(i) $2x-3y+7z=5$, $3x+y-3z=13$, $2x+19y-47z=32$

$$A=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 2 & 19 & -47\end{bmatrix}\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}\quad B=\begin{bmatrix}5\\ 13\\ 32\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3 & 7\\ 3 & 1 & -3\\ 0 & 22 & -54\end{bmatrix}\quad B=\begin{bmatrix}5\\ 13\\ 27\end{bmatrix}$$

$$\begin{array}{r}-47\\ -7\\ \hline -54\end{array}\quad \begin{array}{r}32\\ -5\\ \hline 27\end{array}$$

$R_2 \to R_2 - \frac{3}{2}R_1$

$2\times\frac{3}{2}$

$$\begin{bmatrix}2 & -3 & 7\\ 0 & 11/2 & -\frac{27}{2}\\ 0 & 22 & -54\end{bmatrix}\quad B=\begin{bmatrix}5\\ 11/2\\ 27\end{bmatrix}$$

$\frac{3}{2}(3)\ \frac{1+9}{2}$

$=\frac{11}{2}$

$-3-\frac{21}{2}=-\frac{27}{2}$

$5\times\frac{3}{2}$

$13-\frac{15}{2}$

$$\begin{array}{r}26\\ -15\\ \hline 11\end{array}$$

$\frac{11}{2}\times 4.$ $\begin{array}{r}27\\ -22\\ \hline 5\end{array}$

$$\begin{bmatrix}2 & -3 & 7\\ 0 & 11/2 & -27/2\\ 0 & 0 & 0\end{bmatrix}\quad B=\begin{bmatrix}5\\ 11/2\\ 5\end{bmatrix}$$

Rank of $A=2$ rank of $A:B=3$

$\rho(A)\neq\rho(A:B)$ Incosistency.

ii) $2x-y+3z=8$, $-x+2y+z=4$, $3x+y-4z=0$

$$A=\begin{bmatrix}2 & -1 & 3\\ -1 & 2 & 1\\ 3 & 1 & -4\end{bmatrix}\quad B=\begin{bmatrix}8\\ 4\\ 0\end{bmatrix}\quad X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$$

$$A=\begin{bmatrix}-1 & 2 & 1\\ 2 & -1 & 3\\ 3 & 1 & -4\end{bmatrix}\quad B=\begin{bmatrix}4\\ 8\\ 0\end{bmatrix}$$

$$A=\begin{bmatrix}-1 & 2 & 1\\ 0 & 3 & 5\\ 0 & 7 & -1\end{bmatrix}\quad B=\begin{bmatrix}4\\ 16\\ 12\end{bmatrix}$$

B. Karthikeya

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -\frac{38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$R_3 \to R_3 - \frac{7}{3} R_2$

$= -1 - \frac{35}{3}$

$= -\frac{38}{3}$

$\therefore \rho(A) = \rho(\cancel{B})(A:B)$ consistency.

2 iii)

Q) $4x - y = 12, \; -x + 5y - 2z = 0, \; -2x + 4z = -8$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & +19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$

consistency.

B. Karthi Priya.

b) For what values of λ and μ.

$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$

i) no solution. ii) a unique solution (iii) infinite no. of solution.

$$AX = B$$

Sol:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix}$$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$$

(i) No solution $(\rho(A) \neq \rho(A:B))$

$\lambda = 3,\ \mu \neq 10$

(ii) a unique solution. $\rho(A) = \rho(A:B) = n$

$\lambda \neq 3$, μ any value

(iii) Infinite solution $\rho(A) = \rho(A:B) < n$

$\lambda = 3,\ \mu = 10$

$n \rightarrow$ no. of unknows.

