American Sign Language Image Transcriber

By Benjamin Singleton and Omar Abdelmotaleb

Abstract

In this project, we seek to utilize multiple image classification methods for classifying 28 x 28 resolution images of hand signs to label them with a respective letter. These hand signs are meant to represent a letter in the English language through translation of American Sign Language. The first method we explore is eigensigns, which is our take on the eigenface method using eigen-decomposition on face images but instead on hand sign images. Then, we look at using image centroids, implementing multiple distance calculation methods to find which hand sign is the image closest to, with the hand signs in comparison are the mean images of each label. Next, we use SVD bases, taking the Singular Value Decomposition of the hand signs as well as calculating the residuals to classify which one the hand sign represents the closest. Finally, smoothing is implemented as our last method in an attempt to optimize the performance of our previous models. Overall, we find that some methods are less effective than others in their accuracy of classifying a hand sign to be the correct English letter. Eigensigns and SVD bases yields an accuracy above 80%, while image centroids and our attempted optimization with smoothing yielded no more than 50%.

Introduction

American Sign Language is a popular means of communication for those who may be deaf or hard of hearing. Utilizing hand signs effectively demonstrates letters in the English language which can be used to construct words and sentences. Translating English words to hand signs is a straightforward task which has been accomplished in numerous ways. However, it's not frequently done the other way around. Translating American Sign Language to English is a more challenging task because it requires methods involving image classification. More importantly, attaining a high accuracy is not as simple and straightforward due to the flaws in existing image classification as well as potentional clarity issues with the images themselves. Translating hand sign images to English serves a purpose in allowing those who are not familiar with ASL to understand or learn from hand signs. This can also help in recording American Sign Language in English for those who want to transcribe it.

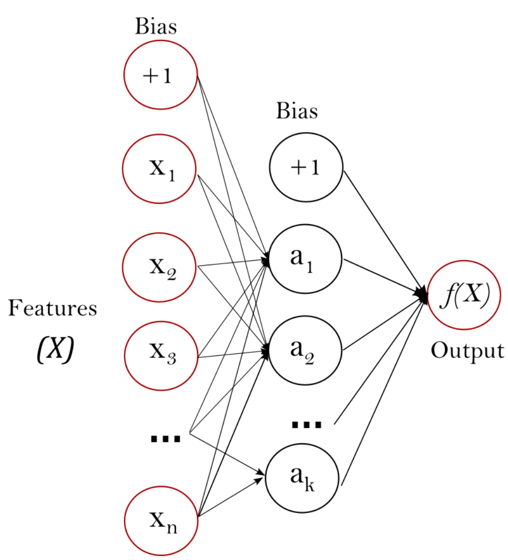
What you have accomplished

Our dataset Sign Language MNIST was retrieved from Kaggle and read using a pandas dataframe. The training dataset is comrpomised of over 27000 rows and the testing dataset of over 7000 rows. The first header both is the label, which is a numerical value corresponding to a letter in the English alphabet. The remaining 784 columns represent an individual pixel (i.e. pixel1, pixel2, ..., pixel784) of which is a 1-dimensional representation of the 28 x 28 image, able to be displayed once reshaped. Each row will represent an individual image.

It is important to note with the dataset that the first first label header's numerical values is missing 9 and 25. This is key to understanding why we work with the number 24 a lot despite the English alphabet having 26 letters. As such, the number 8 corresponds to corresponds to "I", the number 9 corresponds to "J", and the number 10 corresponds to "K". Of course, the number 9 and the number 25 corresponding to "Z" will not show up anywhere in the datasets. American Sign Language has hand signs for J and Z, but they are actually hand movements. Since we are working strictly with images, we can not accurately show hand movements, and as such are not included in training or testing.

Neural Network

As a baseline, we implemented a neural network to classify the images. The classification of handwritten digits using a neural network is a very well researched and developed topic. This is commonly done using the famous MNIST dataset and the accuracy of these models has become quite high. The classification of sign language images is a similar problem and so neural networks should provide us with a solid baseline to gauge the performance of our latter models on. We used scikit learn's Multi-Layer Perceptron (MLP) to implement our neural network. To give a brief overview, an MLP is a supervised learning algorithm that is modeled after the human brain. Input data is fed through a series of layers of interconnected nodes. These node represent neurons in the brain and the connections between them. Each layer in the network transforms the values from the previous layer using a weighted linear summation and then applies the result to an activiation function. The general layout of a neural network is displayed below.



A favoriate mathematician of Ben's has a great series of videos on neural networks if you'd like to learn about them. As mentioned, they work well for the problem of image recognition; however, a drawback of neural networks is that they are black boxes. This means their innerworkings and why they classifiy each image as a given class is unknowable. Just as we can't inspect the neurons in a brain to understand why someone thinks the way they do, we can't inspect a neural network to understand why it classifies a certain way. This provides us with the motivation to explore other methods of classification. The other algorithms we'll employ in this project utilize concepts from numerical linear algebra that we all should be very familar with at this point. As such, how these algorithms classify each image should be much more understanble.

Image Centroids

The centroids method is defined in the textbook as follows:

Image Centroids classification algorithm

Training: Given the manually classified training set, compute the means (centroids) m_i, i = 0, ... , 8, 10, ... , 24$, of all the 24 classes.

Classification: For each digit in the test set, classify it as k if m_k is the closest mean.

As the name suggests, the central idea of the algorithm is to find the centroid (mean) of each class of images. When you want to classify a new image, you can use a metric to determine how similar the new each is to each class centroid. You classify the new image as the centroid it is closest to. In the textbook, it suggests using euclidean distance which we implemented. We also implemented several other distance metrics to see if any preformed better than euclidean distance. We choose manhattan distance, pearson correlation coefficient, and Cosine similarity as well.

SVD Bases

The other method defined in the textbook, SVD bases, is defined as follows:

SVD Bases classification algorithm

Training: For the training set of known images, compute the SVD of each set of images of a label.

Classification: For a given test image, compute its relative residual in all 24 bases. If one residual is significantly smaller than all the others, classify as that. Otherwise, give up.

In this method, you seperate each class into its own matrix and find the singular value decomposition of each class. In the SVD decomposition where $A = U\Sigma V^T$, U is the left singular vector and captures the dominating features of class. The most important of these features will be captured by the first k singular vectors, U_k. k is not a set number and has to be experiment with to find. To classify a new image, we can find the residual between the new image z and U_k of each class using the formula

$$|| \, (I - U_k U_k^T) z \, ||_2$$

The residual between U_k and z that's the smaller tells you which class the new image likely belong to.

Eigensigns

Let X be the training dataset. We compute $\bar{X}$ as the mean of X. Then we find the difference of the two as $\hat{X} = X - \bar{X}$ which will act as the mean-adjusted images. Next we construct the covariance matrix by computing

$$cov = \frac{1}{N}(\hat{X} \cdot \hat{X}^T)$$

where N is the size of X. With the covariance matrix cov, we find the eigen-decomposition yielding us the eigenvalues λ and the eigenvectors v. Afterwards, it's important to sort them and find the top K eigenvectors. We chose $K = 150$. Once that's chosen, we compute the eigensigns themselves as $E = \hat{X}^T \cdot v_{sorted}$. We then normalize to get E_{normal}. In our project, we went ahead and showed the reconstruction method of it before utilizing a package for the image classification. For the reconstruction, the formula used the testing dataset y, giving us

$$y_{final} = (((y - \bar{X}) \cdot E_{normal}) \cdot E_{normal}^T) + \bar{X}$$

For the package, we used sklearn's SVC and PCA methods.

Smoothing

Gaussian smoothing is a method briefly mentioned in our textbook as a way to improve the performance of image classficiation algorithms. Gaussian smoothing, as known as gaussian filtering, involves applying a convolution using a gaussian function to each image. The gaussian function G is defined as

$$G(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

where x distance from the origin in the horizontal axis, y is the distance from the origin in the vertical axis, and σ is the standard deviation os the gaussian distribution. Applying this function to the image will reduce noise as well as detail. The resulting image will appear blurier as σ is increased. At the same time, edges will become more defined. This means the characteristics of each image, such as the arrangement of fingers, may become easier for the algorithms to identity.

Observations

Our training dataset is of 27455 rows x 785 columns, including the header row. The first column is the label representing the English letter as a number, excluding 9 and 25 for J and Z. The remaining 784 columns are the pixels, ranging from 0-255, which is a 1D representation of the 28 x 28 images once reshaped to be 2D. In total, it's 27454 black and white images. Our test dataset is of 7172 x 785 columns, including the header row. This follows the same pattern as the training dataset, so 7171 images in total.

