Assigments

1)

$$A=\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\6&8&7&5\end{bmatrix}=\begin{bmatrix}1&2&3&0\\0&0&-3&2\\0&-4&-8&3\\0&-4&-11&5\end{bmatrix}=\begin{bmatrix}1&2&3&0\\0&0&-3&2\\0&-4&-8&3\\0&0&-3&2\end{bmatrix}$$

$R_2\to R_2-2R_1$ $\quad R_4\to R_4-4R_1$ $\quad R_4\to R_4-R_3$ $\quad R_4\to R_4-R_2$

$R_3\to R_3-3R_1$

$$\begin{bmatrix}1&2&3&0\\0&-4&-8&3\\0&0&-3&2\\0&0&0&0\end{bmatrix}$$

no. of non zero rows.
rank = 3

2)

$$A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}\Rightarrow A^{-1}=\frac{1}{4-1}\begin{bmatrix}2&+1\\+1&2\end{bmatrix}=\begin{bmatrix}\frac{2}{3}&+\frac{1}{3}\\\frac{1}{3}&2/3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{3}-\lambda&1/3\\1/3&\frac{2}{3}-\lambda\end{bmatrix}=\left(\frac{2}{3}-\lambda\right)^2-\left(\frac{1}{3}\right)^2=0$$

$\boxed{a^2-b^2=(a-b)(a+b)}$

$$\left(\frac{1}{3}-\lambda\right)(1-\lambda)=0$$

$\boxed{\text{Eigenvalues } \lambda=1,\ 1/3}$

$\lambda=1$

$$\begin{bmatrix}-1/3&1/3\\1/3&-1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$$

$$-x\frac{1}{3}+y\left(\frac{1}{3}\right)=0$$

$\boxed{\text{eigen vector} = K\begin{bmatrix}1\\1\end{bmatrix}}$

$x=y=K$

$\lambda=1/3$

$$\begin{bmatrix}1/3&1/3\\1/3&1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$x=-y$ $\quad\boxed{\text{Eigen vector}=\begin{bmatrix}K\\-K\end{bmatrix}=K\begin{bmatrix}1\\-1\end{bmatrix}}$

$$Z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$Z - \lambda I = \begin{vmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{vmatrix}$$

$(a^2 - b^2 = (a-b)(a+b)$

$$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$$

$$= (5-\lambda)(7-\lambda) = 0$$

$\boxed{\text{eigen values } \lambda = 5, 7}$

$\lambda = 5$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0$$

$x = y$

$\boxed{\text{Eigen Vector} = K\begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 7$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x = -y$$

$\boxed{\text{eigen vector} = K\begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

$K\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

2) $T: W \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b)+(b-c)x+(c-a)x^2.$$

find the rank and nullity of T.

$\rightarrow$ $$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b)+(b-c)x+(c-a)x^2$$

let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(A) = (a-b)+(b-c)x+(c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree at most 2, denoted as P_2

Rank of T:

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1, and x^2) the rank of T is 3.

The Null space of symmetric matrix

$$T(A) = 0$$

this leads to the system of equations

$$a-b=0 \qquad b-c=0 \qquad c-a=0$$

$\therefore$ $a=b=c$

4) $3x - 0.1y + 0.2z = 7.85$ $\Rightarrow x = \dfrac{7.85 - 0.2z + 0.1y}{3}$

$0.1x + 7y - 0.3z = -19.3$ $\Rightarrow y = \dfrac{-19.3 + 0.3z - 0.1x}{7}$

$0.3x - 0.2y + 10z = 71.4$

$\Rightarrow z = \dfrac{71.4 + 0.2y - 0.3x}{10}$

$x(0) = 0,\ y(0) = 0,\ z(0) = 0$

Iteration-1

$x_{(1)} = \dfrac{7.85}{3} = 2.616.$

$y_{(1)} = \dfrac{-19.3 + -0.1(2.616)}{7} = \dfrac{-19.5616}{7} = 2.794$

$z_{(1)} = \dfrac{71.4 + 0.2(-2.794) - 0.3(2.616)}{10} = \dfrac{70.056}{10}$
$= 7.0056.$