$x, y, z.$

B. Karthikeya

B. Karthikeya

Q) Find for what value of λ Have a solution. & ~~Find~~ $\cancel{\lambda}$ ~~to~~ solve for each value of λ.

$$x+y+z=1$$
$$x+2y+4z=\lambda$$
$$x+4y+10z=\lambda^2$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A)=2$

$\rho(A:B)=2$

$$A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-1-3\lambda+3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix}$$

$$\lambda^2-3\lambda+2=0$$
$$\lambda^2-2\lambda-\lambda+2=0$$
$$\lambda(\lambda-1)-2(\lambda-1)=0$$
$$(\lambda-1)(\lambda-2)=0$$
$$\lambda=1,2$$

$\rho(A)=\rho(A:B)\neq n$
3 → unknowns

<u>Case-1</u> $\lambda=1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$x+y+z=1$$
$$y+3z=0$$

$z=k$

$y=-3k$

$x=1-y-z$

$=1+3k-k=1+2k$

Case - 2 $\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 1$

$y = 1 - 3K$, $z = K$

$x = 1 - 1 + 3K - K = 2K$

d) Find the solⁿ of the system of equations.

$x+3y-2z=0,\ 2x-y+4z=0,\ x-11y+14z=0.$

B. Karthikeya

$$A=\begin{bmatrix}1&3&-2\\2&-1&4\\1&-11&14\end{bmatrix}\quad X=\begin{bmatrix}x\\y\\z\end{bmatrix}\quad B=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&3&-2\\0&-7&8\\0&-14&16\end{bmatrix}=\begin{bmatrix}1&3&-2\\0&-7&8\\0&0&0\end{bmatrix}\ \begin{matrix}0\\0\\0\end{matrix}$$

$\cancel{12z=0}$

$\cancel{z\neq 0}$

$\cancel{-7y=0}$

$\cancel{y=0}$

$\cancel{x\neq 0}$.

$\rho(A)=\rho(A:B)$
$=2\neq n$

Infinite solution

e) Find for what values of λ the given equations $3x+y-\lambda z=0$, $4x-2y-3z=0$, $2\lambda x+4y+\lambda z=0$, may posses non-trivial solution and solve them completely in each case

Infinite

$$A=\begin{bmatrix}3&+1&-\lambda\\4&-2&-3\\2\lambda&4&\lambda\end{bmatrix}=\begin{bmatrix}3&+1&-\lambda\\4&-2&-3\\2\lambda+3&4+1&0\end{bmatrix}$$

$R_3=R_3+R_1$

$$\begin{bmatrix}12&4&-4\lambda\\12&-6&-9\\2\lambda+3&5&0\end{bmatrix}=\begin{bmatrix}12&4&-4\lambda\\0&-10&-9+4\lambda\\2\lambda+3&5&0\end{bmatrix}$$

$R_1=R_1 4\ /\ R_2=R_2 3$ $\qquad R_2=R_2-R_1$

$$=\begin{bmatrix}12&4&-4\lambda&0\\0&-10&-9+4\lambda&0\\2\lambda+3&5&0&0\end{bmatrix}=\begin{bmatrix}12&-6&-9&0\\0&-10&-9+4\lambda&0\\2\lambda+3&5&0&0\end{bmatrix}$$

$R_1=R_1+R_2$

$$\begin{vmatrix}12&-6&-9&0\\0&-10&-9+4\lambda&0\\2\lambda+3&0&\frac{-9+4\lambda}{2}&0\end{vmatrix}$$

$x(2\lambda+3)+5y=0$

$\cancel{-10y+(-9+4\lambda)z=0}$

$\cancel{12x+4y-4\lambda z=0}$

$2\lambda+3=0 \qquad \frac{-9+4\lambda}{2}=0$

$\lambda=\frac{-3}{2} \qquad \lambda=\frac{9}{4}$

$10y=(-9+4\lambda)z$

$z=10y/(-9+4\lambda)$

$=12x+6y-9z=0$

$-10y+(-9+4\lambda)z=0$

$(2\lambda+3)x+5y=0$

$-\frac{(2\lambda+3)x}{5}=y,\ \cancel{-9+4}$

B. Karthikeya

$12x - 6y - 9z = 0$

$12x + \frac{6(2\lambda+3)x}{5} - 9\left(\frac{10y}{(-9+4\lambda)}\right) = 0$

$12x + \frac{6(2\lambda+3)}{5}x + 9\left(\frac{\cancel{10}\,2}{(-9+4\lambda)}\left(\frac{2\lambda+3}{\cancel{5}}\right)x\right) = 0$

