pixel783    178
pixel784    174
Length: 785, dtype: int32
0
Sign:  A
```

In []:

```
sign_means = []
```

```
for i in range(0,26): sign_means.append(df_train.loc[df_train['label'] == i].mean())

temp = math.inf
min_index_index = 0
for i in range(0,26):
    if (np.linalg.norm(sign_means[i][1:] - df_test.loc[1][1:]) < temp):
        min_index_index = i
        temp = np.linalg.norm(sign_means[i][1:] - df_test.loc[1][1:])
print(sign_map[min_index_index])
```

In []:

```
getImageFromTest(1)
```

Sign: F

In []:

```
df_train.shape
```

Out[]: (27455, 785)

In []:

```
true = df_test.iloc[:,0].tolist()
```

In []:

```
true = [sign_map[k] for k in true]
```

In []:

```
df_train.iloc[:,1:]
```

Out[]:

	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	pixel10	...	pixel775	pixel7
0	107	118	127	134	139	143	146	150	153	156	...	207	2
1	155	157	156	156	156	157	156	158	158	157	...	69	1
2	187	188	188	187	187	186	187	188	187	186	...	202	2
3	211	211	212	212	211	210	211	210	210	211	...	235	2
4	164	167	170	172	176	179	180	184	185	186	...	92	1
...	
27450	189	189	190	190	192	193	193	193	193	194	...	132	1
27451	151	154	157	158	160	161	163	164	166	167	...	198	1
27452	174	174	174	174	174	175	175	174	173	173	...	121	1
27453	177	181	184	185	187	189	190	191	191	190	...	119	

	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	pixel10	...	pixel775	pixel7
27454	179	180	180	180	182	181	182	183	182	182	...	108	1

27455 rows × 784 columns

Initial, inefficient implementation

```
In [ ]:
class CentroidCompOld:
    def __init__(self):
        self.sign_means = []
        self.signs = 26

    def train(self,X):
        for i in range(0,26):
            self.sign_means.append(X.loc[X['label'] == i].mean())

    def get_centroids(self):
        return self.sign_means

    def predict(self,X):
        y_pred = []
        for index, row in tqdm(X.iterrows(), total=X.shape[0]):
            temp = math.inf
            min_index_index = 0
            for i in range(0,26):
                diff = np.linalg.norm(self.sign_means[i][1:] - X.loc[index][1:])
                if (diff < temp):
                    min_index_index = i
                    temp = diff
            y_pred.append(sign_map[min_index_index])
        return y_pred
```

More efficient implementation. Reading 6 on clustering methods gave us a good set of measures to implement to compare the class means and test data.

```
In [ ]:
class CentroidComp:
    def __init__(self,method="euclid", verbose=False, debug=False):
        self.sign_means = None
        self.method = method
        self.verbose = verbose
        if self.verbose or self.debug:
            print("Initialized centroid class with method " + self.method)

    def train(self,X):
        self.sign_means = X.groupby('label').mean().to_numpy()
        if self.verbose or self.debug:
            print("Trained model")

    def get_centroids(self):
        return self.sign_means

    def predict(self,X):
        y_pred = []
        if self.verbose or self.debug:
            print("Predicting model with " + self.method)
```

```

if (self.method == "euclid"):
    for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmax(np.dot(self.sign_means,rowi) / (np.linalg.norm(self.sign_means - rowi))

elif (self.method == "cos"):
    for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmin(np.linalg.norm(self.sign_means - rowi),

elif (self.method == "man"):
    for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmin(np.sum(np.abs(self.sign_means - rowi),

elif (self.method == "cor"):
    for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        corr = np.corrcoef(self.sign_means, rowi)
        coefs = corr[:-1, -1]
        y_pred.append(reduced_sign_map[np.argmax(coefs)])

else:
    return "you messed up"

return y_pred

```

```

In [ ]: euclidModel = CentroidComp(method="euclid", verbose=True)
euclidModel.train(X = df_train)
y_pred_euclid = euclidModel.predict(df_test)
print(classification_report(y_pred_euclid, true))

```

Initialized centroid class with method euclid

Trained model

Predicting model with euclid

	precision	recall	f1-score	support
A	0.76	0.56	0.65	448
B	0.66	0.93	0.77	304
C	0.61	0.67	0.64	281
D	0.44	0.47	0.46	232
E	0.66	0.76	0.71	435
F	0.56	0.49	0.52	284
G	0.48	0.51	0.49	329
H	0.58	0.83	0.68	304
I	0.24	0.29	0.27	240
K	0.53	0.30	0.39	575
L	0.66	0.63	0.64	218
M	0.18	0.36	0.24	199
N	0.25	0.37	0.30	196
O	0.60	0.51	0.55	289
P	0.82	0.49	0.62	577
Q	0.83	0.75	0.79	182
R	0.41	0.31	0.36	188
S	0.17	0.10	0.13	398
T	0.50	0.31	0.38	401
U	0.14	0.18	0.15	200
V	0.30	0.38	0.34	279
W	0.28	0.25	0.27	228
X	0.54	0.55	0.54	262

Y	0.18	0.50	0.27	123
accuracy			0.48	7172
macro avg	0.47	0.48	0.46	7172
weighted avg	0.51	0.48	0.48	7172

