Lecture 5. Data Flow Analysis

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Dataflow Analysis

What is dataflow analysis?

- The flow of data values
- The global properties of data at a program point

Reaching Definitions

Available Expressions

Live Variable Analysis

Generalizing Dataflow Analysis

An instance of dataflow problem:

- What is the problem?
- The properties of dataflow problems:
  - backward or forward (determine the direction of the dataflow)
  - may or must (determine how to merge dataflow)
- Dataflow equation: Gen, Kill, In, Out (local information)

Generalizing Dataflow Analysis

- **Goal**: solving dataflow equations —- determine dataflow for the whole program
    - Reaching definitions: for each program point, what are the definitions can reach
    - Available expressions: for each program point, what are the expressions available
    - Live variables: for each program point, what are the variables available
- **Framework**: a set of data propagation algorithms for a set of dataflow problems
- **Key of the algorithm**: Stabilize in presence of loops – fixpoint, using worklist algorithm
- **Data structure for efficiency**: bit vector to represent the global information
    - Reaching definitions: each definition is a bit, each program point has a bitvector
    - Available expressions: each expression is a bit, each program point has a bitvector
    - Live variable analysis: each variable is a bit, each program point has a bitvector

Dataflow Algorithms

- Exhaustive (Dragon Book)
- Demand-driven [1]

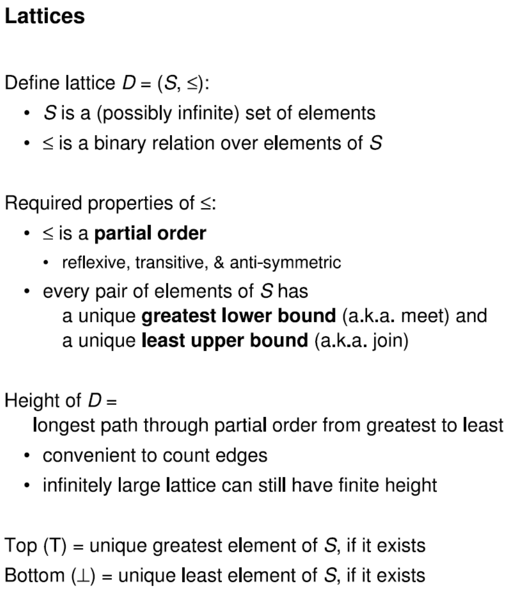
for forward problem, considering all possible paths from the entry to the given point, compute flow value for all the paths, and then meet the flow value at the end of the points

Conservative Analysis

Formal Model to Describe Dataflow Analysis [3, 2]

- *Lattice*: define domain of program properties computed by dataflow analysis (flow value) (the set of elements plus the order of these elements)
- Define *flow functions (transfer functions: how each node affects the flow value)* and *merge functions* over this domain using standard lattice operations

Lattice



Lattices

Define lattice $D = (S, \leq)$:

- S is a (possibly infinite) set of elements
- $\leq$ is a binary relation over elements of S

Required properties of $\leq$:

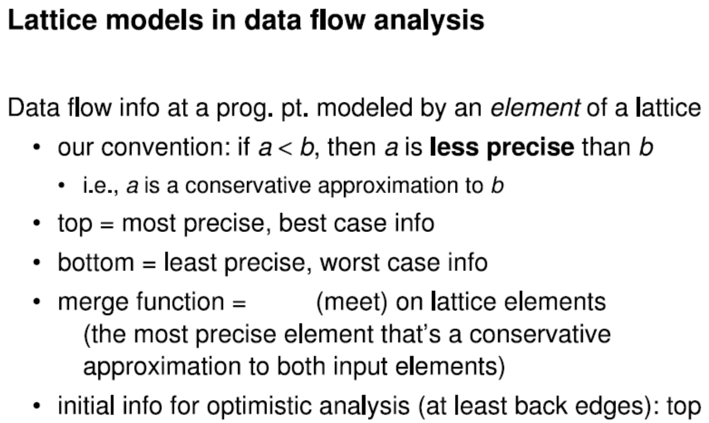
- $\leq$ is a **partial order**
    - reflexive, transitive, & anti-symmetric
- every pair of elements of S has
  a unique **greatest lower bound** (a.k.a. meet) and
  a unique **least upper bound** (a.k.a. join)