$12 + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(\cancel{z}(4\lambda-9)} = 0$

$12(4\lambda-9)5 + \underset{(12\lambda+18)}{6(2\lambda+3)}(4\lambda-9) + 18(5)(2\lambda+3) = 0$

$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$

$48\lambda^2 + 384\lambda - 432 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$\lambda^2 + 9\lambda - \lambda - 9 = 0$

$\lambda(\lambda-1) + 9(\lambda-1) = 0$

$(\lambda+9)(\lambda-1) = 0 \Rightarrow \lambda = 1, \lambda = -9$

$\underline{\underline{\lambda=1}}$ ~~x=~~

$-x = y, \quad z = -2y$

$12x - 6y - 9z = 0$

$12(-y) - 6y - 9(-2y) = 0$

$18y - 18y = 0$

$y \in y = 0$ ~~non~~ trivial solution

$\underline{\lambda = -9}$

$y = \left(\frac{-(2\lambda+3)x}{5}\right)$ $\qquad z = \frac{10y}{(-9+4\lambda)}$

$= -\frac{(-18+3)}{5}x = +\cancel{15}^{3}x$ $\qquad = \frac{10y}{-45} = \frac{2y}{-9}$

$12x - 6y - 9z = 0$

$\cancel{12}^{4}\left(\frac{y}{\cancel{3}}\right) - 6y - \cancel{9}\left(\frac{2y}{-\cancel{9}}\right) = 0$

$4y - 6y + 2y = 0$

$y = 0$ trivial solution

It has no trivial solution

Assignment-2

Q)

1) [1,0,0], [1 1 0], [1 1 1]

$$\begin{bmatrix} c_1+c_2+c_3 & c_2+c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$c_3=0$, $c_2=0$, $c_1=0$

unique solution
Linearly independent.

Q) [6 0 3 1 4 2], [0 -1 2 7 0 5], [12 3 0 -19 8 -11]

$$\begin{bmatrix} 6c_1+12c_3 & -1c_2+3c_3 & 3c_1+2c_2 & c_1+7c_2-19c_3 & 4c_1+8c_3 & 5c_2-11c_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$c_1+2c_3=0$

$-c_2+3c_3=0$

$c_2=3c_3$

$c_1+2c_3=0$

$c_1+7c_2-19c_3=0$

$3c_1+2c_2=0$

$5c_2-11c_3=0$

$15c_3-11c_3=0$

$4c_3=0$

$c_3=0$

$c_2=0$ $\quad$ $c_1=0$

uniquesolution. linearly independent

B Karthikeya.

2) $[7\ -3\ 11\ -6], [-56\ 24\ -88\ 48]$ B. Karthikeya

$[7c_1-56c_2 \quad -3c_1+24c_2 \quad 11c_1-88c_2 \quad -6c_1+48c_2]$

$= [0 \quad 0 \quad 0 \quad 0]$

$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

$7c_1 = 56c_2$

$c_1 = 8c_2$

$c_1 = 8k$

linealy dependent

3) $[-1\ 5\ 0], [16\ 8\ -3]. [-64\ 56\ 9]$

$[-1c_1+16c_2-64c_3 \quad 5c_1+8c_2+56c_3 \quad -3c_2+9c_3]$

$= [0 \quad 0 \quad 0]$

$-3c_2+9c_3=0$

$3.\cancel{9}c_3=\cancel{3}c_2$

$c_2=3c_3$
$=3k$

$-c_1+16c_2-64c_3=0$

$16c_2=64c_3+c_1$

$16(3c_3)=64c_3+c_1$

$c_1=0$

vector $= k \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ linealy dependent