```
In [ ]: cosModel = CentroidComp(method="cos", verbose=True)
cosModel.train(X = df_train)
y_pred_cos = cosModel.predict(df_test)
print(classification_report(y_pred_cos, true))
```

Initialized centroid class with method cos

Trained model

Predicting model with cos

	precision	recall	f1-score	support
A	0.56	0.58	0.57	323
B	0.47	0.93	0.63	220
C	0.53	0.67	0.59	244
D	0.33	0.53	0.41	150
E	0.51	0.63	0.56	407
F	0.30	0.37	0.33	195
G	0.43	0.52	0.47	290
H	0.42	0.77	0.54	237
I	0.36	0.16	0.23	640
K	0.40	0.26	0.32	499
L	0.36	0.66	0.46	114
M	0.11	0.32	0.17	137
N	0.19	0.42	0.26	132
O	0.40	0.43	0.42	229
P	0.80	0.39	0.53	712
Q	0.58	0.17	0.26	560
R	0.13	0.15	0.14	129
S	0.13	0.10	0.11	296
T	0.53	0.21	0.30	613
U	0.05	0.11	0.07	116
V	0.37	0.30	0.33	437
W	0.17	0.17	0.17	201
X	0.42	0.57	0.48	199
Y	0.14	0.52	0.23	92
accuracy			0.38	7172
macro avg	0.36	0.41	0.36	7172
weighted avg	0.44	0.38	0.37	7172

```
In [ ]: manModel = CentroidComp(method="man", verbose=True)
manModel.train(X = df_train)
y_pred_man = manModel.predict(df_test)
print(classification_report(y_pred_man, true))
```

Initialized centroid class with method man

Trained model

Predicting model with man

	precision	recall	f1-score	support
A	0.49	0.55	0.51	295
B	0.31	0.85	0.46	160
C	0.56	0.65	0.61	268
D	0.25	0.63	0.36	99
E	0.47	0.44	0.45	541

F	0.27	0.25	0.26	263
G	0.39	0.41	0.40	333
H	0.33	0.78	0.46	182
I	0.40	0.14	0.21	789
K	0.39	0.22	0.28	584
L	0.33	0.52	0.40	132
M	0.09	0.41	0.15	85
N	0.12	0.40	0.18	88
O	0.36	0.37	0.36	238
P	0.76	0.42	0.54	628
Q	0.51	0.20	0.29	416
R	0.09	0.09	0.09	147
S	0.10	0.10	0.10	245
T	0.54	0.17	0.26	801
U	0.08	0.13	0.10	150
V	0.24	0.39	0.30	213
W	0.14	0.11	0.12	276
X	0.34	0.52	0.41	177
Y	0.13	0.69	0.22	62
accuracy			0.33	7172
macro avg	0.32	0.39	0.31	7172
weighted avg	0.41	0.33	0.33	7172

In []:

```
corModel = CentroidComp(method="cor", verbose=True)
corModel.train(X = df_train)
y_pred_cor = corModel.predict(df_test)
print(classification_report(y_pred_cor, true))
```


Initialized centroid class with method cor

Trained model

Predicting model with cor

	precision	recall	f1-score	support
A	0.88	0.58	0.70	502
B	0.66	0.84	0.74	340
C	0.60	0.50	0.54	369
D	0.51	0.43	0.47	287
E	0.71	0.77	0.74	461
F	0.58	0.41	0.48	349
G	0.53	0.42	0.47	436
H	0.52	0.78	0.62	287
I	0.21	0.32	0.25	191
K	0.56	0.31	0.40	603
L	0.30	0.72	0.42	88
M	0.10	0.24	0.14	165
N	0.29	0.44	0.35	193
O	0.51	0.35	0.42	355
P	0.93	0.52	0.67	614
Q	0.74	0.60	0.66	203
R	0.28	0.33	0.30	120
S	0.34	0.21	0.26	397
T	0.50	0.37	0.43	334
U	0.08	0.13	0.10	162
V	0.30	0.61	0.40	169
W	0.31	0.35	0.33	179
X	0.61	0.57	0.59	285
Y	0.13	0.51	0.20	83
accuracy			0.48	7172
macro avg	0.46	0.47	0.45	7172
weighted avg	0.55	0.48	0.49	7172

Euclid distance and pearson correlation are the best measures at nearly 50% accuracy.

Method 2: SVD Bases

General

Training: For the training set of known signs, compute the SVD of each set of signs of one kind.

Classification: For a given test sign, compute its relative residual in all 10 bases. If one residual is significantly smaller than all others, classify as that one.

Linear Algebra

Training: Compute SVDs of 10 matrices of dimension $m^2 \times n_i$. Each digit is an $m \times m$ digitized image. n_i is the number of training signs i .

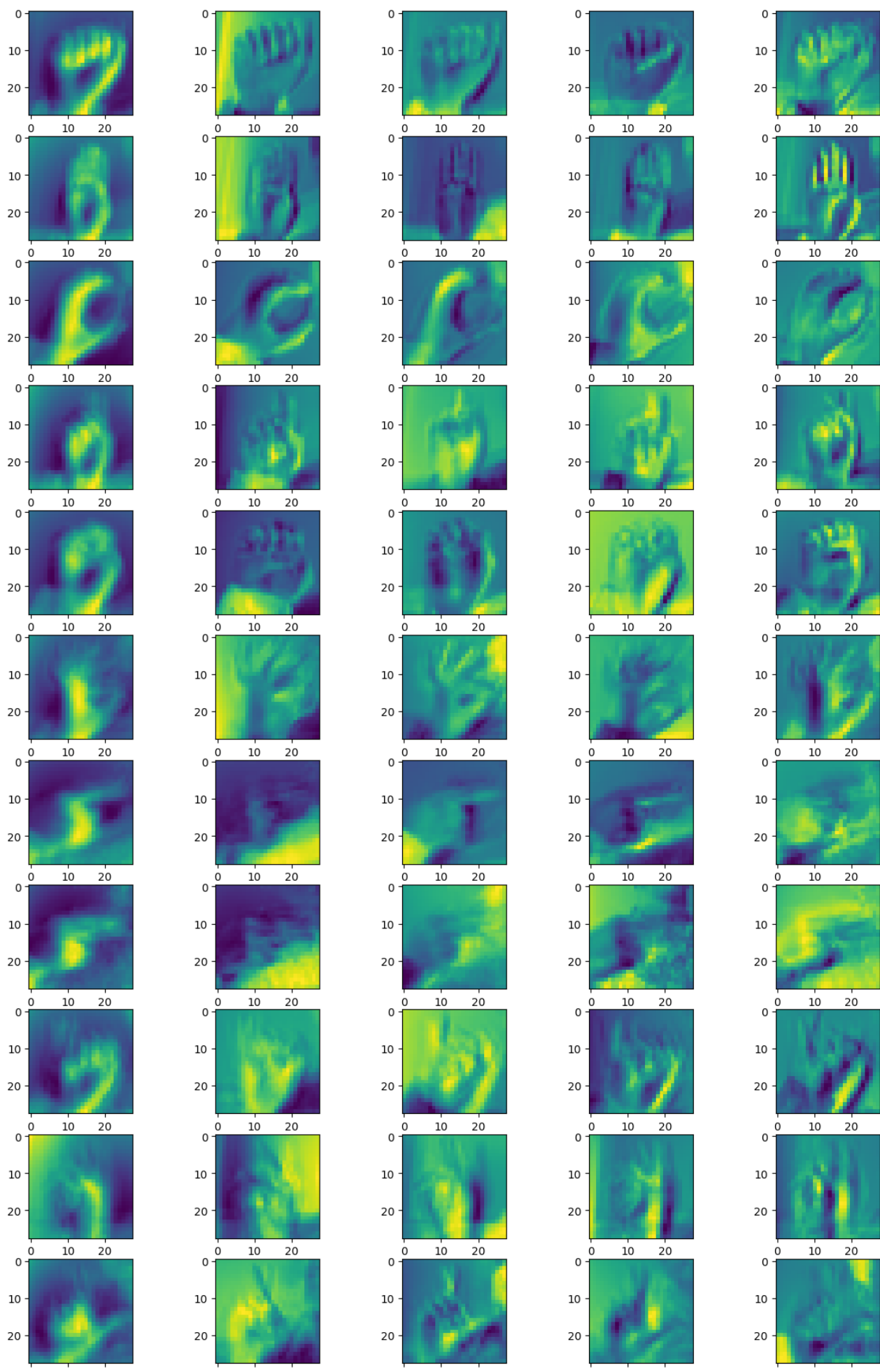
Classification: Compute 10 least squares residuals

TO DO

- Experiment with number of bases
- Determine best number of bases for this dataset

```
In [ ]: train_SVDs = []
letters = df_train.groupby('label')
for name, group in letters:
    letter = group.drop('label', axis=1).values
    train_SVDs.append(np.linalg.svd(letter))
```

