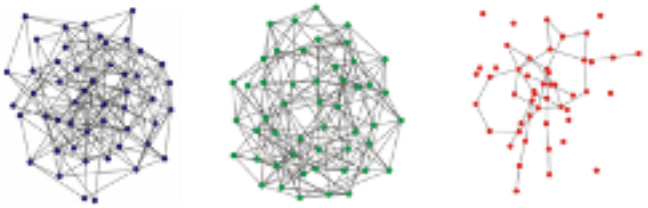

# HOMEWORK ASSIGNMENT 2: METRICS

PHYSICS 5300
NETWORKS THEORY SPRING 2021
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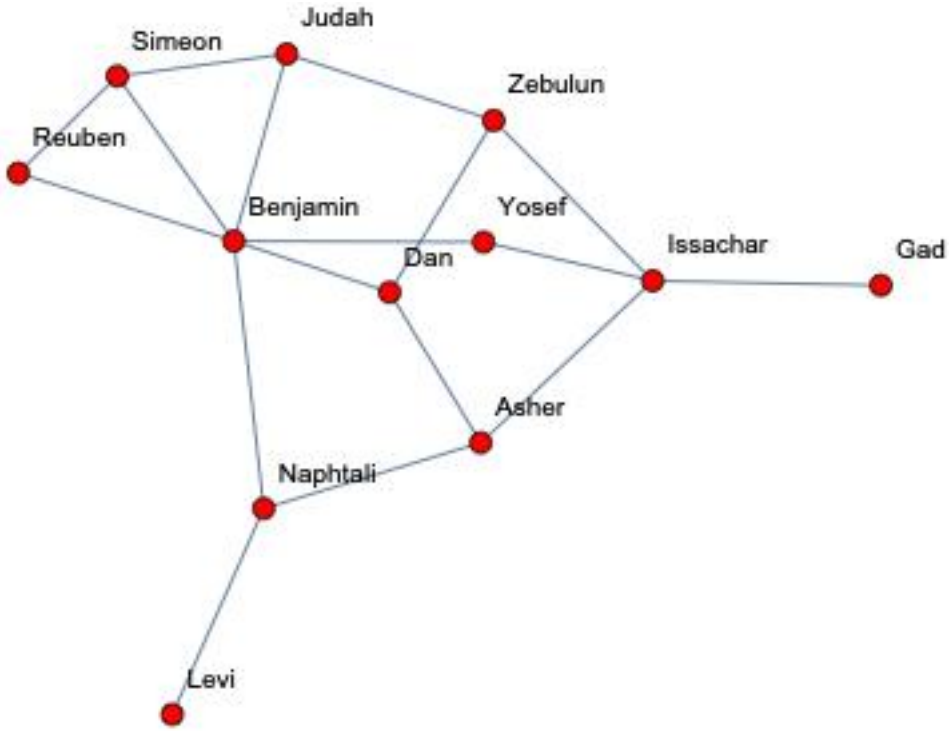
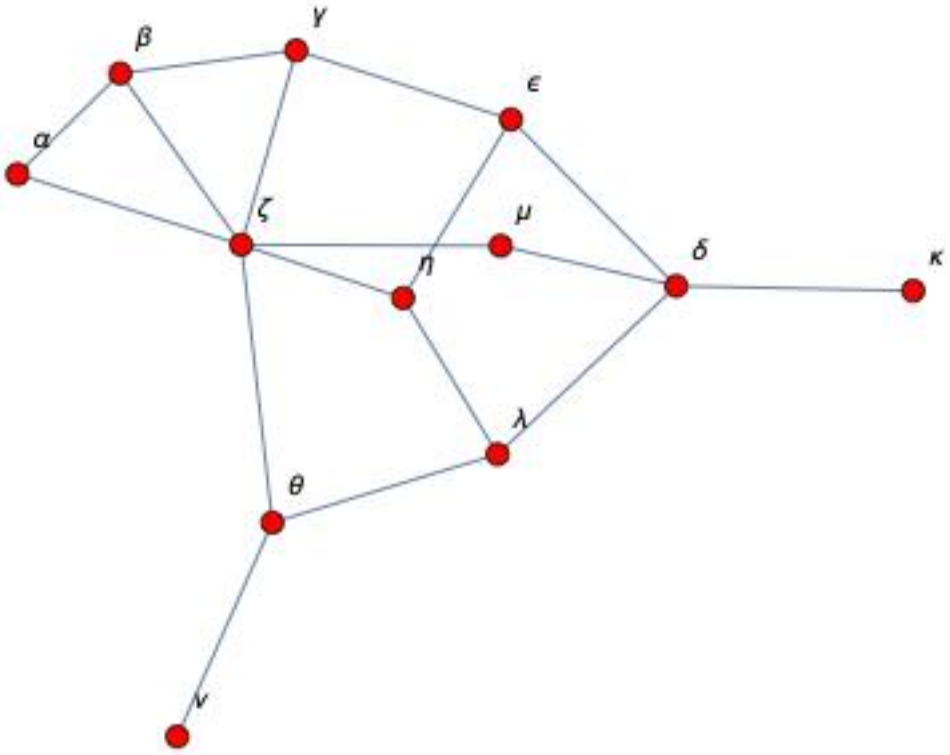
This is a network I just made up (that is a lie, it is the network of friendship among students in a class I taught a while ago, but I changed their names to protect their identity) to some names that you might recognize. In case that is complicated for you, I give you a different version of the same network in which I refer to the sons of Jacob by their Greek names.






