Experiment / Trial	coin flip.	die roll	life time of bulb.
outcome.	H, T	1, 2, ⋯ 6	$t \geq 0$
probability	$\frac{1}{2}$ $\frac{1}{2}$	$\frac{1}{6}$ --- $\frac{1}{6}$	

More Terminology

Def. (sample space) : set of all possible outcomes. (ω).

denoted by Ω

Def. (event) : a subset of Ω.

eg. die is even $\leftarrow$ def in words

$\omega \in \{2, 4, 6\} \leftarrow$ def in math.

event $\{2, 4, 6\}$ occurs $\leftarrow$ probability jargon

$\mathbb{P}(\{2, 4, 6\}) = \mathbb{P}(\omega \in \{2, 4, 6\})$

When we say a event occurs it means.

the outcome is in the event. not the whole

every try gives just one outcome.

$E, F \subset \Omega$ eg $E = \{2, 4, 6\}$ $F = \{4, 5, 6\}$

$E \cup F$: $(E \cup F)$ occurs $=$ $\omega \in E \cup F$ $=$ $\omega \in E$ or $\omega \in F$

$=$ Either E or F occurs.

$E \cap F$ ⋯ $=$ both E and F occurs.

E^c $=$ Not E.

Def. (σ-field) : $\boxed{\text{a collection}}$ of events of interest. ⟺ a collection of information.

① not interested in all events.

② Don't know prob of all events.

lack of information.

Def. (field) : A collection $\mathcal{F}$ of subset of Ω which satisfies the following.

① $\phi \in \mathcal{F}$ $\Omega \in \mathcal{F}$ $\mathbb{P}(\phi) = \mathbb{P}(\omega \in \phi) = 0$.

$\mathbb{P}(\Omega) = \mathbb{P}(\omega \in \Omega) = 1$

② $A \in \mathcal{F}$ $A^c \in \mathcal{F}$

③ If $A, B \in \mathcal{F}$ the $A \cup B \in \mathcal{F}$ $A \cap B \in \mathcal{F}$

closed under taking finite union.

Def σ-field

① $\phi \in \mathcal{F}$

② $A \in \mathcal{F}$ $A^c \in \mathcal{F}$

③. If $A_1, A_2 \cdots \in \mathcal{F}$ $\bigcup_{i=1}^{\infty} A_i \in \mathcal{F}$

closed under taking <u>countable union.</u>

Def. (measurable space) $(\Omega, \mathcal{F})$ the space is ready for us to assign a measure.

Def. (probability measure) $\mathbb{P} : \mathcal{F} \to [0,1]$

① $\mathbb{P}(\phi) = 0$ $\mathbb{P}(\Omega) = 1$

② (countable additivity)

If $A_1, A_2, \cdots$ are disjoint i.e. $A_i \cap A_j = \phi \ \forall i \neq j$

then.

$$\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathbb{P}(A_i).$$

More properties.

① $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

② $\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mathbb{P}(A_i) - \sum_{i<j} \mathbb{P}(A_i \cap A_j) + \cdots$

③ (sub additivity). $\mathbb{P}\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} \mathbb{P}(A_i)$

Def. measure. $\mu : \mathcal{F} \to [0, \infty]$

(i) $\mu(\phi) = 0$

(ii) countable additivity