```
In [ ]: fig = plt.figure(1, figsize = [15, 50], dpi = 100)
c = 1
for j in range(len(reduced_sign_map)):
    for i in range(0,5):
        plt.subplot(24,5,c)
        plt.imshow(train_SVDs[j][2][i:i+1,:].reshape(28,28))
        c = c+1
fig.subplots_adjust(wspace=0.1)
plt.show()
```



In []:

```
# Randomized SVD as provided in notebook 12
def rSVD(X,r,q,p):
    # Step 1: Sample column space of X with P matrix
    ny = X.shape[1]
    P = np.random.randn(ny,r+p) # Gaussian Random Matrix
    Z = X @ P
    for k in range(q): # Power iteration
        Z = X @ (X.T @ Z)

    Q, R = np.linalg.qr(Z,mode='reduced')

    # Step 2: Compute SVD on projected Y = Q.T @ X
    Y = Q.T @ X
    UY, S, VT = np.linalg.svd(Y,full_matrices=0)
    U = Q @ UY

    return U, S, VT
```

In []:

```
# from concurrent.futures import ThreadPoolExecutor, as_completed
# import threading

class SVDBases:
    def __init__(self, bases=5, method='SVD', verbose=False):
        self.verbose = verbose
        self.bases = bases
        self.method = method
        self.SVDs = []
        if self.verbose:
            print("Initialized SVD Bases class")

    def train(self,X):
        letters = X.groupby('label')
        for name,group in letters:
            letter = group.drop('label', axis=1).values
            if (self.method=='SVD'):
                U, s, Vt = np.linalg.svd(letter.T)
                self.SVDs.append(U)
            elif (self.method=='rSVD'):
                U, s, Vt = randomized_svd(letter.T, n_components=10, random_state=0)
                self.SVDs.append(U)
            else:
                print('Something went wrong')

    def predict(self, X):
        y_pred = []
        I = np.eye(784)
        uTu = np.array([np.dot(u[:, :self.bases], u[:, :self.bases].T) for u in self.SVDs])

        # def worker(i):
        #     z = X.iloc[:,1:].loc[i].to_numpy()
        #     y_pred.append(reduced_sign_map[np.argmax(np.linalg.norm(np.dot((I-uTu),z))

        # threads = []
        # with ThreadPoolExecutor(max_workers=1) as executor:
        #     # submit tasks to the executor
        #     futures = [executor.submit(worker, i) for i in range(len(X))]
        #     for future in tqdm(as_completed(futures), total=len(futures)):
```

```

#         result = future.result()

# for i in range(len(X)):
#     t = threading.Thread(target=worker, args=(i,))
#     threads.append(t)

# for i in tqdm(range(len(threads))):
#     if i == 0:
#         threads[i].start()
#     else:
#         threads[i-1].join()
#         threads[i].start()
# for thread in threads:
#     thread.join()

for i in tqdm(range(len(X))):
    z = X.iloc[:,1:].loc[i].to_numpy()

    y_pred.append(reduced_sign_map[np.argmin(np.linalg.norm(np.dot((I-uTu),z),

# y_pred.append(reduced_sign_map[np.argmax(np.dot(self.sign_means,rowi) / (
'''
    min=maxsize
    index = 0
    for i in range(len(reduced_sign_map)):
        print(np.dot(self.SVDs[i][:, :self.bases], self.SVDs[i][:, :self.bases])
        bases = (self.SVDs[i][:, :self.bases])
        diff = np.linalg.norm(np.dot(I-np.dot(bases, bases.T), X.iloc[j, 1:].to
        if (diff < min):
            min = diff
            index = i
        break
    break
    y_pred.append(reduced_sign_map[index])
'''

return y_pred

```

```
In [ ]: SVDmodel = SVDBases(verbose=True, method='SVD', bases=20)
```

Initialized SVD Bases class

```
In [ ]: SVDmodel.train(df_train)
```

```
In [ ]: y_pred = SVDmodel.predict(df_test)
```

100%|██████████| 7172/7172 [05:22<00:00, 22.27it/s]

```
In [ ]: accuracy_score(y_pred, true)
```

```
Out[ ]: 0.8453708867819297
```

```
In [ ]: SVDmodelB5 = SVDBases(verbose=True, method='SVD', bases=5)
SVDmodelB5.train(df_train)
```

```
y_predB5 = SVDmodelB5.predict(df_test)
accuracy_score(y_predB5, true)
```

Initialized SVD Bases class

100%|██████████| 7172/7172 [05:03<00:00, 23.67it/s]

Out[]: 0.6441717791411042