We found Neural Network to serve as basis for a somewhat accurate classification method. The Neural Network we used is an MLP Classifier with hidden layer sizes of 512 and 256. The accuracy it yielded came out to be around **76.3%**.

Image centroids was our least accurate method. We could not get above a 50% accuracy with any of the distance formulas, but this is not a surprise. We speculate that the resolution of the images being only 28 x 28 has a negative impact on the performance. We could get more variation by having more detailed images, but we find with other methods that they yield a higher accuracy despite having such low detail images. Nonetheless, the textbook we referred to already mentions that this method of using distance formulas for finding centroids of images is not very effective overall. To go through each formula:

- **Euclid distance**: 48%
- **Cosine similarity**: 38%
- **Manhattan distance**: 33%
- **Correlation**: 48%

For SVD Bases, we found that having around 20 bases was when the accuracy started to plateau. 5 bases gave us the lowest accuracy and 10 bases gave us the highest increase in accuracy with a difference of **16.6%**. For 25 and 30 bases, it began to decrement with a negligible difference. Here we have a list of the bases and their respective accuracies:

- **5 bases**: 64%
- **10 bases**: 80.6%
- **15 bases**: 83.4%
- **20 bases**: 84.5%
- **25 bases**: 84.4%
- **30 bases**: 83.3%

Eigensigns is where we find the most accurate classification in testing. For starters, the images were able to reconstructed visually well. With this method, we had shortened our sample size to 300. While attempting to make it into the 1000's, we came across a detrimental issue to the method where we came across complex numbers in our eigen-decomposition of the covariance matrix. Unfortunately, high dimensionality was causing a lot of problems with our more manual methods, making the task much harder. We were still able to accomplish reconstruction however despite this obstacle. More importantly, using the sklearn methods for SVC and PCA gave us an **87%** accuracy for image classification.

Smoothing was used to apply a gaussian filter with $\sigma = 1$ as the parameter for the number of deviations. This essentially blurred the images a bit visually, however it increases the definition between different parts of the image and helps remove background noise. Using this method, we saw a 1% to 2% increase in performance for SVD Bases and Eigensigns. This was not the case for Image Centroids though, negatively impacting the accuracy for most of the methods by 5% to 10%.

Conclusion

After utilizing multiple image classification methods, we found that not all of them work the best at least in our case. Using 28 x 28 black and white images leaves not much room for detail, so we have to be as precise as possible. After completely going through each method, we found that Eigensigns and SVD Bases yielded the best results with over 87% and 84.5% accuracies respectively. The Neural Network gave us 76.3% accuracy, which isn't terrible but we'd certainly want better. Same deal goes for the image centroids and smoothing, only giving us a range of 30% to 50% accuracy.

We made attempts to improve the accuracy of the centroids model especially, with utilizing multiple distance methods only to find Euclidean distance to be the best of the bunch. Smoothing did not give us the boost in accuracy we were expecting, but at the same time unsurprising in regards to using distance calculations for image classification since it simply isn't designed for that purpose.

Eigensigns was the most surprising as we were expecting SVD bases to yield the highest accuracy. This is especially due to the fact that the images are only 28 x 28 leaving not much room for detail. Even upon inspection of the eigensigns themselves we saw that they were difficult to visually make out. Despite this, it gave us the highest classification accuracy thanks to sklearn's packages.

Smoothing was a disappointment to us. The textbook highlighted that this would be a great method for increasing the accuracy of our models. While we saw the slight 1% to 2% increase in performance for SVD Bases and Eigensigns, that was not what we were interested in. We wanted to increase the performance of our model for Image Centroids, but instead the got the opposite. We can't say for certain why Image Centroids performed worse with smoothing, but we can speculate it still had something to do with the poor resolution of our images we used with the dataset. Regardless, we weren't expecting to get a higher accuracy than our other methods anyway, but it was a good demonstration nonetheless.

We'd further like to note the limitations we faced regarding runtime performance. We had made attempts to multithread our executions but despite a 50% to 60% increase in speed, we learned the hard way that accuracy gets cut by the number of threads it gets executed over, and as such removed such processes. We could have made the entire execution run faster if we multithreaded in the individual methods to run at the same time, but we this would have taken longer and created more points of failure, not to forget the performance limitations of our own machines for the size of the data we're dealing with.

If we had a larger dataset with higher quality images and a more powerful machine especially for single threaded performance, we may have been able to find better results with our given methods. We wished to have achieved >90% accuracy to make this thing usable in a real-world scenario, however with the current problems as they stand we'll have to be more patient. Overall, we're satisfied with what we accomplished.

Codes

In [ ]:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

#from google.colab import files
#from google.colab import drive
from tqdm import tqdm
import io
from sys import maxsize
from sklearn.utils.extmath import randomized_svd
from scipy.ndimage import gaussian_filter
from scipy.ndimage import gaussian_filter1d
import multiprocessing as mp


from sklearn.svm import SVC
from sklearn.model_selection import train_test_split
from sklearn.decomposition import PCA
from sklearn.metrics import confusion_matrix, classification_report
from sklearn.neural_network import MLPClassifier
```

```
from sklearn.preprocessing import MinMaxScaler
import warnings
warnings.filterwarnings("ignore")

# import tensorflow as tf
```

In [ ]:

```
# Assessment metrics
from sklearn.metrics import classification_report
from sklearn.metrics import accuracy_score
from sklearn.metrics import precision_score
```

In [ ]:

```
import math
```

In [ ]:

```
from PIL import Image
```

In [ ]:

```
sign_map = {
    0: "A",
    1: "B",
    2: "C",
    3: "D",
    4: "E",
    5: "F",
    6: "G",
    7: "H",
    8: "I",
    9: "J",
    10: "K",
    11: "L",
    12: "M",
    13: "N",
    14: "O",
    15: "P",
    16: "Q",
    17: "R",
    18: "S",
    19: "T",
    20: "U",
    21: "V",
    22: "W",
    23: "X",
    24: "Y",
    25: "Z"
}
```

In [ ]:

```
reduced_sign_map = {
    0: "A",
    1: "B",
    2: "C",
    3: "D",
    4: "E",
    5: "F",
    6: "G",
    7: "H",
    8: "I",
```

```
    9: "K",
    10: "L",
    11: "M",
    12: "N",
    13: "O",
    14: "P",
    15: "Q",
    16: "R",
    17: "S",
    18: "T",
    19: "U",
    20: "V",
    21: "W",
    22: "X",
    23: "Y"
}
```

In [ ]:

```
df_train = pd.read_csv('sign_mnist_train.csv')
```

In [ ]:

```
df_test = pd.read_csv('sign_mnist_test.csv')
```

In [ ]:

```
df_train.head(10)
```

Out[ ]:

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pi
0	3	107	118	127	134	139	143	146	150	153	...	207	207	
1	6	155	157	156	156	156	157	156	158	158	...	69	149	
2	2	187	188	188	187	187	186	187	188	187	...	202	201	
3	2	211	211	212	212	211	210	211	210	210	...	235	234	
4	13	164	167	170	172	176	179	180	184	185	...	92	105	
5	16	161	168	172	173	178	184	189	193	196	...	76	74	
6	8	134	134	135	135	136	137	137	138	138	...	109	102	
7	22	114	42	74	99	104	109	117	127	142	...	214	218	
8	3	169	174	176	180	183	185	187	188	190	...	119	118	
9	3	189	189	189	190	190	191	190	190	190	...	13	53	

10 rows × 785 columns

In [ ]:

```
df_test.head(10)
```

Out[ ]:

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pi
0	6	149	149	150	150	150	151	151	150	151	...	138	148	
1	5	126	128	131	132	133	134	135	135	136	...	47	104	

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776	pi
2	10	85	88	92	96	105	123	135	143	147	...	68	166	
3	0	203	205	207	206	207	209	210	209	210	...	154	248	
4	3	188	191	193	195	199	201	202	203	203	...	26	40	
5	21	72	79	87	101	115	124	131	135	139	...	187	189	
6	10	93	100	112	118	123	127	131	133	136	...	173	175	
7	14	177	177	177	177	177	178	179	179	178	...	232	223	
8	3	191	194	196	198	201	203	204	205	205	...	43	57	
9	7	171	172	172	173	173	173	173	173	172	...	199	199	

10 rows × 785 columns

In [ ]:

```
def getImageFromTrain(index):
  row = df_train.loc[index].tolist()
  sign = row[0]
  row = row[1:]
  width, height = 28, 28
  img = Image.new("L", (width, height))
  img.putdata(row)
  newsize = (300, 300)
  img = img.resize(newsize)
  img.show()
  print("Sign: ", sign_map[sign])
```

In [ ]:

```
def getImageFromTest(index):
  row = df_test.loc[index].tolist()
  sign = row[0]
  row = row[1:]
  width, height = 28, 28
  img = Image.new("L", (width, height))
  img.putdata(row)
  newsize = (300, 300)
  img = img.resize(newsize)
  img.show()
  print("Sign: ", sign_map[sign])
```

In [ ]:

```
def getImage(row):
  print(row)
  sign = row['label']
  print(sign)
  row = row[1:]
  width, height = 28, 28
  img = Image.new("L", (width, height))
  img.putdata(row)
  newsize = (300, 300)
  img = img.resize(newsize)
  img.show()
  print("Sign: ", sign_map[sign])
```

In [ ]:

```
getImage(df_train.loc[2])
```

```
label         2
pixel1      187
pixel2      188
pixel3      188
pixel4      187
           ...
pixel780    199
pixel781    198
pixel782    195
pixel783    194
pixel784    195
Name: 2, Length: 785, dtype: int64
2
Sign:  C
```

Method 0: Neural Network

In [ ]:

```
mlp = MLPClassifier(hidden_layer_sizes=(512, 256), verbose=True)
```

In [ ]:

```
x_train = df_train.iloc[:,1:]
y_train = df_train.iloc[:,:1]
x_test = df_test.iloc[:,1:]
y_test = df_test.iloc[:,:1]
```

In [ ]:

```
scaler = MinMaxScaler()
```

In [ ]:

```
x_train = scaler.fit_transform(x_train)
x_test = scaler.fit_transform(x_test)
```

In [ ]:

```
mlp.fit(x_train, y_train.values.ravel())
```

```
Iteration 1, loss = 2.22737248
Iteration 2, loss = 1.18025719
Iteration 3, loss = 0.84407521
Iteration 4, loss = 0.59013930
Iteration 5, loss = 0.43724693
```

```
Iteration 6, loss = 0.32700000
Iteration 7, loss = 0.23352503
Iteration 8, loss = 0.16559877
Iteration 9, loss = 0.11884729
Iteration 10, loss = 0.08792733
Iteration 11, loss = 0.05763120
Iteration 12, loss = 0.04607492
Iteration 13, loss = 0.03391879
Iteration 14, loss = 0.03126376
Iteration 15, loss = 0.06739533
Iteration 16, loss = 0.01336139
Iteration 17, loss = 0.01205062
Iteration 18, loss = 0.00986606
Iteration 19, loss = 0.00981036
Iteration 20, loss = 0.00720068
Iteration 21, loss = 0.00604480
Iteration 22, loss = 0.22586252
Iteration 23, loss = 0.08225233
Iteration 24, loss = 0.01872063
Iteration 25, loss = 0.01361165
Iteration 26, loss = 0.01045963
Iteration 27, loss = 0.00855691
Iteration 28, loss = 0.00751814
Iteration 29, loss = 0.00621460
Iteration 30, loss = 0.00510116
Iteration 31, loss = 0.00462611
Iteration 32, loss = 0.00421034
Iteration 33, loss = 0.00351637
Iteration 34, loss = 0.00322533
Iteration 35, loss = 0.00264344
Iteration 36, loss = 0.00241634
Iteration 37, loss = 0.00223854
Iteration 38, loss = 0.00218337
Iteration 39, loss = 0.00190175
Iteration 40, loss = 0.00172835
Iteration 41, loss = 0.00167255
Iteration 42, loss = 0.00152440
Iteration 43, loss = 0.00171780
Iteration 44, loss = 0.00133137
Iteration 45, loss = 0.00116314
Iteration 46, loss = 0.00112198
Iteration 47, loss = 0.00107562
Iteration 48, loss = 0.77095920
Iteration 49, loss = 0.08349379
Iteration 50, loss = 0.03974592
Iteration 51, loss = 0.03026192
Iteration 52, loss = 0.02057078
Iteration 53, loss = 0.01623949
Iteration 54, loss = 0.01218967
Iteration 55, loss = 0.00998020
Iteration 56, loss = 0.00945859
Training loss did not improve more than tol=0.000100 for 10 consecutive epochs. Stoppin
g.
```

Out[ ]:
```
MLPClassifier(hidden_layer_sizes=(512, 256), verbose=True)
```

In [ ]:
```
y_pred_ANN = mlp.predict(x_test)
```

In [ ]:
```
accuracy_score(y_test, y_pred_ANN)
```

Out[ ]:
```
0.7721695482431679
```

Method 1: Image Centroids

In [ ]:

```
df_train_A = df_train.loc[df_train['label'] == 0]
```

In [ ]:

```
df_train.loc[2]
```

Out[ ]:

```
label         2
pixel1      187
pixel2      188
pixel3      188
pixel4      187
           ...
pixel780    199
pixel781    198
pixel782    195
pixel783    194
pixel784    195
Name: 2, Length: 785, dtype: int64
```

In [ ]:

```
df_train_A.mean().astype(int)
```

Out[ ]:

```
label         0
pixel1      164
pixel2      165
pixel3      162
pixel4      161
           ...
pixel780    184
pixel781    182
pixel782    182
pixel783    178
pixel784    174
Length: 785, dtype: int32
```

The getImage started only working for means when you also convert to type int. I swear it wasn't like this before and I don't know what changed

In [ ]:

```
getImage(df_train_A.mean().astype(int))
```

```
label         0
pixel1      164
pixel2      165
pixel3      162
pixel4      161
           ...
pixel780    184
pixel781    182
pixel782    182
pixel783    178
pixel784    174
Length: 785, dtype: int32
0
Sign:  A
```

In [ ]:

```
sign_means = []
```

In [ ]:

```
getImageFromTest(1)
```

```
Sign:  F
```

In [ ]:

```
df_train.shape
```

Out[ ]:

```
(27455, 785)
```

In [ ]:

```
true = df_test.iloc[:,0].tolist()
```

In [ ]:

```
true = [sign_map[k] for k in true]
```

In [ ]:

```
df_train.iloc[:,1:]
```

Out[ ]:

	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	pixel10	...	pixel775	pixel7
0	107	118	127	134	139	143	146	150	153	156	...	207	2
1	155	157	156	156	156	157	156	158	158	157	...	69	1
2	187	188	188	187	187	186	187	188	187	186	...	202	2
3	211	211	212	212	211	210	211	210	210	211	...	235	2
4	164	167	170	172	176	179	180	184	185	186	...	92	1
...	...	...	...	...	...	...	...	...	...	...	...	...	
27450	189	189	190	190	192	193	193	193	193	194	...	132	1
27451	151	154	157	158	160	161	163	164	166	167	...	198	1
27452	174	174	174	174	174	175	175	174	173	173	...	121	1
27453	177	181	184	185	187	189	190	191	191	190	...	119	
27454	179	180	180	180	182	181	182	183	182	182	...	108	1

27455 rows × 784 columns

Initial, inefficient implementation

In [ ]:

```
class CentroidCompOld:
  def __init__(self):
    self.sign_means = []
    self.signs = 26

  def train(self,X):
    for i in range(0,26):
      self.sign_means.append(X.loc[X['label'] == i].mean())

  def get_centroids(self):
    return self.sign_means
```

```
  def predict(self,X):
    y_pred = []
    for index, row in tqdm(X.iterrows(), total=X.shape[0]):
        temp = math.inf
        min_index_index = 0
        for i in range(0,26):
          diff = np.linalg.norm(self.sign_means[i][1:] - X.loc[index][1:])
          if (diff < temp):
            min_index_index = i
            temp = diff
        y_pred.append(sign_map[min_index_index])
    return y_pred
```

More efficient implementation. Reading 6 on clustering methods gave us a good set of measures to implement to compare the class means and test data.

In [ ]:

```
class CentroidComp:
  def __init__(self,method="euclid", verbose=False, debug=False):
    self.sign_means = None
    self.method = method
    self.verbose = verbose
    if self.verbose or self.debug:
      print("Initialized centroid class with method " + self.method)

  def train(self,X):
    self.sign_means = X.groupby('label').mean().to_numpy()
    if self.verbose or self.debug:
      print("Trained model")

  def get_centroids(self):
    return self.sign_means

  def predict(self,X):
    y_pred = []
    if self.verbose or self.debug:
      print("Predicting model with " + self.method)

    if (self.method == "euclid"):
      for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmax(np.dot(self.sign_means,rowi) / (np.lin

    elif (self.method == "cos"):
      for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmin(np.linalg.norm(self.sign_means - rowi,

    elif (self.method == "man"):
      for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        y_pred.append(reduced_sign_map[np.argmin(np.sum(np.abs(self.sign_means - rowi),

    elif (self.method == "cor"):
      for i in range(len(X)):
        rowi = X.iloc[:,1:].loc[i].to_numpy()
        corr = np.corrcoef(self.sign_means, rowi)
        coefs = corr[:-1, -1]
        y_pred.append(reduced_sign_map[np.argmax(coefs)])
```

