

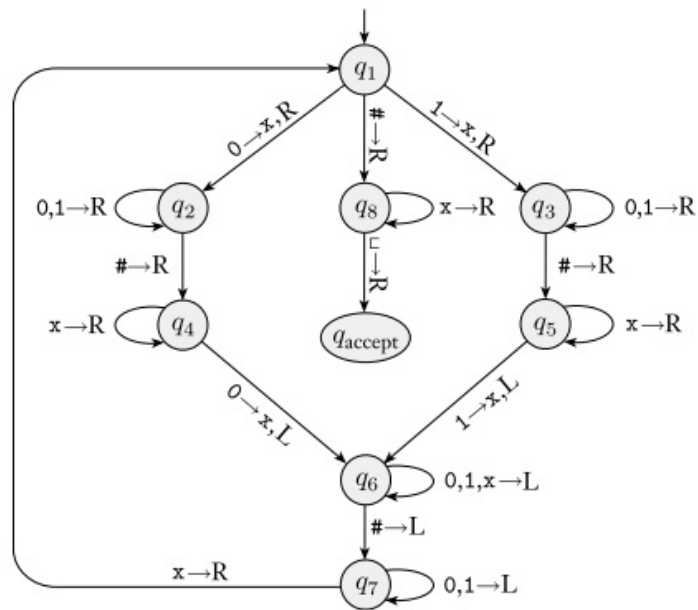
**3.2** This exercise concerns TM  $M_1$ , whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that  $M_1$  enters when started on the indicated input string.

- <sup>A</sup>a. 11.  
b. 1#1.  
c. 1##1.  
d. 10#11.  
e. 10#10.

**EXAMPLE 3.9** -----

The following is a formal description of  $M_1 = (Q, \Sigma, \Gamma, \delta, q_1, q_{\text{accept}}, q_{\text{reject}})$ , the Turing machine that we informally described (page 167) for deciding the language  $B = \{w\#w \mid w \in \{0,1\}^*\}$ .

- $Q = \{q_1, \dots, q_8, q_{\text{accept}}, q_{\text{reject}}\}$ ,
- $\Sigma = \{0, 1, \#\}$ , and  $\Gamma = \{0, 1, \#, \times, \sqcup\}$ .
- We describe  $\delta$  with a state diagram (see the following figure).
- The start, accept, and reject states are  $q_1$ ,  $q_{\text{accept}}$ , and  $q_{\text{reject}}$ , respectively.



a.)	Input	↑ ↑
	Estado	conf
0	$q_1$	$q_1 \uparrow \uparrow$
↑	$q_3$	$x q_3 \uparrow$
2	$q_3$	$x \uparrow q_3$
Final		<u><math>x \uparrow</math></u>

c.) Inp    ↑ ## ↑

0	$q_1$	$q_1 \uparrow \# \# T$
↑	$q_3$	X $q_3 \uparrow \# \# T$
2	$q_5$	X $\uparrow q_5 \uparrow \# T$
$T_{\text{final}}$		X $\uparrow \# \# T$

	$\bar{c}$	out
0	$q_1$	$q_1 \uparrow \# \uparrow$
1	$q_3$	$X q_3 \# \uparrow$
2	$q_5$	$X \# q_5 \uparrow$
3	$q_6$	$X q_6 \# X$
4	$q_7$	$q_7 X \# X$
5	$q_1$	$X q_1 \# X$
6	$q_8$	$X \# q_8 X$
7	$q_8$	$X \# X q_8$

Acceptable

0	$q_1$	$q_1 10 \# 11$
1	$q_{15}$	$x q_3 0 \# 11$
2	$q_{13}$	$x 0 q_3 \# 11$
3	$q_{15}$	$x 0 \# q_5 11$
4	$q_{16}$	$x 0 q_0 \# x 1$
5	$q_7$	$x q_7 0 \# x 1$
6	$q_7$	$q_7 x 0 \# x 1$
7	$q_1$	$x q_1 0 \# x 1$
8	$q_2$	$x x q_2 \# x 1$
9	$q_4$	$x x \# q_4 x 1$
10	$q_4$	$x x \# x q_4 1$
Final		$x x \# x 1$

0	$q_1$	$q_1 10 \neq 10$
1	$q_3$	$x q_3 0 \neq 10$
2	$q_3$	$x 0 q_3 \neq 10$
3	$q_5$	$x 0 \neq q_5 10$
4	$q_6$	$x 0 q_6 \neq x 0$
5	$q_7$	$x q_7 0 \neq x 0$
6	$q_7$	$q_7 x 0 \neq x 0$
7	$q_1$	$x q_1 0 \neq x 0$
8	$q_2$	$x x q_2 \neq x 0$
9	$q_4$	$x x \neq q_4 x 0$
10	$q_4$	$x x \neq x q_4 0$
11	$q_6$	$x x \neq q_6 x x$
12	$q_6$	$x x q_6 \neq x x$
13	$q_7$	$x q_7 x \neq x x$
14	$q_1$	$x x q_1 \neq x x$
15	$q_8$	$x x \neq q_8 x x$
16	$q_8$	$x x \neq x q_8 x$
17	$q_9$	$x x \neq x x q_9$
18	<del>Acceptance</del>	

**3.6** In Theorem 3.21, we showed that a language is Turing-recognizable iff some enumerator enumerates it. Why didn't we use the following simpler algorithm for the forward direction of the proof? As before,  $s_1, s_2, \dots$  is a list of all strings in  $\Sigma^*$ .

- $E =$  “Ignore the input.
1. Repeat the following for  $i = 1, 2, 3, \dots$
  2.   Run  $M$  on  $s_i$ .
  3.   If it accepts, print out  $s_i$ .”

**THEOREM 3.21** -----

A language is Turing-recognizable if and only if some enumerator enumerates it.

**PROOF** First we show that if we have an enumerator  $E$  that enumerates a language  $A$ , a TM  $M$  recognizes  $A$ . The TM  $M$  works in the following way.

- $M =$  “On input  $w$ :
1. Run  $E$ . Every time that  $E$  outputs a string, compare it with  $w$ .
  2. If  $w$  ever appears in the output of  $E$ , *accept*.”

Clearly,  $M$  accepts those strings that appear on  $E$ 's list.  
Now we do the other direction. If TM  $M$  recognizes a language  $A$ , we can construct the following enumerator  $E$  for  $A$ . Say that  $s_1, s_2, s_3, \dots$  is a list of all possible strings in  $\Sigma^*$ .

- $E =$  “Ignore the input.
1. Repeat the following for  $i = 1, 2, 3, \dots$
  2.   Run  $M$  for  $i$  steps on each input,  $s_1, s_2, \dots, s_i$ .
  3.   If any computations accept, print out the corresponding  $s_j$ .”

If  $M$  accepts a particular string  $s$ , eventually it will appear on the list generated by  $E$ . In fact, it will appear on the list infinitely many times because  $M$  runs from the beginning on each string for each repetition of step 1. This procedure gives the effect of running  $M$  in parallel on all possible input strings.

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3.8 Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet  $\{0,1\}$ .

- a.  ~~$\{w \mid w \text{ contains an equal number of 0s and 1s}\}$~~   
b.  $\{w \mid w \text{ contains twice as many 0s as 1s}\}$

↳ exemplo

Input 010

q<sub>0</sub> q0 + 0

q<sub>1</sub> xq 10

q<sub>2</sub> xyq0

q<sub>3</sub> xqyy

q<sub>4</sub> qxyy

q<sub>5</sub> yyy

q<sub>6</sub> yyqy

q<sub>7</sub> yyyq

Accept