For most of the questions below we chose to present the data as an unweighted undirected simple graph. If we deviate from this we will indicate so, towards the end of the assignment

## 1. Getting started

- Characterize this network by writing its adjacency matrix.
- Check that it satisfies the Perron-Frobenius theorem: that the largest eigenvalue is unique, and the corresponding eigenvector has all positive entries.

## 2. Who is the most "important" student? (Centralities)

Use the network illustrated here and calculate the following centralities for each node:

- a) Degree centrality

- b) Eigenvector centrality

- c) Katz centrality

- d) PageRank centrality.

  For these last two cases you will need to decide on a judicious choice of the parameter $\alpha$ Verify if the results you get are very sensitive to the value $\alpha$, or if the relative ranking of the nodes changes according to that.

- e) The closeness centrality. Obtain also the mean distance between all the nodes in the network.

- f) The betweenness centrality. Be careful how you normalize. If you use any standard package make sure that you understand what choice of normalization is used.

- g) Display all the centralities you calculated in a table like this with one row per student

| Name | Degree | Eigenvector | Katz | PageRank | Closeness | Betweenness |
|---|---|---|---|---|---|---|
| Simeon | | | | | | |
| Judah | | | | | | |
| Etc | | | | | | |
| Etc | | | | | | |
| Etc | | | | | | |
| Spread | | | | | | |

- h) Consider the ratio between the largest value and the smallest value of the centrality of the nodes. This is called the spread of that particular centrality. Which of the centrality measures gives you a larger spread between the nodes? Which one gives you a smaller one? Is that what you expected?

- i) Try to see if there are centralities that give you similar results (in the relative ranking) indicating that they tend to measure similar properties of the nodes.

## 3. Who is a friend of whom? (Groups of vertices)

- a) Identify all the possible 3- cliques and 4-cliques in this network
- b) Identify any 2-plex in this network
- c) Identify any 2-core and 3-core in this network. Is there any 4-core?
- d) Identify any 2-components in this network. Is there any 3-component?
- e) Calculate the global clustering coefficient of this network
- f) Calculate the local clustering coefficient of each node of this network. Is the Watts-Strogatz clustering coefficient of this network equal to the global clustering coefficient determined in question e)?
- g) List the local clustering coefficients and the degrees of all the nodes, in order of degrees (smaller to higher) Do you detect any correlation between degree and clustering coefficient, as discussed in class? Can you guess why?

| Name | Degree | Local clustering coeffcient |
|---|---|---|
| | | |
| | | |
| | | |
| | | |
| | | |
| | | |

## 4. Can we all (not) get along? (signed networks)

- Can you find an assignation of friendship/un-friendship bonds in this network so that the network is structurally balanced? In other words, take each of the 17 bonds and assign a sign to it: ( Reuben likes Simeon so the bond $Reuben \longleftrightarrow Simeon$ gets a (+) sign, etc.)

- If so, show how you can clusterize the network for that particular assignation , in the sense of the Harary theorem.

  NOTE: Do not cheat !! Do not pick an assignation in which everybody likes everybody else. That is trivial. Do not pick an assignation with very few dislikes (for example there are several possibilities if you only include 2 dislikes and all the remaining 15 are likes).

- Is there a structurally balanced assignation with 10 (+) bonds and 7 (-) bonds? Or is that a violation of Harary?s theorem?. If there is one display it and clusterize the network.

## 5. Who looks like whom? (Similarities between nodes)

- a) Find the cosine similarity for all the pairs of nodes in this network. Display them in a table.
  Which are the most similar and the most dissimilar nodes? Can you see why?

- b) Find the Pearson correlation coefficient among all the possible pairs of nodes in this network. Display them in a table.
  Which are the most similar and the most dissimilar nodes? Can you see why? Are your results similar to the previous case?