Iteration-2

$x_{(2)} = \dfrac{7.85 - 0.2(7.00) + 0.1(2.79)}{3} = \dfrac{6.675}{3} = 2.057$

$y_{(2)} = \dfrac{-19.3 + 0.3(7.00) - 0.1(2.05)}{7} = -2.487$

$z_{(2)} = \dfrac{71.4 + 0.2(-2.487) - 0.3(2.27)}{10} = 7.023$

Iteration-3

$x(3) = \dfrac{7.85 - 0.2(7.023) + 0.1(-2.487)}{3} = 2.064$

$y(3) = \dfrac{-19.3 + 0.3(7.023) - 0.3(2.064)}{7} = -2.544.$

$z(3) = \dfrac{71.4 + 0.2(-2.54) - 0.3(2.064)}{10} = 7.027$

5) B

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Define Consistent and inconsistent system of Equations. Hence Solve the following system of equations if consistent.
$x+3y+2z=0$, $2x-y+3z=0$; $3x-5y+4z=0$, $x+17y+4z=0$

Sol:- **Consistent**: A system of equations is Consistent if it has at least one Solution, meaning the equation have a Common solution.

In Consistent:- A system of equations is inconsistent if it has no solution, meaning the equations do not intersect at any point are Contradictory.

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$C = [A:B]$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$R_4 \to R_4 + R_3$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3 - 2R_2$

$\text{Rank}(C) = \text{Rank}(A) \neq$ no. of unknowns

infinite Solutions.

$AX = B$.

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + 3y + 2z = 0$$

$$-7y - z = 0$$

$$-7y = z$$

$$\boxed{y = \frac{-z}{7}}$$

let $\boxed{z = K}$.

$$x + 3\left(\frac{-K}{7}\right) + 2(K) = 0$$

$$7(x) + (-3K) + 14K = 0$$

$$\boxed{x = -\frac{11K}{7}}$$

6) $T: P_2 \to P_2$ is linear transformation.

$$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$$

→ $T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$ is a linear transformation, we ~~need~~ need to check two properties.

1) Additivity. $T(u+v) = T(u)+T(v)$

2) homogeneity of degree 1:

$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

1) $T(u+v) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$

$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$

$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$

$= (a_1+1)+(b_1+1)x+(c_1+1)x^2 + (a_2+1)+(b_2+1)x + (c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

so function is additive.

∴ Homogeneity of Degree 1:

$T(KV)=T(K(a+bx+c))$

$=T(Ka+Kbx+KC)=(Ka+1)+(Kb+1)x+(Kc+1)$

$=K(a+1)+K(b+1)x+K(c+1)x^2$

$=KT(a+bx+c)$

so, the function is homogeneous of degree 1.

∴ it indeed. linear transformed

7) $S=\{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determine sub space - spanned by S.

→ $S=\{(1,2,3),(3,1,0),(-2,1,3)\}$

can be arranged as a matrix

$$A=\begin{bmatrix}1 & 3 & -2\\ 2 & 1 & 1\\ 3 & 0 & 3\end{bmatrix}$$

Now, let's perform row reduction to obtained the Echelon form:

$$\begin{bmatrix}1 & 3 & -2\\ 2 & 1 & 1\\ 3 & 0 & 3\end{bmatrix}$$

$R_2 \leftarrow R_2-2R_1$ and $R_3 \leftarrow R_3-3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \to R_3 + \frac{9R_2}{5} \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Third row of zeros indicates that the vectors in S are linearly dependent.

for basis of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

(1,3,2) and (0,−5,5) these vectors form a basis for the subspace spanned by S.

∴ Dimension of subspace spanned by S = 2.

∴ Set S is not a basis of R^3 because the row-reduced form has a row of zeros.

∴ the basis for the subspace spanned by S is {(1,3,−2), (0,−5,5)}

∴ The dimension of the subspace is 2

8) using Jocobis method (perform 3 iterations) solve.

