$$5x_1 + 3x_2 - x_3 + 2x_4 = 10$$
$$x_1 - 2x_2 + 14x_3 + x_4 = 22$$
$$7x_1 + x_2 - 21x_3 + 5x_4 = 41$$
$$10x_1 - 5x_2 + 4x_3 + 7x_4 = 5$$

$$A = \begin{bmatrix} 5 & 3 & -1 & 2 \\ 1 & -2 & 14 & 1 \\ 7 & 1 & -21 & 5 \\ 10 & -5 & 4 & 7 \end{bmatrix} \quad b = \begin{bmatrix} 10 \\ 22 \\ 41 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = A^{-1} b$$

if A^{-1} exists

$$2x + 4y = 6 \quad / \cdot 1.5 \qquad 3x + 6y = 9$$
$$3x + 6y = 9$$

Linear independence: If you can express one of the equations as a combination of other equations, then your coefficient matrix is linearly dependent/singular.

$$2x + 4y + z = 6 \quad (1)$$
$$3x + 6y - 3z = 9 \quad (2)$$
$$5x + 10y - 2z = 15 \quad (3)$$

$$(1) + (2) = (3)$$

Linear dependence: if $\exists$ any λ, γ such that $\lambda(1) + \gamma(2) = (3) \Rightarrow$ singular

The determinant:

2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A) = a \cdot d - b \cdot c$$

3x3 matrix $B = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

$$\det(B) = aei + bfg + cdh - ceg - bdi - afh$$

Sarrus' rule: Subtract / Add

$$\det(A) = 2 \cdot 4 - 7 \cdot 1 = 1$$
$$\det(B) = 12.5 - e \cdot \pi = 60 - e\pi$$
$$\det(C) = 5 \cdot \tfrac{1}{5} - 0.1 \cdot 10 = 0$$
$$\det(D) = 2.4.1 + 7.3.3 + 1.1.2 - 7.1.1 - 2.3.2 - 1.4.3 = 42$$

$$\det(E) = 1.2.3 + 0.5.4 + 3.0.1 - 0.0.3 - 1.5.1 - 3.2.4 = -23$$