```
In [ ]: SVDmodelB10 = SVDBases(verbose=True, method='SVD', bases=10)
SVDmodelB10.train(df_train)
y_predB10 = SVDmodelB10.predict(df_test)
accuracy_score(y_predB10, true)
```

Initialized SVD Bases class

100%|██████████| 7172/7172 [05:11<00:00, 23.00it/s]

Out[]: 0.8064696040156163

```
In [ ]: SVDmodelB15 = SVDBases(verbose=True, method='SVD', bases=15)
SVDmodelB15.train(df_train)
y_predB15 = SVDmodelB15.predict(df_test)
accuracy_score(y_predB15, true)
```

Initialized SVD Bases class

100%|██████████| 7172/7172 [05:15<00:00, 22.72it/s]

Out[]: 0.8339375348577802

```
In [ ]: SVDmodelB20 = SVDBases(verbose=True, method='SVD', bases=20)
SVDmodelB20.train(df_train)
y_predB20 = SVDmodelB20.predict(df_test)
accuracy_score(y_predB20, true)
```

Initialized SVD Bases class

100%|██████████| 7172/7172 [05:15<00:00, 22.70it/s]

Out[]: 0.8453708867819297

```
In [ ]: SVDmodelB25 = SVDBases(verbose=True, method='SVD', bases=25)
SVDmodelB25.train(df_train)
y_predB25 = SVDmodelB25.predict(df_test)
accuracy_score(y_predB25, true)
```

Initialized SVD Bases class

100%|██████████| 7172/7172 [05:19<00:00, 22.42it/s]

Out[]: 0.8441160066926938

```
In [ ]: SVDmodelB30 = SVDBases(verbose=True, method='SVD', bases=30)
SVDmodelB30.train(df_train)
y_predB30 = SVDmodelB30.predict(df_test)
accuracy_score(y_predB30, true)
```

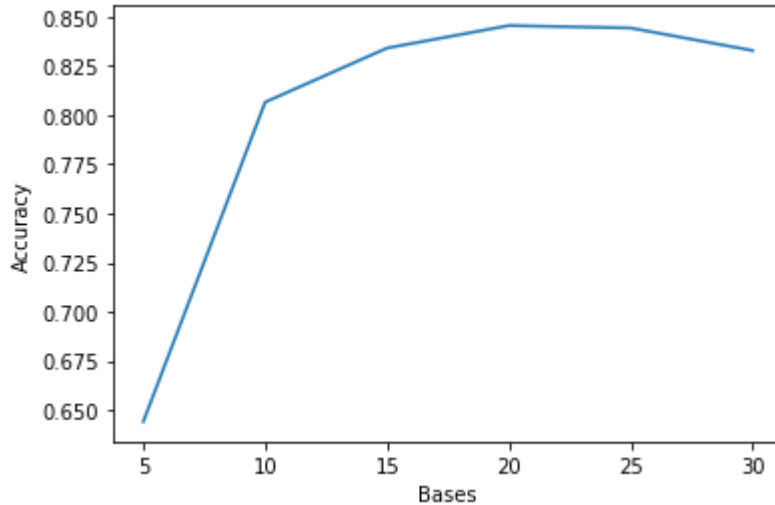
Initialized SVD Bases class

100%|██████████| 7172/7172 [04:59<00:00, 23.97it/s]

Out[]: 0.8326826547685443

```
In [ ]: x = [5,10,15,20,25,30]
y = [0.6441717791411042, 0.8064696040156163,0.8339375348577802,0.8453708867819297,0.844
plt.plot(x,y)
plt.xlabel("Bases")
plt.ylabel("Accuracy")
```

```
Out[ ]: Text(0, 0.5, 'Accuracy')
```



The best accuracy is achieved with around 20 bases, $20/784 \approx 2.5\%$ of the total bases.

```
In [ ]: uTu = np.array([np.dot(u[:, :20], u[:, :20].T) for u in SVDmodel.SVDs])
```