```
    else:
      return "you messed up"

    return y_pred
```

In [ ]:

```
euclidModel = CentroidComp(method="euclid", verbose=True)
euclidModel.train(X = df_train)
y_pred_euclid = euclidModel.predict(df_test)
print(classification_report(y_pred_euclid, true))
```

```
Initialized centroid class with method euclid
Trained model
Predicting model with euclid
              precision    recall  f1-score   support

           A       0.76      0.56      0.65       448
           B       0.66      0.93      0.77       304
           C       0.61      0.67      0.64       281
           D       0.44      0.47      0.46       232
           E       0.66      0.76      0.71       435
           F       0.56      0.49      0.52       284
           G       0.48      0.51      0.49       329
           H       0.58      0.83      0.68       304
           I       0.24      0.29      0.27       240
           K       0.53      0.30      0.39       575
           L       0.66      0.63      0.64       218
           M       0.18      0.36      0.24       199
           N       0.25      0.37      0.30       196
           O       0.60      0.51      0.55       289
           P       0.82      0.49      0.62       577
           Q       0.83      0.75      0.79       182
           R       0.41      0.31      0.36       188
           S       0.17      0.10      0.13       398
           T       0.50      0.31      0.38       401
           U       0.14      0.18      0.15       200
           V       0.30      0.38      0.34       279
           W       0.28      0.25      0.27       228
           X       0.54      0.55      0.54       262
           Y       0.18      0.50      0.27       123

    accuracy                           0.48      7172
   macro avg       0.47      0.48      0.46      7172
weighted avg       0.51      0.48      0.48      7172
```

In [ ]:

```
cosModel = CentroidComp(method="cos", verbose=True)
cosModel.train(X = df_train)
y_pred_cos = cosModel.predict(df_test)
print(classification_report(y_pred_cos, true))
```

```
Initialized centroid class with method cos
Trained model
Predicting model with cos
              precision    recall  f1-score   support

           A       0.56      0.58      0.57       323
           B       0.47      0.93      0.63       220
           C       0.53      0.67      0.59       244
           D       0.33      0.53      0.41       150
           E       0.51      0.63      0.56       407
           F       0.30      0.37      0.33       195
```

```
           G       0.43      0.52      0.47       290
           H       0.42      0.77      0.54       237
           I       0.36      0.16      0.23       640
           K       0.40      0.26      0.32       499
           L       0.36      0.66      0.46       114
           M       0.11      0.32      0.17       137
           N       0.19      0.42      0.26       132
           O       0.40      0.43      0.42       229
           P       0.80      0.39      0.53       712
           Q       0.58      0.17      0.26       560
           R       0.13      0.15      0.14       129
           S       0.13      0.10      0.11       296
           T       0.53      0.21      0.30       613
           U       0.05      0.11      0.07       116
           V       0.37      0.30      0.33       437
           W       0.17      0.17      0.17       201
           X       0.42      0.57      0.48       199
           Y       0.14      0.52      0.23        92

    accuracy                           0.38      7172
   macro avg       0.36      0.41      0.36      7172
weighted avg       0.44      0.38      0.37      7172
```

In [ ]:

```
manModel = CentroidComp(method="man", verbose=True)
manModel.train(X = df_train)
y_pred_man = manModel.predict(df_test)
print(classification_report(y_pred_man, true))
```

```
Initialized centroid class with method man
Trained model
Predicting model with man
              precision    recall  f1-score   support

           A       0.49      0.55      0.51       295
           B       0.31      0.85      0.46       160
           C       0.56      0.65      0.61       268
           D       0.25      0.63      0.36        99
           E       0.47      0.44      0.45       541
           F       0.27      0.25      0.26       263
           G       0.39      0.41      0.40       333
           H       0.33      0.78      0.46       182
           I       0.40      0.14      0.21       789
           K       0.39      0.22      0.28       584
           L       0.33      0.52      0.40       132
           M       0.09      0.41      0.15        85
           N       0.12      0.40      0.18        88
           O       0.36      0.37      0.36       238
           P       0.76      0.42      0.54       628
           Q       0.51      0.20      0.29       416
           R       0.09      0.09      0.09       147
           S       0.10      0.10      0.10       245
           T       0.54      0.17      0.26       801
           U       0.08      0.13      0.10       150
           V       0.24      0.39      0.30       213
           W       0.14      0.11      0.12       276
           X       0.34      0.52      0.41       177
           Y       0.13      0.69      0.22        62

    accuracy                           0.33      7172
   macro avg       0.32      0.39      0.31      7172
weighted avg       0.41      0.33      0.33      7172
```

In [ ]:

```
corModel = CentroidComp(method="cor", verbose=True)
corModel.train(X = df_train)
y_pred_cor = corModel.predict(df_test)
print(classification_report(y_pred_cor, true))
```

```
Initialized centroid class with method cor
Trained model
Predicting model with cor
              precision    recall  f1-score   support

           A       0.88      0.58      0.70       502
           B       0.66      0.84      0.74       340
           C       0.60      0.50      0.54       369
           D       0.51      0.43      0.47       287
           E       0.71      0.77      0.74       461
           F       0.58      0.41      0.48       349
           G       0.53      0.42      0.47       436
           H       0.52      0.78      0.62       287
           I       0.21      0.32      0.25       191
           K       0.56      0.31      0.40       603
           L       0.30      0.72      0.42        88
           M       0.10      0.24      0.14       165
           N       0.29      0.44      0.35       193
           O       0.51      0.35      0.42       355
           P       0.93      0.52      0.67       614
           Q       0.74      0.60      0.66       203
           R       0.28      0.33      0.30       120
           S       0.34      0.21      0.26       397
           T       0.50      0.37      0.43       334
           U       0.08      0.13      0.10       162
           V       0.30      0.61      0.40       169
           W       0.31      0.35      0.33       179
           X       0.61      0.57      0.59       285
           Y       0.13      0.51      0.20        83

    accuracy                           0.48      7172
   macro avg       0.46      0.47      0.45      7172
weighted avg       0.55      0.48      0.49      7172
```

Euclid distance and pearson correlation are the best measures at nearly 50% accuracy.

Method 2: SVD Bases

General

Training: For the training set of known signs, compute the SVD of each set of signs of one kind.

Classification: For a given test sign, compute its relative residual in all 10 bases. If one residual is significantly smaller than all others, classifiy aas that one.

Linear Algebra

Training: Compute SVDs of 10 matrices of dimenion m^2 x n_i. Each digit is an mxm digitized image. n_i is the number of training signs i.

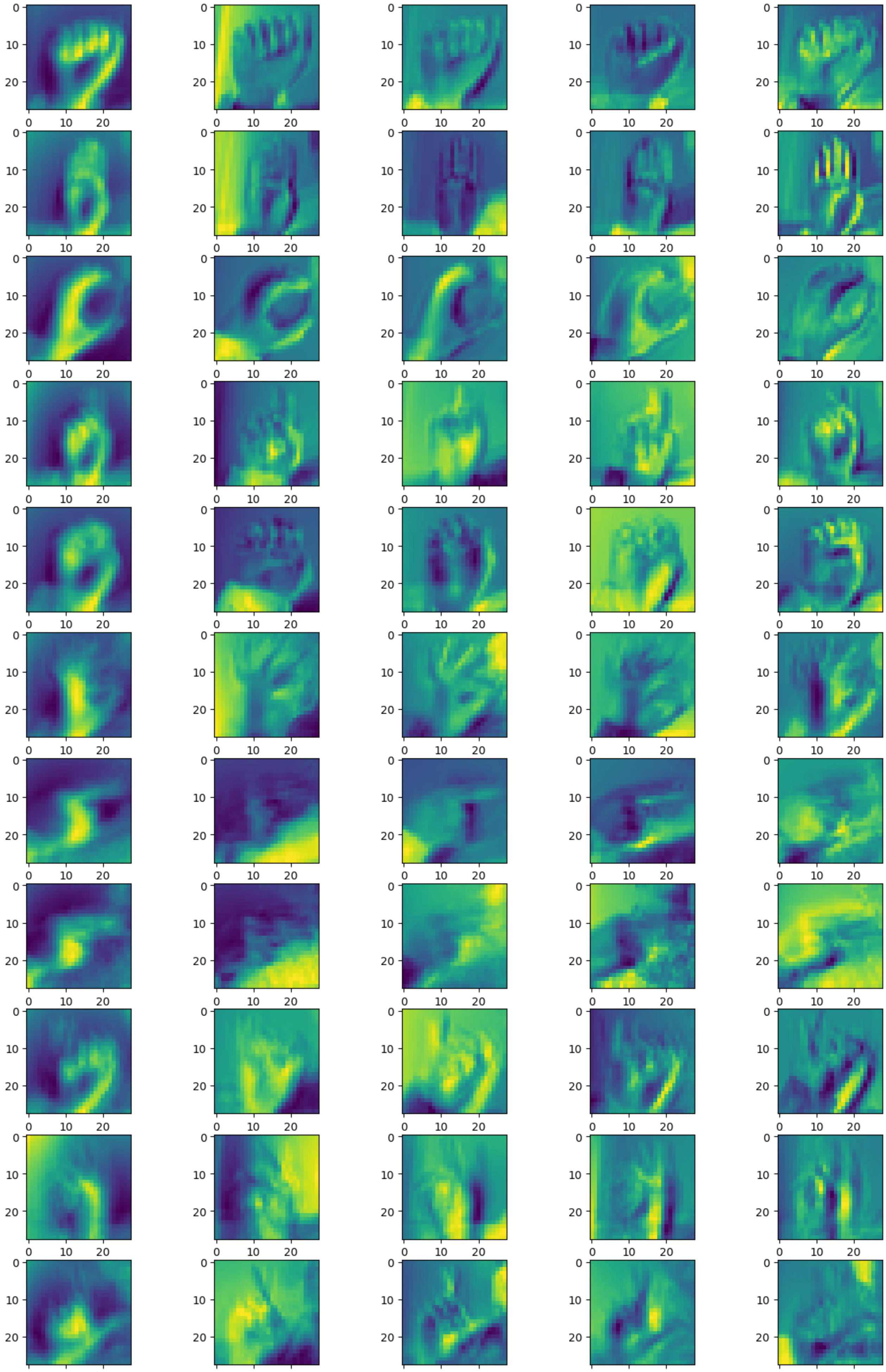
Classification: Compute 10 least squares residuals

In [ ]:

```
train_SVDs = []
letters = df_train.groupby('label')
for name,group in letters:
  letter = group.drop('label', axis=1).values
  train_SVDs.append(np.linalg.svd(letter))
```

In [ ]:

```
fig = plt.figure(1, figsize = [15, 50], dpi = 100)
c = 1
for j in range(len(reduced_sign_map)):
  for i in range(0,5):
    plt.subplot(24,5,c)
    plt.imshow(train_SVDs[j][2][i:i+1,:].reshape(28,28))
    c = c+1
fig.subplots_adjust(wspace=0.1)
plt.show()
```



In [ ]:

```
# Randomized SVD as provided in notebook 12
def rSVD(X,r,q,p):
  # Step 1: Sample column space of X with P matrix
  ny = X.shape[1]
  P = np.random.randn(ny,r+p) # Gaussian Random Matrix
  Z = X @ P
  for k in range(q): # Power iteration
      Z = X @ (X.T @ Z)

  Q, R = np.linalg.qr(Z,mode='reduced')

  # Step 2: Compute SVD on projected Y = Q.T @ X
  Y = Q.T @ X
  UY, S, VT = np.linalg.svd(Y,full_matrices=0)
  U = Q @ UY

  return U, S, VT
```

In [ ]:

```
# from concurrent.futures import ThreadPoolExecutor, as_completed
# import threading

class SVDBases:
    def __init__(self, bases=5, method='SVD', verbose=False):
        self.verbose = verbose
        self.bases = bases
        self.method = method
        self.SVDs = []
        if self.verbose:
            print("Initialized SVD Bases class")

    def train(self,X):
        letters = X.groupby('label')
        for name,group in letters:
            letter = group.drop('label', axis=1).values
            if (self.method=='SVD'):
                U, s, Vt = np.linalg.svd(letter.T)
                self.SVDs.append(U)
            elif (self.method=='rSVD'):
                U, s, Vt = randomized_svd(letter.T, n_components=10, random_state=0)
                self.SVDs.append(U)
            else:
                print('Something went wrong')

    def predict(self, X):
        y_pred = []
        I = np.eye(784)
        uTu = np.array([np.dot(u[:, :self.bases], u[:, :self.bases].T) for u in self.SV

        for i in tqdm(range(len(X))):
            z = X.iloc[:,1:].loc[i].to_numpy()

            y_pred.append(reduced_sign_map[np.argmin(np.linalg.norm(np.dot((I-uTu),z),

            # y_pred.append(reduced_sign_map[np.argmax(np.dot(self.sign_means,rowi) / (
            '''
            min=maxsize
            index = 0
```

```
            for i in range(len(reduced_sign_map)):
                print(np.dot(self.SVDs[i][:, :self.bases], self.SVDs[i][:, :self.bases]
                bases = (self.SVDs[i][:, :self.bases])
                diff = np.linalg.norm(np.dot(I-np.dot(bases, bases.T), X.iloc[j, 1:].to
                if (diff < min):
                    min = diff
                    index = i
                break
            break
            y_pred.append(reduced_sign_map[index])
            '''

        return y_pred
```

In [ ]:

```
SVDmodelB5 = SVDBases(verbose=True, method='SVD', bases=5)
SVDmodelB5.train(df_train)
y_predB5 = SVDmodelB5.predict(df_test)
accuracy_score(y_predB5, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:03<00:00, 23.66it/s]
```

Out[ ]:

```
0.6441717791411042
```

In [ ]:

```
print(classification_report(y_predB5, true))
```

	precision	recall	f1-score	support
A	0.92	0.72	0.81	426
B	0.83	0.75	0.79	474
C	0.85	0.87	0.86	305
D	0.67	0.58	0.62	282
E	0.74	0.77	0.75	476
F	0.75	0.81	0.78	228
G	0.72	0.92	0.81	274
H	0.85	0.86	0.85	429
I	0.64	0.70	0.67	262
K	0.44	0.49	0.46	298
L	0.95	0.98	0.96	204
M	0.40	0.53	0.46	298
N	0.26	0.34	0.30	222
O	0.76	0.78	0.77	238
P	0.93	1.00	0.96	322
Q	0.73	0.94	0.82	128
R	0.15	0.07	0.09	321
S	0.71	0.37	0.49	468
T	0.71	0.65	0.68	272
U	0.37	0.29	0.33	337
V	0.32	0.78	0.45	139
W	0.32	0.23	0.27	281
X	0.92	0.61	0.73	405
Y	0.23	0.94	0.38	83
accuracy			0.64	7172
macro avg	0.63	0.67	0.63	7172
weighted avg	0.67	0.64	0.64	7172

In [ ]:

```
SVDmodelB10 = SVDBases(verbose=True, method='SVD', bases=10)
SVDmodelB10.train(df_train)
```

```
y_predB10 = SVDmodelB10.predict(df_test)
accuracy_score(y_predB10, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:04<00:00, 23.56it/s]
```

Out[ ]: 0.8064696040156163

In [ ]:

```
print(classification_report(y_predB10, true))
```

```
              precision    recall  f1-score   support

           A       1.00      0.91      0.95       363
           B       0.85      0.88      0.87       420
           C       1.00      0.86      0.92       361
           D       1.00      0.78      0.88       315
           E       0.92      0.93      0.92       490
           F       0.77      0.96      0.86       199
           G       0.85      0.89      0.87       334
           H       0.91      0.92      0.92       430
           I       0.95      0.93      0.94       294
           K       0.76      0.74      0.75       341
           L       0.85      1.00      0.92       178
           M       0.70      0.77      0.73       358
           N       0.63      0.83      0.72       223
           O       0.89      0.93      0.91       237
           P       1.00      1.00      1.00       347
           Q       0.99      1.00      1.00       163
           R       0.29      0.25      0.27       167
           S       0.98      0.61      0.75       396
           T       0.68      0.80      0.73       210
           U       0.53      0.39      0.45       357
           V       0.43      0.59      0.50       250
           W       0.53      0.47      0.50       230
           X       0.87      0.79      0.83       295
           Y       0.64      1.00      0.78       214

    accuracy                           0.81      7172
   macro avg       0.79      0.80      0.79      7172
weighted avg       0.82      0.81      0.81      7172
```

In [ ]:

```
SVDmodelB15 = SVDBases(verbose=True, method='SVD', bases=15)
SVDmodelB15.train(df_train)
y_predB15 = SVDmodelB15.predict(df_test)
accuracy_score(y_predB15, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:13<00:00, 22.88it/s]
```

Out[ ]: 0.8339375348577802

In [ ]:

```
print(classification_report(y_predB15, true))
```

```
              precision    recall  f1-score   support

           A       1.00      0.92      0.96       358
           B       0.90      0.96      0.93       405
           C       1.00      0.93      0.96       335
           D       1.00      0.83      0.91       295
           E       0.96      0.89      0.92       536
```