$3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$,

with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

→ $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + z(7) = 16$

with initial values

$x = 1, y = 1, z = 1$

Iteration-1.

$$x^{(1)} = \frac{23 + 6y^{(0)} - 2(z)^{(0)}}{3} \approx 9.0$$

$$y^{(1)} = \frac{-15 + 4(x^{(0)}) + z^{(0)}}{1} \approx -9.0$$

$$z^{(1)} = \frac{16 - x^{(0)} - 3(y^{(1)}) +}{7} = \cancel{\frac{16 - 9 - 3(2)}{7}} \quad 2.0$$

Iteration-2

$$x^{(2)} = \frac{23 + 6y^{(1)} - 2(z^{(1)})}{3} = \cancel{\frac{23+6(2) - 2(-8)}{3}} \cancel{= \frac{165}{3}} \cancel{= 55}$$

$$= 5.0$$

$$y^{(2)} = \frac{-15 + 4(x)^{(1)} + z^{(1)}}{1} = -5.0$$

$$z^{(2)} = \frac{16 - x^{(1)} + 3y^{(1)}}{7} \approx 3.0.$$

Iteration-3

$$x^{(3)} = \frac{23 + 6y^{(2)} - 2z^{(2)}}{3} = 6.0$$

$$y^{(3)} = \frac{-15 + 4x^{(2)} + z^{(2)}}{1} \approx -6.0$$

$$z^{(3)} = \frac{16 - x^{(2)} + 3y^{(2)}}{7} \approx 2.0$$

9) Explain one application of matrix operations in image processing with example.

Sol:- Matrix Magic in Image Processing: Convoluting Takes Center stage.

Digital images are essentially grids of tiny squares called pixels, each holding an intensity value (grayscale) or color information (RGB). To manipulate these images effectively, image processing relies heavily on the power of matrices.

1) Image as a Matrix

We represent the image as a matrix, where each pixel's value becomes an element in the grid.

2) Kernel: The Tiny Powerhouse:

We define a small matrix called a Kernel or filter. Different Kernel designs produce different results.

3) The Convolution Slide:
We slide the Kernel across the image matrix, one element at a time.

4) Summing up for a New Value Pixel:
The multiplied values from the element-wise multiplication are then summed up.

5) Iterating Across the Image
We repeat this process (sliding, multiplication, summation) for every position of the Kernel within the image matrix.

Here's an example of how convolution works using a simple image filtering task:

→ let's Consider a small 3x3 gray scale image matrix:

$$\begin{bmatrix} 100 & 150 & 200 \\ 120 & 180 & 220 \\ 110 & 160 & 190 \end{bmatrix}$$

→ Now, we want to apply a simple 3x3 filter/Kernel to this image. Let's take a blur filter

Kernel: $\begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$

→ To apply this filter, we perform element-wise multipulication of the filter Kernel with the corresponding pixels in the image matrix and then sum up the results.

ient if it has not intersect of

32 - 5y 3z=0

This sum becomes the new value for the ~~filter~~ ~~kernel~~
Central pixel in the output image.

$$\left(100\times\frac{1}{9}\right)+\left(150\times\frac{1}{9}\right)+\left(200\times\frac{1}{9}\right)+\left(120\times\frac{1}{9}\right)+\left(180\times\frac{1}{9}\right)+\left(220\times\frac{1}{9}\right)$$

$$+\left(110\times\frac{1}{9}\right)+\left(160\times\frac{1}{9}\right)+\left(190\times\frac{1}{9}\right)=159$$

→ Each pixel value is weighted average of its neighbouring pixels, effectively creating a blurred version of the original image.

→ Convolution allows for the application of various filters to image, enabling tasks such as blurring, sharpening, edge detection and more.

10) Give a brief description of linear transformation for Computer vision for rotating 2D image.

Sol: Linear transformations are a powerful tool in Computer vision for tasks like image rotation. there's a breakdown.

1) Image as a Vector.
we can represent a 2D image as a collection of points, where each point corresponding to a pixel's location (x, y coordinates)

2) Rotation Matrix:
To rotate the image, we use a specific rotation matrix.
3) This matrix encodes the rotation angle and how it affects the x and y coordinates of each point (pixel) in the image.

3) Transformation Magic
By multiplying the image vector (representing all pixel locations) with the rotation matrix, we perform a linear transformation on all the points simultaneously.

Affine transformation include operations such as rotation, translation, scaling and shearing. when rotating a 2D image the affine transformation involves applying a rotation matrix to Coordinates of each ~~plate~~ pixel in the image

For rotating a 2D image clockwise by an angle θ, the rotation matrix R is typically used

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$

Each pixel Coordinate (x, y) in the original image is multiplied by this rotation matrix to obtain the new coordinates (x', y') in the the rotated image. The new pixel value at (x', y') is then determined by interpolating the values of neighboring pixels in the original image.