4) $[1\ -1\ 1]; [1\ 1\ -1]; [-1\ 1\ 1]. [0\ 1\ 0]$

$[c_1+c_2-c_3 \quad -c_1+c_2+c_3+c_4 \quad c_1-c_2+c_3]$

$c_1+c_2-c_3=0$

$c_1+c_2=c_3$

$c_1+c_1+\cancel{c_3}=\cancel{c_3}$

$2c_1=0 \Rightarrow c_1=0.$

$-c_1+c_2+c_3+c_4=0$

$c_1=c_2+c_3+c_4$

$c_4=-2k$

$c_1-c_2+c_3=0$

$c_2=c_1+c_3.$

$c_2=c_3=k.$

vector $= k\begin{bmatrix} 0 \\ 1 \\ 1 \\ \end{bmatrix}$

linealy dependent

5) [2 -4] , [1 9] , [3 5]

$[2c_1 + c_2 + 3c_3 \quad -4c_1 + 9c_2 + 5c_3] = [0 \quad 0]$

$2c_1 + c_2 + 3c_3 = 0$

$4c_1 + 2c_2 + 6c_3 = 0.$

$9c_2 + 5c_3 + 2c_2 + 6c_3 = 0$

$11c_2 + 11c_3 = 0$

$c_2 = -c_3 = -K.$

$4c_1 = 9c_2 + 5c_3$

$4c_1 = 9(K) - 5K$

$4c_1 = 4K$

$c_1 = K.$

$$\text{Vector} = \begin{bmatrix} K \\ K \\ -K \end{bmatrix} = K\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

linearly dependent

a) [3 -2 0 4] , [5 0 0 +1] ; [-6 1 0 1]

[2 0 0 3]

$[3c_1 + 5c_2 - 6c_3 + 2c_4 \quad -2c_1 + c_3 \quad 0 \quad 4c_1 + c_2 + c_3 + 3c_4] = [0 \quad 0 \quad 0 \quad 0]$

$-2c_1 + c_3$

$2c_1 = c_3$

$c_1 = K$

$c_3 = 2c_1$

$= 2K$

$4c_1 + c_2 + c_3 + 3c_4 = 0$

$20c_1 + \cancel{5c_2} + 5c_3 + 15c_4 = 0$

$3c_1 + \cancel{5c_2} - 6c_3 + 2c_4 = 0$

$- \quad - \quad + \quad -$

$17c_1 + 11c_3 - 13c_4 = 0$

$17K + 11(2K) = 13c_4$

$(17 + 22)K = 13c_4$

$\frac{3\ \cancel{39}K}{\cancel{13}} = c_4 = 3K$

$4c_1 + c_2 + c_3 + 3c_4$

$4(K) + c_2 + 2K + 3(3K) = 0$

$c_2 + 15K = 0$

$c_2 = -15K.$

$$\text{Vector} = \begin{bmatrix} K \\ -15K \\ 2K \\ 3K \end{bmatrix} = K\begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$$

linearly dependent

B. Karthikeya.

7) $[3\ 4\ 7], [2\ 0\ 3], [8\ 2\ 3], [5\ 5\ 6]$

$[3c_1+2c_2+8c_3+5c_4 \quad 4c_1+2c_3+5c_4 \quad 7c_1+3c_2+3c_3+6c_4]$
$=[0\ 0\ 0]$

$3c_1+2c_2+8c_3+5c_4=0$
$4c_1+2c_3+5c_4=0$

$-c_1+2c_2+6c_3=0$

$c_1=2c_2+6c_3$

$5c_1=10c_2+30c_3$

$7c_1+3c_2+3c_3+6c_4=0$

$14c_1+6c_2+6c_3+12c_4=0$
$9c_1+6c_2+24c_3+15c_4=0$

$5c_1-18c_3-3c_4=0$

$5c_1=18c_3+3c_4$

$10c_2+30c_3=18c_3+3c_4$

$12c_3=3c_4-10c_2$

$4c_1+2c_3+5c_4=0$
$36c_1+18c_3+45c_4=0$
$5c_1-18c_3-3c_4=0$

$41c_1+42c_4=0$

$41c_1=-42c_4$

$c_1=\frac{-42}{41}c_4=\frac{-42}{41}k$

$$=\begin{bmatrix}c_1\\c_2\\c_3\\c_4\end{bmatrix}=k\begin{bmatrix}\frac{-42}{41}\\\frac{10543}{492}\\1\\\frac{1506}{216}\end{bmatrix}$$

linearly dependent

$18c_3=5c_1-3c_4$

$c_3=5\left(\frac{-42}{41}\right)k-\cancel{3}(k)\left(\frac{1}{\cancel{18}_6}\right)$

