Mechanical Design: putting physics into practice

Synoptic Extension Topic

A-level keywords:

- Moments
- Hooke's Law
- Angular Velocity
- Magnetic Fields
- Experimental Design

Samuel Morse [1]

Explanatory Notes

This question applies a range of physical concepts practically in the design and operation of a famous device: the Morse recevier.

It aims to be interesting, to improve physical understanding, and to encourage 'thinking like an engineer' — here, this means considering the practical aspects of design, acquiring data about the device to analyse its operation, and making pragmatic approximations for easier analysis.

Complete worked answers, common pitfalls, and further discussion are included.

Sources

Samuel Morse portrait, shown above:
https://commons.wikimedia.org/wiki/File:Samuel_F.B._Morse,_inventor_of_the_telegraph_-_NARA_-_526779.jpg, Mathew Brady
Figure 4 image: https://en.wikipedia.org/wiki/File:Morse-Code.svg, PvtKing

Questions

The first step towards automated recording of data in physics was the invention of the telegraph system by Samuel Morse in 1838-9 – specifically, the telegraph receiver. Although Morse's goal was remote communication, this device was the first example of mechanical recording of electrical signals onto pen and paper.

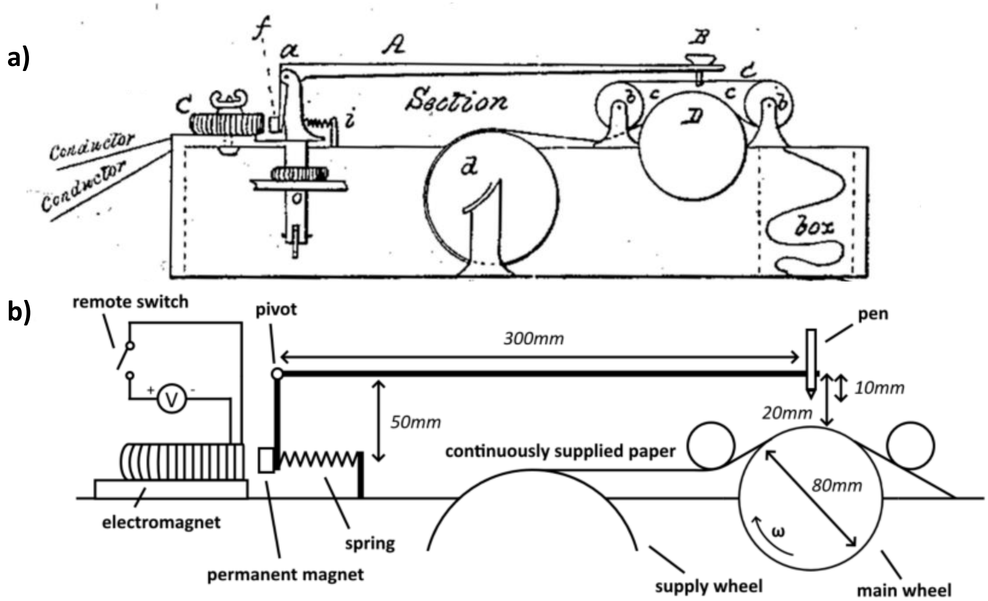


Figure 1: a) Morse's original telegraph receiver design from his 1840 patent.
b) A student's diagram of her replica of Morse's receiver.

To operate the device, a remote user closes a switch. This completes the circuit on the left-hand side of Figure 1, causing the current to flow through the electromagnet and so exerting a force on the permanent magnet. The central lever assembly then rotates clockwise about its pivot, moving the pen down and marking the rolling paper.

A student finds Morse's original patent, available freely online. She decides to try to replicate the key elements of the receiver.

(a) Draw all forces acting on the free body diagram of the lever mechanism reproduced below and label them. *You do not have to calculate the numerical values of the forces.* [3]

(b) If the pen moves to make contact with the paper, approximately how many millimetres does the spring extend from its current position? *You can assume that both the pen and the spring move a small distance, in roughly straight lines.* [1]

Having constructed the wooden lever and pivot assembly for her replica, the student obtains some data about her components.