```
In [ ]: uTu
```

```
Out[ ]: array([[ 7.19129670e-03,  4.52196944e-03,  3.60780517e-03, ...,
                -1.03601969e-03, -2.27461501e-03, -3.56974713e-03],
               [ 4.52196944e-03,  8.83245430e-03,  9.58501962e-03, ...,
                9.86102163e-04,  8.08700103e-04,  2.58516003e-04],
               [ 3.60780517e-03,  9.58501962e-03,  1.32391753e-02, ...,
                1.94050201e-03,  1.72099215e-03,  5.42853447e-04],
               ...,
               [-1.03601969e-03,  9.86102163e-04,  1.94050201e-03, ...,
                5.15943525e-02,  5.27875617e-02,  5.20806514e-02],
               [-2.27461501e-03,  8.08700103e-04,  1.72099215e-03, ...,
                5.27875617e-02,  5.94579162e-02,  6.22272366e-02],
               [-3.56974713e-03,  2.58516003e-04,  5.42853447e-04, ...,
                5.20806514e-02,  6.22272366e-02,  6.90081298e-02]],
               [ 1.38476633e-02,  1.22620228e-02,  9.16047197e-03, ...,
                2.31892171e-03,  4.39931395e-04,  1.02553576e-05],
               [ 1.22620228e-02,  1.49804044e-02,  1.04964898e-02, ...,
                -1.30449979e-03, -1.73217268e-03, -1.50256410e-03],
               [ 9.16047197e-03,  1.04964898e-02,  9.79193935e-03, ...,
                3.11007557e-03,  2.40445323e-03,  1.29213985e-03],
               ...,
               [ 2.31892171e-03, -1.30449979e-03,  3.11007557e-03, ...,
                6.16780825e-02,  5.70018933e-02,  5.09161675e-02],
               [ 4.39931395e-04, -1.73217268e-03,  2.40445323e-03, ...,
                5.70018933e-02,  5.54322203e-02,  5.11694273e-02],
               [ 1.02553576e-05, -1.50256410e-03,  1.29213985e-03, ...,
                5.09161675e-02,  5.11694273e-02,  5.35236952e-02]]],
```

```

[[ 1.06696901e-02,  9.38971129e-03,  7.86502202e-03, ...,
  -1.32870382e-03, -1.04063475e-03, -5.70677486e-04],
 [ 9.38971129e-03,  8.45202317e-03,  7.28742380e-03, ...,
  -7.65970596e-04, -4.85248794e-04, -2.06053336e-05],
 [ 7.86502202e-03,  7.28742380e-03,  6.60682275e-03, ...,
  -2.68682657e-04,  4.94406636e-05,  4.94925704e-04],
 ...,
 [-1.32870382e-03, -7.65970596e-04, -2.68682657e-04, ...,
  9.79308521e-03,  9.48301148e-03,  8.49973084e-03],
 [-1.04063475e-03, -4.85248794e-04,  4.94406636e-05, ...,
  9.48301148e-03,  1.00431340e-02,  9.07402820e-03],
 [-5.70677486e-04, -2.06053336e-05,  4.94925704e-04, ...,
  8.49973084e-03,  9.07402820e-03,  1.10309845e-02]],

...,

[[ 9.92890369e-03,  9.05003940e-03,  6.18167798e-03, ...,
  2.53922212e-03,  1.71417082e-03,  4.65251664e-04],
 [ 9.05003940e-03,  9.00682426e-03,  6.38720729e-03, ...,
  3.62969055e-04,  1.74955892e-04, -1.03216063e-03],
 [ 6.18167798e-03,  6.38720729e-03,  6.03851293e-03, ...,
  7.94853851e-04,  1.30099055e-03,  9.88040369e-04],
 ...,
 [ 2.53922212e-03,  3.62969055e-04,  7.94853851e-04, ...,
  5.77242646e-02,  5.95334613e-02,  5.84528740e-02],
 [ 1.71417082e-03,  1.74955892e-04,  1.30099055e-03, ...,
  5.95334613e-02,  6.73324719e-02,  6.92923630e-02],
 [ 4.65251664e-04, -1.03216063e-03,  9.88040369e-04, ...,
  5.84528740e-02,  6.92923630e-02,  7.82728178e-02]],

[[ 1.23830160e-02,  1.03595976e-02,  9.28293815e-03, ...,
  -1.52726821e-03, -1.80742082e-03, -3.78247277e-03],
 [ 1.03595976e-02,  9.09570376e-03,  8.18347262e-03, ...,
  -1.28660632e-03, -1.38256149e-03, -3.17226149e-03],
 [ 9.28293815e-03,  8.18347262e-03,  7.44967914e-03, ...,
  -1.66320177e-03, -1.70450382e-03, -3.15188405e-03],
 ...,
 [-1.52726821e-03, -1.28660632e-03, -1.66320177e-03, ...,
  4.21571970e-02,  3.95821610e-02,  3.72283694e-02],
 [-1.80742082e-03, -1.38256149e-03, -1.70450382e-03, ...,
  3.95821610e-02,  4.01142946e-02,  3.95842613e-02],
 [-3.78247277e-03, -3.17226149e-03, -3.15188405e-03, ...,
  3.72283694e-02,  3.95842613e-02,  4.41434531e-02]],