```
           F       0.76      0.82      0.79       229
           G       0.88      0.94      0.91       327
           H       0.90      0.95      0.92       413
           I       0.92      0.87      0.89       307
           K       0.79      0.76      0.78       347
           L       0.98      1.00      0.99       204
           M       0.64      0.86      0.74       294
           N       0.78      0.91      0.84       247
           O       0.80      0.84      0.82       237
           P       1.00      1.00      1.00       347
           Q       1.00      0.98      0.99       167
           R       0.42      0.36      0.39       169
           S       0.93      0.61      0.74       375
           T       0.75      0.82      0.78       228
           U       0.53      0.56      0.54       250
           V       0.55      0.66      0.60       286
           W       0.64      0.53      0.58       250
           X       0.87      0.74      0.80       314
           Y       0.76      1.00      0.86       252

    accuracy                           0.83      7172
   macro avg       0.82      0.82      0.82      7172
weighted avg       0.84      0.83      0.83      7172
```

In [ ]:

```python
SVDmodelB20 = SVDBases(verbose=True, method='SVD', bases=20)
SVDmodelB20.train(df_train)
y_predB20 = SVDmodelB20.predict(df_test)
accuracy_score(y_predB20, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:02<00:00, 23.71it/s]
```

Out[ ]:

```
0.8453708867819297
```

In [ ]:

```python
print(classification_report(y_predB20, true))
```

```
              precision    recall  f1-score   support

           A       1.00      0.94      0.97       354
           B       0.90      0.92      0.91       423
           C       1.00      0.94      0.97       330
           D       1.00      0.80      0.89       305
           E       0.99      0.90      0.95       549
           F       0.85      0.92      0.88       229
           G       0.94      0.94      0.94       347
           H       0.90      0.99      0.95       396
           I       0.92      0.88      0.90       303
           K       0.82      0.82      0.82       331
           L       0.97      1.00      0.98       202
           M       0.68      0.91      0.78       294
           N       0.84      0.91      0.88       268
           O       0.83      0.83      0.83       247
           P       1.00      1.00      1.00       347
           Q       1.00      0.97      0.98       169
           R       0.58      0.34      0.43       244
           S       0.95      0.71      0.81       326
           T       0.75      0.82      0.78       226
           U       0.51      0.62      0.56       216
           V       0.49      0.67      0.57       251
           W       0.63      0.47      0.54       274
           X       0.84      0.72      0.78       309
```

```
           Y       0.70      1.00      0.82       232

    accuracy                           0.85      7172
   macro avg       0.84      0.84      0.83      7172
weighted avg       0.86      0.85      0.84      7172
```

In [ ]:

```python
SVDmodelB25 = SVDBases(verbose=True, method='SVD', bases=25)
SVDmodelB25.train(df_train)
y_predB25 = SVDmodelB25.predict(df_test)
accuracy_score(y_predB25, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:08<00:00, 23.24it/s]
```

Out[ ]:

```
0.8441160066926938
```

In [ ]:

```python
print(classification_report(y_predB25, true))
```

```
              precision    recall  f1-score   support

           A       1.00      0.93      0.96       357
           B       0.90      0.97      0.94       401
           C       1.00      0.94      0.97       330
           D       1.00      0.75      0.86       327
           E       0.96      0.90      0.93       528
           F       0.83      0.92      0.87       223
           G       0.95      0.94      0.94       349
           H       0.95      1.00      0.98       416
           I       0.92      0.94      0.93       281
           K       0.83      0.83      0.83       330
           L       0.93      1.00      0.96       194
           M       0.68      0.86      0.76       310
           N       0.77      0.91      0.84       245
           O       0.85      0.91      0.88       229
           P       1.00      1.00      1.00       347
           Q       1.00      0.87      0.93       188
           R       0.62      0.34      0.44       268
           S       1.00      0.72      0.84       341
           T       0.75      0.86      0.80       216
           U       0.52      0.61      0.56       228
           V       0.47      0.65      0.55       253
           W       0.67      0.52      0.58       264
           X       0.81      0.69      0.74       313
           Y       0.70      0.99      0.82       234

    accuracy                           0.84      7172
   macro avg       0.84      0.84      0.83      7172
weighted avg       0.86      0.84      0.84      7172
```

In [ ]:

```python
SVDmodelB30 = SVDBases(verbose=True, method='SVD', bases=30)
SVDmodelB30.train(df_train)
y_predB30 = SVDmodelB30.predict(df_test)
accuracy_score(y_predB30, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [05:18<00:00, 22.52it/s]
```

Out[ ]:

```
0.8326826547685443
```

In [ ]:

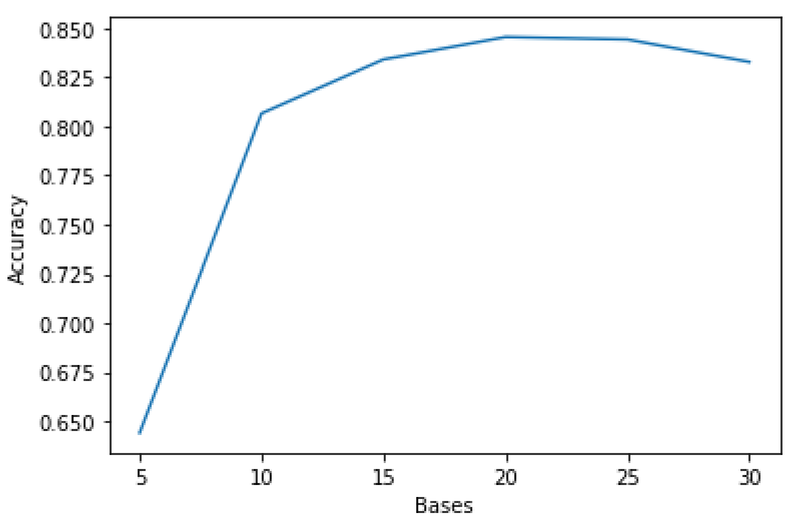
```
print(classification_report(y_predB30, true))
```

```
              precision    recall  f1-score   support

           A       1.00      0.94      0.97       351
           B       0.90      0.98      0.94       397
           C       1.00      0.94      0.97       330
           D       1.00      0.75      0.86       328
           E       0.96      0.92      0.94       518
           F       0.83      0.92      0.87       224
           G       0.95      0.94      0.94       349
           H       0.94      1.00      0.97       412
           I       0.86      1.00      0.92       249
           K       0.84      0.85      0.84       327
           L       0.94      1.00      0.97       197
           M       0.68      0.86      0.76       310
           N       0.70      0.91      0.79       224
           O       0.84      0.90      0.87       229
           P       1.00      1.00      1.00       347
           Q       1.00      0.82      0.90       200
           R       0.40      0.21      0.28       270
           S       1.00      0.62      0.77       394
           T       0.75      0.78      0.77       237
           U       0.53      0.60      0.57       236
           V       0.47      0.62      0.54       261
           W       0.62      0.51      0.56       250
           X       0.78      0.69      0.74       301
           Y       0.70      1.00      0.82       231

    accuracy                           0.83      7172
   macro avg       0.82      0.82      0.81      7172
weighted avg       0.84      0.83      0.83      7172
```

In [ ]:

```
x = [5,10,15,20,25,30]
y = [0.6441717791411042, 0.8064696040156163,0.8339375348577802,0.8453708867819297,0.844
plt.plot(x,y)
plt.xlabel("Bases")
plt.ylabel("Accuracy")
```

Out[ ]: Text(0, 0.5, 'Accuracy')



The best accuracy is achieved with around 20 bases, 20/784=~2.5% of the total bases.

Method 3: Eigensigns

In [ ]:

```
sample_size = 300
train = df_train.values[:,1:]
train = train[:sample_size,]
test = df_test.values[:,1:]
test = test[:200,]
# Label at index 0
# 784 pixels = 28 x 28
# print(first.reshape(28,28))
first = train[6]
# plt.imshow(first.reshape(28,28), cmap="gray")
labels = df_train.values[:,0]
print(np.sort(labels))
# sign_train_mean = train.mean(axis=0)
# sign_test_mean = test.mean(axis=0)
# plt.imshow(sign_test_mean.reshape(28,28), cmap="gray")
```

```
[ 0  0  0 ... 24 24 24]
```

In [ ]:

```
train_mean = train.mean(axis=0)
train_pca = np.subtract(train, train_mean)

example = train_pca[6]
r = np.asarray(example).reshape(28,28)
# plt.imshow(r, cmap="gray")
```

In [ ]:

```
train_pca_t = np.transpose(train_pca)
# Y * Y_t / size_train = the covariance matrix
yy_t = np.dot(train_pca, train_pca_t)
n_train, _ = train.shape
cov = np.divide(yy_t, n_train)
```

In [ ]:

```
eigenvalues, eigenvectors = np.linalg.eig(cov)
```

In [ ]:

```
# Top K = 150 eigensigns computed
K = 150

eigenvalues_index_sorted = np.argsort(eigenvalues)[::-1]
eigenvalues_sorted       = eigenvalues[eigenvalues_index_sorted][0:K]
# eigenvectors_sorted    = eigenvectors[:, eigenvalues_index_sorted]
eigenvectors_sorted      = eigenvectors[:,eigenvalues_index_sorted][:,0:K]
```

In [ ]:

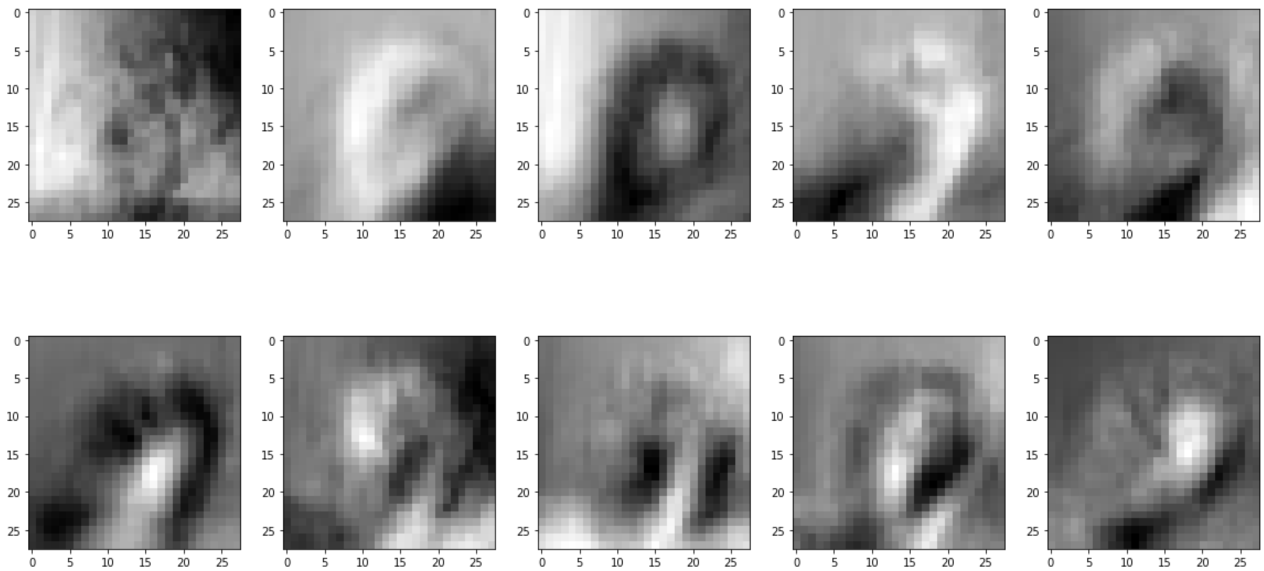
```
eigensigns = np.dot(train_pca_t, eigenvectors_sorted)
```

In [ ]:

```
eigensigns_normal = eigensigns / np.linalg.norm(eigensigns, axis=0)
```

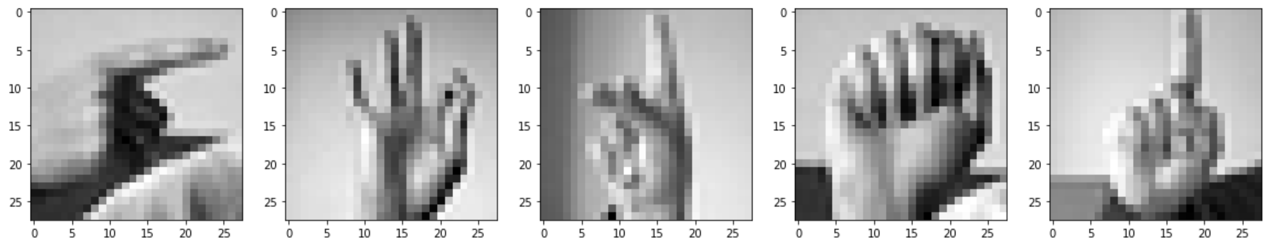
In [ ]:

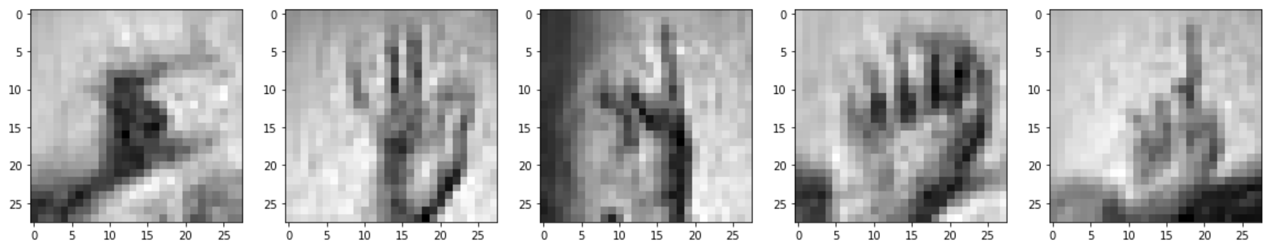
```
fig = plt.figure(figsize=(20, 10))
for i in range(10):
    v = eigensigns_normal[:, i]
    r = np.asarray(v).reshape(28, 28)
    fig.add_subplot(2, 5, i + 1)
    plt.imshow(r, cmap='gray')

plt.savefig(f"eigensign_figures/K={K}_eigensign.pdf")
```



In [ ]:

```
eigensigns_normal_k_t = np.transpose(eigensigns_normal)

# test_t       = test          - train_mean
# test_t_e     = test_t        @ eigensigns_normal
# test_t_e_et = test_t_e      @ eigensigns_normal_k_t
# test_final   = test_t_e_et   + train_mean

test_final = (((test - train_mean) @ eigensigns_normal) @ eigensigns_normal_k_t) + trai

fig_r = plt.figure(figsize=(20, 10))
i = 1

for v in test:
    r = np.asarray(v).reshape(28, 28)
    fig_r.add_subplot(2, 5, i)
    i += 1
    plt.imshow(r, cmap="gray")
    if i > 5:
        break

for v in test_final:
    r = np.asarray(v).reshape(28, 28)
    fig_r.add_subplot(2, 5, i)
    i += 1
    plt.imshow(r, cmap="gray")
    if i > 10:
        break

plt.savefig(f"eigensign_figures/K={K}_test_vs_reconstructed.pdf")
```



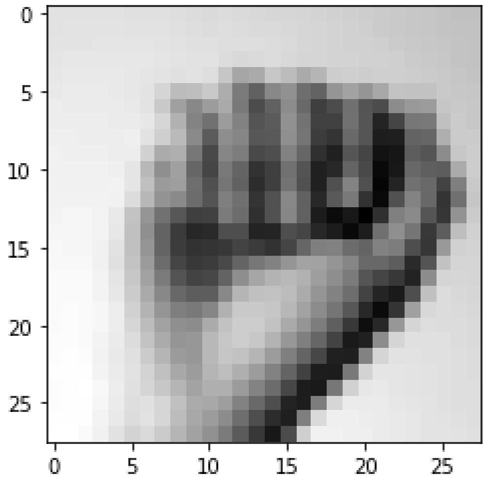


In [ ]:

```python
test_image_A = Image.open("test_images/A.jpg")
test_image_B = Image.open("test_images/B.jpg")
test_image_L = Image.open("test_images/L.jpg")

test_image_A = np.array(test_image_A.convert("L"))
test_image_B = np.array(test_image_B.convert("L"))
test_image_L = np.array(test_image_L.convert("L"))


test_image_A_1D = test_image_A.ravel()
test_image_B_1D = test_image_B.ravel()
test_image_L_1D = test_image_L.ravel()

def ops(test_img):
    return (((test_img - train_mean) @ eigensigns_normal) @ eigensigns_normal_k_t) + tr

test_image_A_final = ops(test_image_A_1D)
test_image_B_final = ops(test_image_B_1D)
test_image_L_final = ops(test_image_L_1D)


distances = np.sqrt(np.sum((df_test.values[:,1:] - test_image_A_final)**2, axis=1))