$=\frac{-(210(6)-6(41))}{(41)(6)}k=\frac{(1260+216)k}{216}$

$=\frac{1506k}{216}$

$\frac{\cancel{6}\times36+35}{36}$

$\frac{-42}{41}k=2c_2+\cancel{6}\left(\cancel{6}\left(\frac{251}{3\cancel{6}_6}\right)\right)k$

$2c_2=-\frac{42k}{41}-\frac{251}{6}k$

$c_2=-\left(\frac{252+10291}{2\times246}\right)k=\frac{10543k}{492}$

B. Karthikeya

Eigen Values

1) $A=\begin{bmatrix}-2\\2\\-1\end{bmatrix}$

$=$

1)

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} \qquad A - \lambda I = 0$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = \begin{array}{l} (-2-\lambda)(-\lambda(1-\lambda)+6(-2)) \\ -2(-\lambda(2)+6(-1)) \\ -3(-4+1-\lambda) \end{array}$$

$$= (-2-\lambda)(-\lambda+\lambda^2-12) + 2(+\lambda+6) + 3(+3+\lambda)$$

$$= +2\lambda - 2\lambda^2 + 24 + \lambda^2 - \lambda^3 + 12\lambda + 2\lambda + 12 + 9 + 3\lambda \qquad \begin{array}{r} 21 \\ 24 \\ \hline 45 \end{array}$$

$$0 = 19\lambda - \lambda^2 - \lambda^3 + 45$$

$$\lambda^3 + \lambda^2 - 19\lambda - 45 = 0$$

2)

$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \quad A - \lambda I = 0$$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = \begin{array}{l} (4-\lambda)(1-\lambda)(1-\lambda) + \\ 1(+2(1-\lambda)) \end{array}$$

$$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$$

$$= (4 - \lambda - 4\lambda + \lambda^2 + 2)$$

$$= (\lambda^2 - 5\lambda + 6)$$

$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

$\lambda = -1, 2, 3.$

Eigenvalues.

$\lambda = -1$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$5x + z = 0$

$-2x + 2y = 0$

$-2x + 2z = 0$

$\underline{10x + 2z = 0}$

$-12x = 0$

$x = 0$

$z = 0$

$y = 0$

Eigen vector = $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$2x + z = 0$

$-2x - y = 0$

$-2x - z = 0$

$x = K$

$z = 2K$

$y = -2K$

Eigen vector = $K \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$$

$x + z = 0$

$-2x - 2y = 0$

$-2x - 2z = 0$

$x = -z = -K$

$z = K$

$y = -x$

$y = K$

Vector = $\begin{bmatrix} -K \\ K \\ K \end{bmatrix}$

$= +K \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3) = $\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$

$\therefore A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$

$(5-\lambda)(-\lambda)(3-\lambda) = 0$

$\lambda = 0, 3, 5$

$\lambda = 0$

$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

$5x = 0 \Rightarrow x = 0$

$-x + 3z = 0$

$z = 0$

$\text{Vector} = K \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = 3$ $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} =$

$2x = 0$

$-3y = 0$

$-x = 0$

$\text{vector} = K \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\lambda = 5$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$-5y = 0$

$-x - 2z = 0$

$x = -2z = -2K$

$\text{vector} = \begin{bmatrix} -2K \\ 0 \\ K \end{bmatrix} = K \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$

4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$+\lambda((3-\lambda)(+2+\lambda)) = 0$

$\lambda = 0, 3, -2$

$\lambda = 0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} =$ $\quad 3y + \cancel{3}4z = 0$

$-z = 0$

$z = 0, y = 0$

vector $= K\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 3$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$ $\quad 4z = 0$

$-5z = 0$

vector $= K\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = -2$ $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$ $\quad 5y + 4z = 0$

$5y = -4z = -4K$

$y = -\frac{4}{5}K$

$K\begin{bmatrix} 0 \\ -\frac{4}{5} \\ 1 \end{bmatrix}$