Height of D =
longest path through partial order from greatest to least

- convenient to count edges
- infinitely large lattice can still have finite height

Top (T) = unique greatest element of S, if it exists
Bottom ($\perp$) = unique least element of S, if it exists

Lattice



Lattice models in data flow analysis

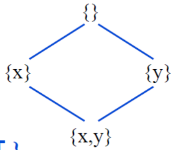
Data flow info at a prog. pt. modeled by an *element* of a lattice

- our convention: if $a < b$, then a is **less precise** than b
  - i.e., a is a conservative approximation to b
- top = most precise, best case info
- bottom = least precise, worst case info
- merge function =   (meet) on lattice elements (the most precise element that's a conservative approximation to both input elements)
- initial info for optimistic analysis (at least back edges): top

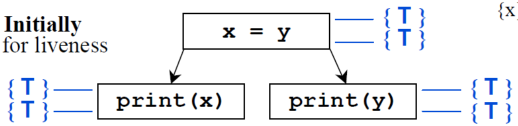
From Lattice to Dataflow Problems

Remember what these flow values represent

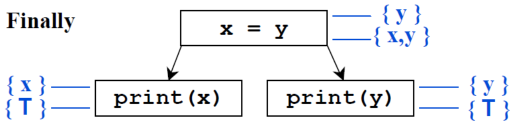
– At each program point a lattice element represents an in[] set or an out[] set



Initially for liveness



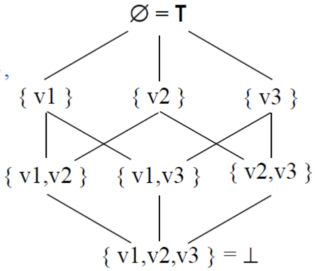
Finally



From Lattice to Dataflow Problems

Example: Liveness analysis with 3 variables
S = {v1, v2, v3}

- V: $2^S = \{\{v1,v2,v3\}, \{v1,v2\},\{v1,v3\},\{v2,v3\}, \{v1\},\{v2\},\{v3\}, \varnothing\}$
- Meet (^): $\cup$
  - $\leq$: $\supseteq$
  - Top(T): $\varnothing$
  - Bottom ($\perp$): υ
- F: $\{f_n(X) = Gen_n \cup (X - Kill_n), \forall n\}$



Transfer Functions

- Let L = dataflow information lattice
- Transfer function $F_I : L \rightarrow L$ for each instruction I
  - Describes how I modifies the information in the lattice
  - If in[I] is info before I and out[I] is info after I, then

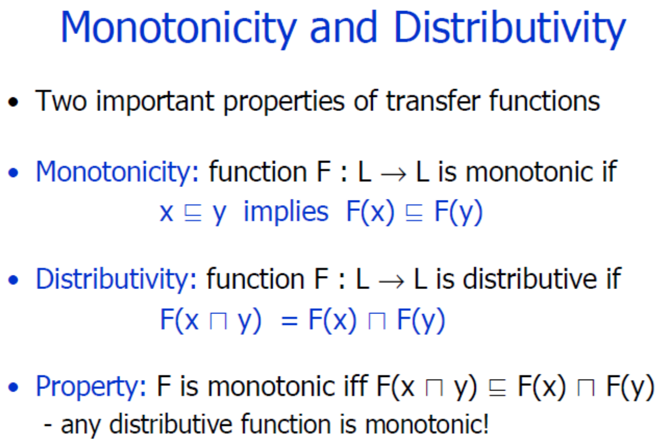
    Forward analysis: $out[I] = F_I(in[I])$