[[ 9.28463279e-03,  8.48413622e-03,  7.81059055e-03, ...,
  3.71251517e-03,  3.55377212e-03,  4.89884070e-03],
 [ 8.48413622e-03,  7.81896757e-03,  7.24108448e-03, ...,
  3.39651846e-03,  3.13627816e-03,  4.33059414e-03],
 [ 7.81059055e-03,  7.24108448e-03,  6.78004440e-03, ...,
  2.94729661e-03,  2.83224287e-03,  4.00257398e-03],
 ...,
 [ 3.71251517e-03,  3.39651846e-03,  2.94729661e-03, ...,
  5.67613649e-02,  5.84557911e-02,  5.67809533e-02],
 [ 3.55377212e-03,  3.13627816e-03,  2.83224287e-03, ...,
  5.84557911e-02,  6.50617773e-02,  6.35925872e-02],
 [ 4.89884070e-03,  4.33059414e-03,  4.00257398e-03, ...,
  5.67809533e-02,  6.35925872e-02,  6.83603909e-02]]])

```

In []:

```
I = np.eye(784)
```



```
In [ ]: z = df_train.iloc[:,1:].loc[0].to_numpy()
```

```
In [ ]: reduced_sign_map[np.argmin(np.linalg.norm(np.dot((I-uTu),z), axis=1))]
```

```
Out[ ]: 'D'
```

```
In [ ]: df_train.loc[0]
```

```
Out[ ]: label      3
pixel1      107
pixel2      118
pixel3      127
pixel4      134
...
pixel780    206
pixel781    206
pixel782    204
pixel783    203
pixel784    202
Name: 0, Length: 785, dtype: int64
```

```
In [ ]: # for i in lst:
#       print(np.linalg.norm(i))
```

```
In [ ]: df_train.loc[0]
```

```
Out[ ]: label      3
pixel1      107
pixel2      118
pixel3      127
pixel4      134
...
pixel780    206
pixel781    206
pixel782    204
pixel783    203
pixel784    202
Name: 0, Length: 785, dtype: int64
```

```
In [ ]: df_train.shape
```

```
Out[ ]: (27455, 785)
```

```
In [ ]: print(classification_report(y_pred, true))
```

	precision	recall	f1-score	support
A	1.00	0.94	0.97	354
B	0.90	0.92	0.91	423
C	1.00	0.94	0.97	330
D	1.00	0.80	0.89	305
E	0.99	0.90	0.95	549
F	0.85	0.92	0.88	229
G	0.94	0.94	0.94	347

H	0.90	0.99	0.95	396
I	0.92	0.88	0.90	303
K	0.82	0.82	0.82	331
L	0.97	1.00	0.98	202
M	0.68	0.91	0.78	294
N	0.84	0.91	0.88	268
O	0.83	0.83	0.83	247
P	1.00	1.00	1.00	347
Q	1.00	0.97	0.98	169
R	0.58	0.34	0.43	244
S	0.95	0.71	0.81	326
T	0.75	0.82	0.78	226
U	0.51	0.62	0.56	216
V	0.49	0.67	0.57	251
W	0.63	0.47	0.54	274
X	0.84	0.72	0.78	309
Y	0.70	1.00	0.82	232
accuracy			0.85	7172
macro avg	0.84	0.84	0.83	7172
weighted avg	0.86	0.85	0.84	7172

```
In [ ]: y_pred[4]
```

```
Out[ ]: 'D'
```

```
In [ ]: df_test.loc[4]
```

```
Out[ ]: label      3
pixel1      188
pixel2      191
pixel3      193
pixel4      195
...
pixel780     46
pixel781     49
pixel782     46
pixel783     46
pixel784     53
Name: 4, Length: 785, dtype: int64
```

Method 3: Smoothing

At the end of chapter 10, Elden recommends image smoothing as a way to improve performance. We attempted to apply some smoothing methods to improve the performance of the models we've made.

```
In [ ]: images = df_train.iloc[:,1:].values.reshape(-1, 28, 28)
```

```
In [ ]: filtered_images = np.zeros_like(images)
for i in range(len(images)):
    filtered_images[i] = gaussian_filter(images[i], sigma=1)
```

```
In [ ]: filtered_data = filtered_images.reshape(-1, 784)
```

```
In [ ]: filtered_df_train = pd.DataFrame(filtered_data)
```

```
In [ ]: filtered_df_train.insert(0, 'label', df_train['label'])
```

```
In [ ]: filtered_df_train.columns
```

```
Out[ ]: Index(['label',      0,      1,      2,      3,      4,      5,      6,
              7,      8,
              ...
              774,    775,    776,    777,    778,    779,    780,    781,
              782,    783],
              dtype='object', length=785)
```

```
In [ ]: getImage(filtered_df_train.loc[1])
```

```
label      6
0         156
1         156
2         156
3         157
...
779        131
780        134
781        123
782        122
783        129
Name: 1, Length: 785, dtype: int64
6
Sign:  G
```

```
In [ ]: model3 = CentroidComp(method="cor", verbose=True)
```

Initialized centroid class with method cor