(c) Describe a way the student could find the spring constant of her spring. [4]

The student weighs a block of the wood she has used to construct the lever arms, finding its density. She looks at the manufacturer’s data book for the ferrite magnet she has bought and finds the schematic shown in Figure 2. She tabulates her measurements in Table 1. She also enters additional dimensions of her wooden lever arms that are not shown in her diagram into the table.

Thickness of lever arm sections, t	10 mm
Depth into page of lever arm sections, d	20 mm
Spring stiffness, k	100 N/m
Mass of pen, m_p	15 g
Density of wood, ρ_w	0.7 g/cm^3
Angular velocity of main wheel ω	1.5 rad/s

Table 1: Measurements of the replica.

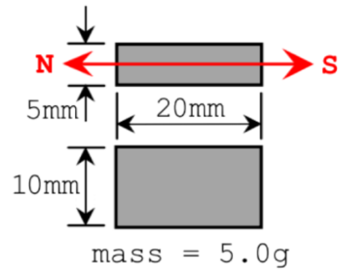


Figure 2: Datasheet schematic of magnet. The largest face is mounted against the wood.

(d) Assuming the pivot is frictionless and the switch is open,

i) Which components’ weights are likely to be significant in the mechanical operation of the lever? Why? [2]

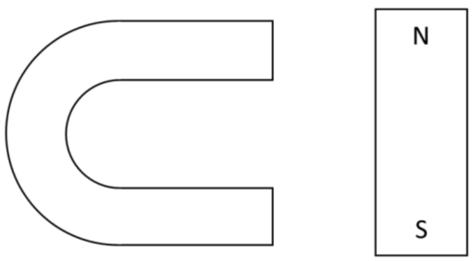
ii) Neglecting components identified as insignificant in i), calculate the extension of the spring from its unstretched length while the device is in this configuration. [3]

The switch is now closed.

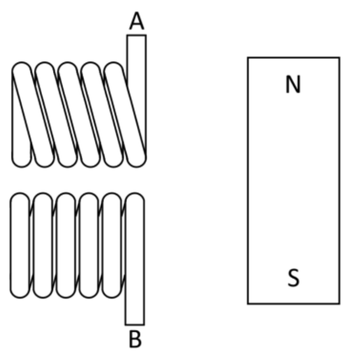
(e) Shown below are top-down views of the electromagnet and permanent magnet. The student connects the circuit so that current flows from A to B.

i) By comparison to attraction between a pair of bar magnets, or otherwise, explain the direction of the force acting on the permanent magnet. *As part of your explanation, you should annotate one or both of the diagrams below.* [3]

ii) Has the student connected the circuit correctly, or should she reverse the wiring? [1]



Top-down view of electromagnet



Top-down view, initial wire coils of electromagnet

The student would like to know the force that needs to be applied to the permanent magnet in order to mark the paper.

(f) Consider the situation when the pen is contact with the paper.

i) Do the moments due to weight acting on the lever mechanism change significantly? [1]

ii) There is now a contact force acting on the pen. Does it need to be considered when calculating the required magnetic force to maintain the pen in this position? [1]

(g) Calculate the approximate minimum required magnetic force on the permanent magnet to allow the device to operate successfully. [2]

The student sets the current flowing through the electromagnet accordingly and installs a roll of paper on the wheel assembly. She decides to test the device with her friend.

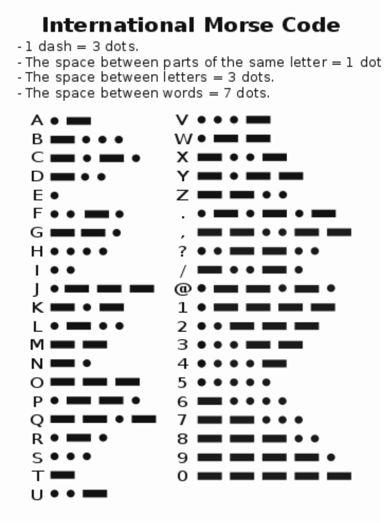


Figure 4:
The dot and dash codes for letters and symbols in International Morse Code.
Reference to this figure is not required to answer any questions.