  Make sure you understand which particular normalization is used when you obtain those values.

## 6. Is this group divided? (Assortativity)

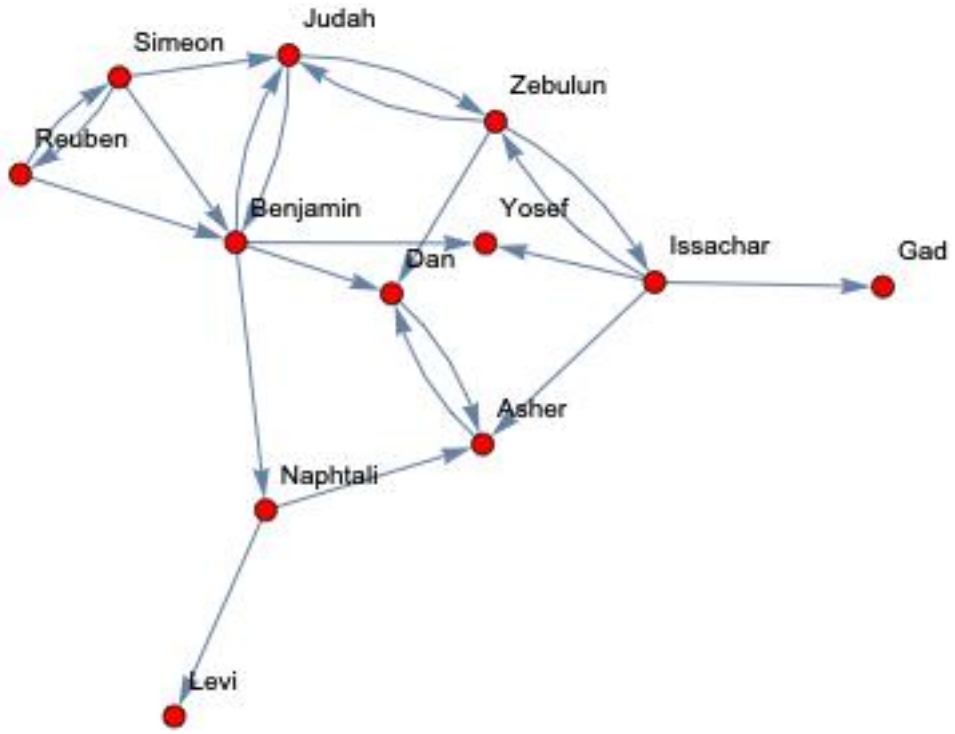
- a) Calculate the assortativity coefficient of this network (normalized modularity) respect to the following two enumerative characteristics:

  1) The sons of Leah (Reuben, Simeon, Levi, Judah, Issachar, and Zebulun) versus the other six brothers.

  2) The six older brothers (Reuben, Simeon, Levi, Judah, Dan, Naphtali) versus the six younger ones (Gad, Asher, Issachar, Zebulun, Joseph and Benjamin).

- b) Calculate the assortativity coefficient of this network respect to the degree of the nodes. Is the network stratified per degree? If so, can you plot the network rearranging the position of the nodes to show this visually?

## 7. One way street (Directed Networks)




This is the actual friendship network of the class in question. The version we used in all the questions above was what was obtained from this graph by symmetrizing the network (in other words ignoring the tips of the arrows and replacing any resulting double link between nodes, by a single link )

Now let us consider this directed network as given here and answer the following questions:

- a) what is the reciprocity degree of this network?
- b) List all the strongly connected components (SCC) on this network.
- c) For each one of those SCC, list their out-component and their in-component
- d) Pick one node at random (the one you prefer, but not one of degree 1 since that is too easy ) and list the shortest distance between the node you chose and each one of the other nodes in the network. In the case of the undirected network you will get one number per each distance; in the case of the directed network you might get

two different numbers: the distance from your node to someone, and the distance from that someone to your node. Present the results in a table and compare.

- e) BONUS QUESTION: Transform this **DIRECTED** network in an **undirected but WEIGHTED** network with the following simple rule.

  i) If there is no arrow between any two nodes, there will still be no connection between those nodes.

  ii) if there is just one arrow between two nodes, there will an undirected link of strength 1 between them

  iii) if there are two arrows between any two nodes, there will be an undirected link of strength 2 between those two nodes.

  In this new WEIGHTED NETWORK calculate again the shortest distance between the node you picked before and all the other nodes, and compare with the previous result. Remember that the length of a path is now not just the number of links along it, but the sum of all the weights of those links, so if you are separated from someone by three links of length one, that is a shorter distance than being separated from him by two links of length two each.