# Find the index of the row with the smallest distance
min_idx = np.argmin(distances)

closest_image = df_test.values[:,1:][min_idx].reshape((28, 28))

print("Label: ", df_test.values[:,0][min_idx])
# plt.imshow(test_image_A_final.reshape(28,28), cmap="gray")
# plt.imshow(test_image_B_final.reshape(28,28), cmap="gray")
# plt.imshow(test_image_L_final.reshape(28,28), cmap="gray")
plt.imshow(closest_image, cmap="gray")

# for t in test_final:
#     print(t.shape)
#     break
```

```
Label:  0
```

Out[ ]: `<matplotlib.image.AxesImage at 0x1b0763fb320>`



Using existing packages to accomplish classification

In [ ]:

```
eigensigns_X = df_train.drop("label", axis=1).values
eigensigns_y = df_train["label"].values
```

In [ ]:

```
# X_train, X_test, y_train, y_test = train_test_split(eigensigns_X, eigensigns_y)
X_train = df_train.drop('label',axis=1).values
y_train = df_train['label'].values
X_test = df_test.drop('label',axis=1).values
y_test = df_test['label'].values
eigensigns_pca = PCA(n_components=50).fit(X_train) # 672 in total
```

In [ ]:

```
X_train_pca = eigensigns_pca.transform(X_train)
```

In [ ]:

```
classifier = SVC().fit(X_train_pca, y_train)
```

In [ ]:

```
X_test_pca = eigensigns_pca.transform(X_test)
predictions = classifier.predict(X_test_pca)
```

In [ ]:

```
print(classification_report(y_test, predictions))
```

```
              precision    recall  f1-score   support

           0       0.94      1.00      0.97       331
           1       1.00      1.00      1.00       432
           2       0.87      0.99      0.93       310
           3       0.91      1.00      0.95       245
           4       0.96      1.00      0.98       498
           5       0.89      0.89      0.89       247
           6       0.95      0.90      0.93       348
           7       0.97      0.96      0.96       436
           8       0.78      0.91      0.84       288
          10       0.84      0.63      0.72       331
          11       0.88      1.00      0.94       209
          12       0.91      0.77      0.84       394
          13       0.90      0.75      0.82       291
          14       0.99      0.93      0.96       246
```

```
          15       1.00      1.00      1.00       347
          16       0.95      1.00      0.98       164
          17       0.40      0.67      0.50       144
          18       0.77      0.78      0.78       246
          19       0.87      0.69      0.77       248
          20       0.72      0.74      0.73       266
          21       0.83      0.65      0.73       346
          22       0.58      0.88      0.70       206
          23       0.83      0.83      0.83       267
          24       0.92      0.76      0.83       332

    accuracy                           0.87      7172
   macro avg       0.86      0.86      0.86      7172
weighted avg       0.88      0.87      0.87      7172
```

Method 4: Smoothing

At the end of chapter 10, Elden recommends image smoothing as a way to improve performance. We attempted to apply some smoothing methods to improve the performance of the models we've made.

In [ ]:

```
images = df_train.iloc[:,1:].values.reshape(-1, 28, 28)
```

In [ ]:

```
filtered_images = np.zeros_like(images)
for i in range(len(images)):
    filtered_images[i] = gaussian_filter(images[i], sigma=1)
```

In [ ]:

```
filtered_data = filtered_images.reshape(-1, 784)
```

In [ ]:

```
filtered_df_train = pd.DataFrame(filtered_data)
```

In [ ]:

```
filtered_df_train.insert(0, 'label', df_train['label'])
```

In [ ]:

```
filtered_df_train.columns
```

Out[ ]:

```
Index(['label',       0,       1,       2,       3,       4,       5,       6,
             7,       8,
       ...
           774,     775,     776,     777,     778,     779,     780,     781,
           782,     783],
      dtype='object', length=785)
```

In [ ]:

```
getImage(filtered_df_train.loc[1])
```

```
label      6
0        156
1        156
2        156
3        157
        ...
779      131
```

```
780      134
781      123
782      122
783      129
Name: 1, Length: 785, dtype: int64
6
Sign:  G
```

In [ ]:

```
model3 = CentroidComp(method="cor", verbose=True)
```

```
Initialized centroid class with method cor
```

In [ ]:

```
model3.train(X = filtered_df_train)
```

```
Trained model
```

In [ ]:

```
y_pred3 = model3.predict(df_test)
```

```
Predicting model with cor
```

In [ ]:

```
accuracy_score(y_pred3, true)
```

Out[ ]:

```
0.45984383714445065
```

In [ ]:

```
def smooth_row(row):
    smoothed_row = gaussian_filter1d(row, sigma=1)
    return pd.Series(smoothed_row, index=row.index)
```

In [ ]:

```
smoothed_df_train = df_train.apply(smooth_row, axis=1)
```

In [ ]:

```
smoothed_df_train['label'] = df_train['label']
```

In [ ]:

```
smoothed_df_train
```

Out[ ]:

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776
0	3	80	111	125	133	138	142	146	149	152	...	206	206
1	6	111	147	155	156	156	156	156	157	157	...	99	117
2	2	132	176	186	187	186	186	186	187	186	...	202	201
3	2	149	199	210	211	211	210	210	210	210	...	234	233
4	13	120	158	169	172	175	178	180	183	184	...	91	100
...	...	...	...	...	...	...	...	...	...	...	...	...	..
27450	13	136	179	189	190	191	192	192	193	193	...	127	131
27451	23	114	146	155	158	159	161	162	164	165	...	198	198
27452	18	127	164	173	174	174	174	174	173	173	...	132	174
27453	17	131	171	182	185	186	188	189	190	190	...	108	70

	label	pixel1	pixel2	pixel3	pixel4	pixel5	pixel6	pixel7	pixel8	pixel9	...	pixel775	pixel776
27454	23	133	170	179	180	181	181	181	182	182	...	115	137

27455 rows × 785 columns

In [ ]:
```
getImage(smoothed_df_train.loc[0])
```

```
label         3
pixel1       80
pixel2      111
pixel3      125
pixel4      133
           ...
pixel780    205
pixel781    205
pixel782    204
pixel783    203
pixel784    202
Name: 0, Length: 785, dtype: int64
3
Sign:   D
```

In [ ]:
```
SVDmodelB20 = SVDBases(verbose=True, method='SVD', bases=20)
SVDmodelB20.train(smoothed_df_train)
y_predB20 = SVDmodelB20.predict(df_test)
accuracy_score(y_predB20, true)
```

```
Initialized SVD Bases class
100%|██████████| 7172/7172 [04:53<00:00, 24.46it/s]
```

Out[ ]:
```
0.8646123814835471
```

In [ ]:
```
eigensigns_X = smoothed_df_train.drop("label", axis=1).values
eigensigns_y = smoothed_df_train["label"].values
```

In [ ]:
```
# X_train, X_test, y_train, y_test = train_test_split(eigensigns_X, eigensigns_y)
X_train = smoothed_df_train.drop('label',axis=1).values
y_train = smoothed_df_train['label'].values
X_test = df_test.drop('label',axis=1).values
y_test = df_test['label'].values
eigensigns_pca = PCA(n_components=50).fit(X_train) # 672 in total
```

In [ ]:
```
X_train_pca = eigensigns_pca.transform(X_train)
```

In [ ]:
```
classifier = SVC().fit(X_train_pca, y_train)
```

In [ ]:
```
X_test_pca = eigensigns_pca.transform(X_test)
predictions = classifier.predict(X_test_pca)
```

In [ ]:

```
print(classification_report(y_test, predictions))
```

```
              precision    recall  f1-score   support

           0       0.94      1.00      0.97       331
           1       1.00      1.00      1.00       432
           2       0.91      0.99      0.95       310
           3       0.86      0.99      0.92       245
           4       0.96      1.00      0.98       498
           5       0.92      0.97      0.95       247
           6       0.94      0.94      0.94       348
           7       0.99      0.95      0.97       436
           8       0.81      0.93      0.87       288
          10       0.86      0.75      0.80       331
          11       0.90      1.00      0.95       209
          12       0.89      0.84      0.87       394
          13       0.90      0.71      0.80       291
          14       1.00      0.97      0.98       246
          15       1.00      1.00      1.00       347
          16       0.99      1.00      0.99       164
          17       0.37      0.69      0.48       144
          18       0.88      0.82      0.85       246
          19       0.88      0.69      0.77       248
          20       0.80      0.75      0.77       266
          21       0.89      0.63      0.74       346
          22       0.54      0.80      0.64       206
          23       0.85      0.82      0.84       267
          24       0.93      0.76      0.84       332

    accuracy                           0.88      7172
   macro avg       0.88      0.88      0.87      7172
weighted avg       0.90      0.88      0.89      7172
```

Works Cited

Eldén, Lars. Matrix Methods in Data Mining and Pattern Recognition. Society for Industrial and Applied Mathematics, 2007.