The standard scheme for communication over the device uses Morse Code, a sequence of 'dots' and 'dashes'. By convention, the length of a dash is three times the length of a dot. The student wants to communicate approximately 10 words per minute, which requires a dot to have a time duration of 100 milliseconds.

Assume the paper does not slip on the wheels for the following questions.

(h) Adopting this convention, what would be the length of a dash in centimetres on the paper? [2]

(i) Approximately how many metres of paper would be required to send a 15-word message? [1]

Extension Question:

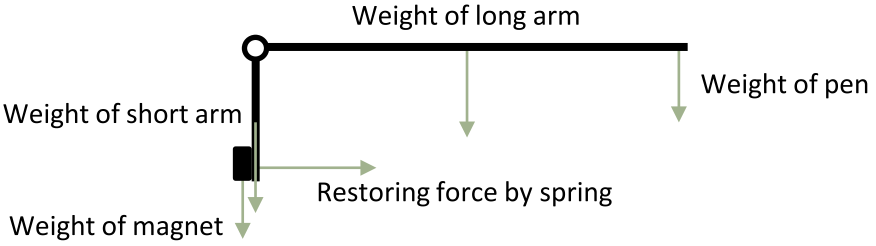
(j) Several approximations have been made to calculate the force in (g). List as many as you can think of. Have these approximations made a significant difference to the answer, and how would the answer change if they were accounted for?

Quick Answers

(b) 1.7mm; (d) i) pen and long wooden arm are significant, ii) 17mm; (e) ii) wiring is already correct; (f) i) no, ii) no; (g) 0.17 N; (h) 18 mm; (i) 5.4 m.

Worked Answers

(a)



If the weights have been combined into a single force applied at the centre of mass, this is fine, as long as it is in a sensible location and clearly labelled as such. The restoring force should have its arrow pointing to the right. Optionally, a magnetic force acting to the left on the permanent magnet can be included.

(b) Both arms are swept through the same angle. The pen moves $20 - 10 = 10\text{ mm}$, since it moves in an approximately straight line. Therefore the angle swept through, θ, is given by

$$\theta = \frac{arc\ length}{radius} \approx \frac{10}{300} \approx 0.033\text{ radians}$$

Which means that the spring extends by approximately $(50\text{ mm}) \times (0.033) \approx 1.7\text{mm}$ in a straight line.

(c) A sensible variant of the following steps should be followed:

- Hang the spring vertically. Record its resting length.
- Add weights with a hook, increasing in increments of eg. 200g (any reasonable increment will do), recording deflection each time.
- Draw a graph of force (=mg) vs. extension from unstretched length.
- The spring constant is obtained by calculating the gradient.

(d) i)

$$m_{magnet} = 5\text{ g (from datasheet)}$$
$$m_{pen} = 15\text{ g (from table)}$$
$$\begin{aligned} m_{long\ arm} &= \rho_w \times t \times d \times length \\ &= (0.7\text{ g/cm}^3) \times (1\text{ cm}) \times (2\text{ cm}) \times (30\text{ cm}) \\ &= 42\text{ g} \end{aligned}$$

Although the weights of the magnet and the short arm of the mechanism are comparable in magnitude to the long arm and the pen, neither will contribute significantly to the mechanical operation of the lever. This is because their perpendicular distances from the pivot point are very small so the moments they generate will be very small in comparison to those generated by the other forces.

Worked Answers (continued)

(d) ii) For equilibrium, the sum of the moments must be equal to zero. Neglecting the moments due to the weights of the short arm and magnet,

$$M_{\circlearrowright} = M_{\circlearrowleft}$$

$$\Rightarrow m_{pen}\ g r_{pen} + m_{arm} g r_{c.o.m} \approx F_{spring} r_{spring} = k x r_{spring}$$

$$\Rightarrow x \approx \frac{m_{pen} g r_{pen} + m_{arm} g r_{c.o.m}}{k r_{spring}}$$

$$\Rightarrow x \approx \frac{(0.015\ \mathrm{kg})(9.81\ \mathrm{N/kg})(300\ \mathrm{mm}) + (0.028\ \mathrm{kg})(9.81\ \mathrm{N/kg})(150\ \mathrm{mm})}{(100\ \mathrm{N/m})(50\ \mathrm{mm})}$$

$$\Rightarrow x \approx 0.017\ \mathrm{m} = 17\ \mathrm{mm}$$

(e) i) Several explanations are possible. The following is perhaps the simplest:

- Consider a single wire loop in the branch of the electromagnet adjacent to A.
- Applying the right-hand rule for magnetic fields with current flowing from A indicates that the magnetic field is oriented into the end of the magnet. Similarly, the field points out of the end of the B-adjacent branch of the magnet.
- This makes the A-adjacent end the South pole, and the B-adjacent end the North pole.
- By comparing this to a pair of bar magnets, North poles are adjacent to South poles, so the permanent magnet is attracted, i.e. the magnetic force acts towards the left.

(e) ii) The wiring is correct, since the permanent magnet is attracted in this configuration.

(f) i) No. Since the pen and the lever arm centre of mass move almost straight down, their perpendicular distances from the pivot point to not change significantly, so their resulting moments don't either.

(f) ii) No. The force will have a friction component and a normal reaction component. The friction component will point to the right due to the direction of motion of the paper, and the normal component will point upwards, so both create a moment opposing the magnet's. However, this moment will only be applied while in contact with the paper, and it is only large enough to support the pen.

Worked Answers (continued)

(g)

$$F_{magnet} r_{magnet} = \Delta F_{spring} r_{spring}$$
$$\Rightarrow F_{magnet} = \Delta F_{spring} = k\Delta x$$

An approximate value of Δx has already been obtained in part (b): $1.7\ \text{mm} = 0.0017\ \text{m}$. This gives a value for F_{magnet} of $(100\ \text{N/m})(0.0017\ \text{m}) = 0.17\ \text{N}$.

This calculation can also be done including the effects of all the other forces. However, it is much easier to use the principle of superposition, and only consider the changes in forces as above.

(h) The paper moves at the same speed as the surface of the wheel, since there is no slip:

$$v = \omega r = \omega \frac{D}{2} = (1.5\ \text{rad/s})(0.040\ \text{m}) = 0.060\ \text{m/s}$$

A dot involves the pen making contact for $100\ \text{ms} = 0.1\ \text{s}$, so its length is

$$l = vt = (0.060\ \text{m/s})(0.1\ \text{s}) = 0.0060\ \text{m} = 6.0\ \text{mm}$$

Therefore the length of a dash is three times that, or $18\ \text{mm}$.

(i) 10 words are sent per minute, so sending 15 words takes:

$$\frac{15\ \text{words}}{10\ \text{words/min}} = 1.5\ \text{min} = 90\ \text{s}$$

The length of paper to go past in that time is therefore

$$l = vt = (0.060\ \text{m/s})(90\ \text{s}) = 5.4\ \text{m}$$

That's a lot of paper! This shows that Morse Code is not an ideal way to communicate large amounts of information.

(j) Some of the approximations made and their effects are discussed in this non-exhaustive list:

- Small angle assumption to neglect moment changes and approximate motion to straight lines. This can affect the answer both ways. However, the approximation is a good one here, and not a significant source of error. For $\theta = 0.033$, the true distance travelled by the ends depends instead on $\sin\theta = 0.03299$, giving an error of 0.02%. Similarly, the distance to the weight moments is reduced by only 0.06%.
- Neglect of mass of magnet and its movement $\Rightarrow$ counteracting moment $\Rightarrow$ larger force required. The small mass of the magnet means that this correction is not very important. You might like to calculate the percentage error from this source for (d) ii).
- Non-ideal pivot $\Rightarrow$ friction moment $\Rightarrow$ larger force required. The impact of this would depend on the quality of the pivot, but it is likely to cause more error than the previous two issues.
- Bending (sag) of the long wooden arm $\Rightarrow$ smaller distance travelled $\Rightarrow$ smaller force required. Especially if the beam is thin, this could have a significant effect.