Algoritmo

① Dividir matrices de entrada A, B y de salida en

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

donde los K_{nm} son las submatrices correspondientes de tamaño $\frac{n}{2} \times \frac{n}{2}$

② Crear submatrices $S_1, \dots, S_{10}$ de tamaño $\frac{n}{2} \times \frac{n}{2}$ donde cada uno es suma o diferencia de las matrices del paso ①

③ Con submatrices de paso ① y ② computar de manera recursiva los productos $P_1, \dots, P_7$ que resulten en matrices de tamaño $\frac{n}{2} \times \frac{n}{2}$

④ Computar submatrices $C_{11}, C_{12}, C_{21}, C_{22}$ combinando las matrices P.

Detalles

②

$$\begin{aligned}
S_1 &= B_{12} - B_{22} \\
S_2 &= A_{11} + A_{12} \\
S_3 &= A_{21} + A_{22} \\
S_4 &= B_{21} - B_{11} \\
S_5 &= A_{11} + A_{22} \\
S_6 &= B_{11} + B_{22} \\
S_7 &= A_{12} - A_{22} \\
S_8 &= B_{21} + B_{22} \\
S_9 &= A_{11} - A_{21} \\
S_{10} &= B_{11} + B_{12}
\end{aligned}$$

③

$$\begin{aligned}
P_1 &= A_{11} S_1 \\
P_2 &= S_2 B_{22} \\
P_3 &= S_3 \cdot B_{11} \\
P_4 &= A_{22} S_4 \\
P_5 &= S_5 \cdot S_6 \\
P_6 &= S_7 \cdot S_8 \\
P_7 &= S_9 S_{10}
\end{aligned}$$

④

$$\begin{aligned}
C_{11} &= P_5 + P_4 - P_2 + P_6 \\
C_{12} &= P_1 + P_2 \\
C_{21} &= P_3 + P_4 \\
C_{22} &= P_5 + P_1 - P_3 - P_7
\end{aligned}$$

Que después de manipular queda

$$\begin{aligned}
C_{11} &= A_{11} B_{11} + A_{12} B_{21} \\
C_{12} &= A_{11} B_{12} + A_{12} B_{22} \\
C_{21} &= A_{21} B_{11} + A_{22} B_{21} \\
C_{22} &= A_{22} B_{22} + A_{21} B_{12}
\end{aligned}$$

Por lo que, dadas las matrices

$$A = \begin{pmatrix} 1 & 3 \\ 7 & 5 \end{pmatrix} \quad B = \begin{pmatrix} 6 & 8 \\ 4 & 2 \end{pmatrix}$$

se da que

$$A_{11} = (1) \quad A_{21} = (3)$$
$$A_{12} = (7) \quad A_{22} = (5)$$
$$B_{11} = (6) \quad B_{21} = (8)$$
$$B_{12} = (4) \quad B_{22} = (2)$$

Y pasando a la versión extendida de C, tenemos que

$$\begin{aligned}
C_{11} &= A_{11} B_{11} + A_{12} B_{21} \\
&= 1 \cdot 6 + 3 \cdot 4 = 18
\end{aligned}$$

$$\begin{aligned}
C_{12} &= A_{11} B_{12} + A_{12} B_{22} \\
&= 1 \cdot 8 + 3 \cdot 2 = 14
\end{aligned}$$

$$\begin{aligned}
C_{21} &= A_{21} B_{11} + A_{22} B_{21} \\
&= 7 \cdot 6 + 5 \cdot 4 = 62
\end{aligned}$$

$$\begin{aligned}
C_{22} &= A_{21} B_{12} + A_{22} B_{22} \\
&= 7 \cdot 8 + 5 \cdot 2 = 66
\end{aligned}$$

De manera que

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} 18 & 14 \\ 62 & 66 \end{bmatrix}$$

Pseudocodigo

```
Strassen(matA, matB)
    szA = len(matA)

    #Suponemos recibimos matrices cuadradas
    # y tenemos método len
    if szA ≥ 1 entonces
        #Creamos submatrices
        nSz = szA/2

        A11 = A[:nSz, :nSz]
        A12 = A[:nSz, nSz:]
        A21 = A[nSz:, :nSz]
        A22 = A[nSz:, nSz:]
        #Repetimos para B
         ...
        #Calculamos los productos
        P1 = Strassen(A11, B12) - Strassen(A11, B22)
        P2 = Strassen(A11, B22) + Strassen(A12, B22)
        P3 = Strassen(A21, B11) + Strassen(A22, B11)
        P4 = Strassen(A22, B21) - Strasse A22, B11)
        P5 = Strassen(A11, B11) + Strassen(A11, B22) + Strassen(A22, B11) + Strassen(A22, B22)
        P6 = Strassen(A12, B21) + Strassen(A12, B22) - Strassen(A22, B21) - Strassen(A22, B22)
        P7 = Strassen(A11, B11) + Strassen(A11, B12) - Strassen(A21, B11) - Strassen(A21, B12)
        #Calculamos submatrices C
        C11 = P5 + P4 - P2 + P6
        C12 = P1 + P2
        C21 = P3 + P4
        C22 = P5 + P1 - P3 - P7
        #Creamos matriz a regresar
        C[:nSz, :nSz] = C11
        C[:nSz, nSz:] = C12
        C[nSz:, :nSz] = C21
        C[nSz:, nSz:] = C22

        return C
    else:
        return A[0,0] * B[0,0]

    Fin if
Fin metodo
```

Muestra que

$T(n) = T(n-1) + n \Leftrightarrow O(n^2)$

Muestra que

$$T(n) = T\left(\frac{n}{2}\right) + 1 \Leftrightarrow O(\lg n)$$