```
In [ ]: model3.train(X = filtered_df_train)
```

Trained model

```
In [ ]: y_pred3 = model3.predict(df_test)
```

Predicting model with cor

```
In [ ]: accuracy_score(y_pred3, true)
```

```
Out[ ]: 0.45984383714445065
```

```
In [ ]:
```

```
In [ ]: def smooth_row(row):
         smoothed_row = gaussian_filter1d(row, sigma=1)
         return pd.Series(smoothed_row, index=row.index)
```

```
In [ ]: smoothed_df_train = df_train.apply(smooth_row, axis=1)
```

```
In [ ]: smoothed_df_train['label'] = df_train['label']
```

```
In [ ]: smoothed_df_train
```

```
Out[ ]:
```

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776
0	3	80	111	125	133	138	142	146	149	152	...	206	206
1	6	111	147	155	156	156	156	156	157	157	...	99	117
2	2	132	176	186	187	186	186	186	187	186	...	202	201
3	2	149	199	210	211	211	210	210	210	210	...	234	233
4	13	120	158	169	172	175	178	180	183	184	...	91	100
...
27450	13	136	179	189	190	191	192	192	193	193	...	127	131
27451	23	114	146	155	158	159	161	162	164	165	...	198	198
27452	18	127	164	173	174	174	174	174	173	173	...	132	174
27453	17	131	171	182	185	186	188	189	190	190	...	108	70
27454	23	133	170	179	180	181	181	181	182	182	...	115	137

27455 rows × 785 columns



```
In [ ]: getImage(smoothed_df_train.loc[0])
```

```
label      3
pixel1     80
pixel2    111
pixel3    125
pixel4    133
...
pixel780   205
pixel781   205
pixel782   204
pixel783   203
pixel784   202
Name: 0, Length: 785, dtype: int64
3
Sign: D
```

Method 4: Neural Network

```
In [ ]: mlp = MLPClassifier(hidden_layer_sizes=(512, 256), verbose=True)
```

```
In [ ]: x_train = df_train.iloc[:,1:]
        y_train = df_train.iloc[:,1]
        x_test = df_test.iloc[:,1:]
        y_test = df_test.iloc[:,1]
```

```
In [ ]: scaler = MinMaxScaler()
```

```
In [ ]: x_train = scaler.fit_transform(x_train)
        x_test = scaler.fit_transform(x_test)
```

```
In [ ]: mlp.fit(x_train, y_train.values.ravel())
```

```
Iteration 1, loss = 2.16389120
Iteration 2, loss = 1.19031912
Iteration 3, loss = 0.81820537
Iteration 4, loss = 0.61433841
Iteration 5, loss = 0.45371412
Iteration 6, loss = 0.31863615
Iteration 7, loss = 0.25137237
Iteration 8, loss = 0.17598662
Iteration 9, loss = 0.11911251
Iteration 10, loss = 0.08234499
Iteration 11, loss = 0.06275498
Iteration 12, loss = 0.04330383
Iteration 13, loss = 0.03373408
Iteration 14, loss = 0.02872066
Iteration 15, loss = 0.02159015
Iteration 16, loss = 0.01624898
Iteration 17, loss = 0.01115089
Iteration 18, loss = 0.00961224
Iteration 19, loss = 0.00823164
Iteration 20, loss = 0.00804878
Iteration 21, loss = 0.22303521
Iteration 22, loss = 0.01631741
Iteration 23, loss = 0.01187284
Iteration 24, loss = 0.00983561
Iteration 25, loss = 0.00784937
Iteration 26, loss = 0.00671869
Iteration 27, loss = 0.00582804
Iteration 28, loss = 0.00515621
Iteration 29, loss = 0.00433231
Iteration 30, loss = 0.00421594
Iteration 31, loss = 0.00373799
Iteration 32, loss = 0.47204357
Iteration 33, loss = 0.06375120
Iteration 34, loss = 0.03214963
Iteration 35, loss = 0.02100711
Iteration 36, loss = 0.01426431
Iteration 37, loss = 0.01184296
Iteration 38, loss = 0.01100657
Iteration 39, loss = 0.00818186
Iteration 40, loss = 0.00755506
Iteration 41, loss = 0.00663645
Iteration 42, loss = 0.00544698
Training loss did not improve more than tol=0.000100 for 10 consecutive epochs. Stopping.
```

Out[]:

```
▼ MLPClassifier  
MLPClassifier(hidden_layer_sizes=(512, 256), verbose=True)
```

In []:

```
y_pred_ANN = mlp.predict(x_test)
```

In []:

```
accuracy_score(y_test, y_pred_ANN)
```

Out[]: 0.7643614054657

Works Cited

Eldén, Lars. Matrix Methods in Data Mining and Pattern Recognition. Society for Industrial and Applied Mathematics, 2007.