    Backward analysis: $in[I] = F_I(out[I])$
- Transfer function $F_B : L \rightarrow L$ for each basic block B
  - Is composition of transfer functions of instructions in B
  - If in[B] is info before B and out[B] is info after B, then

    Forward analysis: $out[B] = F_B(in[B])$

    Backward analysis: $in[B] = F_B(out[B])$

Transfer Functions

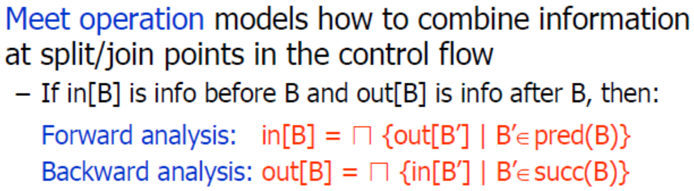


Monotonicity and Distributivity

- Two important properties of transfer functions
- Monotonicity: function F : L $\rightarrow$ L is monotonic if
  $x \sqsubseteq y$ implies $F(x) \sqsubseteq F(y)$
- Distributivity: function F : L $\rightarrow$ L is distributive if
  $F(x \sqcap y) = F(x) \sqcap F(y)$
- Property: F is monotonic iff $F(x \sqcap y) \sqsubseteq F(x) \sqcap F(y)$
  - any distributive function is monotonic!

- Give a flow function, more conservative input leads to more conservative output
- IDFA is guaranteed to terminate if the flow function is monotonic and the lattice has a finite height

Flow Functions



Meet operation models how to combine information at split/join points in the control flow

- If in[B] is info before B and out[B] is info after B, then:

  Forward analysis: $in[B] = \sqcap \{out[B'] \mid B' \in pred(B)\}$

  Backward analysis: $out[B] = \sqcap \{in[B'] \mid B' \in succ(B)\}$

WorkList Algorithm

```
$in[B_S] = X_0$
$out[B] = \top$, for all B
worklist = set of all basic blocks B

Repeat
    Remove a node B from the worklist
    $in[B] = \sqcap \{out[B'] \mid B' \in pred(B)\}$
    $out[B] = F_B(in[B])$
    if out[B] has changed, then
        $worklist = worklist \cup succ(B)$
Until $worklist = \emptyset$
```

Correctness

Worklist algorithm is correct

- Maintains the invariant that

  $in[B] = \sqcap \{out[B'] \mid B' \in pred(B)\}$

  $out[B] = F_B(in[B])$

  for all the blocks B not in the worklist
- At the end, worklist is empty

Termination

- Key observation: at each iteration, information decreases in the lattice

  $$\text{in}_{k+1}[B] \sqsubseteq \text{in}_k[B] \text{ and } \text{out}_{k+1}[B] \sqsubseteq \text{out}_k[B]$$

  where $\text{in}_k[B]$ is info before B at iteration k and $\text{out}_k[B]$ is info after B at iteration k

- Proof by induction:
  - Induction basis: true, because we start with top element, which is greater than everything
  - Induction step: use monotonicity of transfer functions and meet operation

- Information forms a chain: $\text{in}_1[B] \sqsupseteq \text{in}_2[B] \sqsupseteq \text{in}_3[B] \ldots$

Evelyn Duesterwald, Rajiv Gupta, and Mary Lou Soffa.
A practical framework for demand-driven interprocedural data flow analysis.
ACM Transactions on Programming Languages and Systems, 19:992–1030, 1998.

John B. Kam and Jeffrey D. Ullman.
Global data flow analysis and iterative algorithms.
J. ACM, 23(1):158–171, January 1976.

Gary A. Kildall.
A unified approach to global program optimization.
In *Proceedings of the 1st Annual ACM SIGACT-SIGPLAN Symposium on Principles of Programming Languages*, POPL '73, pages 194–206, New York, NY, USA, 1973. ACM.