Assigments

1)

$$A=\begin{bmatrix}1&2&3&0\\2&4&3&2\\3&2&1&3\\6&8&7&5\end{bmatrix}=\begin{bmatrix}1&2&3&0\\0&0&-3&2\\0&-4&-8&3\\0&-4&-11&5\end{bmatrix}=\begin{bmatrix}1&2&3&0\\0&0&-3&2\\0&-4&-8&3\\0&0&-3&2\end{bmatrix}$$

$R_2\to R_2-2R_1$ $\quad R_4\to R_4-4R_1$ $\quad R_4\to R_4-R_3$ $\quad R_4\to R_4-R_2$

$R_3\to R_3-3R_1$

$$\begin{bmatrix}1&2&3&0\\0&-4&-8&3\\0&0&-3&2\\0&0&0&0\end{bmatrix}$$

no. of non zero rows.

rank $=3$

2)

$$A=\begin{bmatrix}2&-1\\-1&2\end{bmatrix}\Rightarrow A^{-1}=\frac{1}{4-1}\begin{bmatrix}2&+1\\+1&2\end{bmatrix}=\begin{bmatrix}\frac{2}{3}&+\frac{1}{3}\\\frac{1}{3}&2/3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{3}-\lambda&1/3\\1/3&\frac{2}{3}-\lambda\end{bmatrix}=\left(\frac{2}{3}-\lambda\right)^2-\left(\frac{1}{3}\right)^2=0$$

$$\boxed{a^2-b^2=(a-b)(a+b)}$$

$$\left(\frac{1}{3}-\lambda\right)(1-\lambda)=0$$

$$\boxed{\text{Eigenvalues } \lambda=1,\ 1/3}$$

$\lambda=1$

$$\begin{bmatrix}-1/3&1/3\\1/3&-1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=0$$

$$-x\frac{1}{3}+y\left(\frac{1}{3}\right)=0$$

$$\boxed{\text{eigen vector}=K\begin{bmatrix}1\\1\end{bmatrix}}$$

$$x=y=K$$

$\lambda=1/3$

$$\begin{bmatrix}1/3&1/3\\1/3&1/3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$x=-y \quad \boxed{\text{Eigen vector}=\begin{bmatrix}K\\-K\end{bmatrix}=K\begin{bmatrix}1\\-1\end{bmatrix}}$$

$z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

$z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$

$(a^2 - b^2 = (a-b)(a+b))$

$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$

$= (5-\lambda)(7-\lambda) = 0$

$\boxed{\text{Eigen values } \lambda = 5, 7}$

$\lambda = 5$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0$

$x = y$

$K \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\boxed{\text{Eigen Vector} = K \begin{bmatrix} 1 \\ 1 \end{bmatrix}}$

$\lambda = 7$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x = -y$

$K \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\boxed{\text{eigen vector} = K \begin{bmatrix} 1 \\ -1 \end{bmatrix}}$

4)

$3x - 0.1y - 0.2z = 7.85.$

$0.1x + 7y - 0.3z = -19.3$

$0.3x - 0.2y + 10z = 71.4$

$x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$

$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$

$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$

Iteration-1:

$z = y = 0$

$x^{(1)} = \frac{7.85}{3} = 2.61$

$z = 0$

$y = \frac{1}{7}\left(-19.3 - 0.1\left(\frac{7.85}{3}\right)\right) = 2.79$

$y(1) = 2.79$

$z = \frac{1}{10}[71.4 - 0.3(2.61) + 0.2(2.79)$

$z = \frac{1}{10}(71.4 - 0.783 + 0.558)$

$z(1) = 7.1175$

Iteration-2

$$x_{(2)} = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$$

$$= 2.9255$$

$$y_{(2)} = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$$

$$= 3.0123$$

$$z_{(2)} = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10} = 7.0132$$

Iteration-3

$$x^{(3)} = \frac{7.85 \cdot 0.1(2.9255) - 0.2(7.0132)}{3} \approx 3.0032$$

$$y^{(3)} = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7} = 3.001$$

$$z^{(3)} = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10} = 7.00$$

$x = 3.0032$, $y = 3.0001$, $z \approx 7.000$.

5) Define Consistent and in Consistent

$x+3y+2z=0,\ 2x-y+3z=0,\ 3x-5y+4z=0,\ x+17y+4z=0$

Sol: $A=\begin{bmatrix}1&3&2\\2&-1&3\\3&-5&4\\1&17&4\end{bmatrix}$ after performing row reduction we obtained Echelon form.

$A=\begin{bmatrix}1&3&2\\0&-7&-1\\0&-14&-2\\0&14&2\end{bmatrix}$ $R_2\to R_2-2R_1$

$R_3\to R_3-3R_1$

$R_4\to R_4-R_1$

$\begin{bmatrix}1&3&2&0\\0&-7&-1&0\\0&-14&-2&0\\0&0&0&0\end{bmatrix}\to$ $R_3\to R_3-2R_2$

$\begin{bmatrix}1&3&2&0\\0&-7&-1&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$ This Corresponds to the system

$x+3y+2z=0$

$-7y-z=0$

Now let's Express the Variables in terms of parameters. let $y=t$;

$x=-3t,\ z=-7t$

so, the system has infinitely many solution give by

$x=-3t,\ y=t,\ z=-7t$

The sytem is Consistent and dependent

2) $T: W \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2.$$

find the rank and nullity of T.

$\to$ $$T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-a)x^2$$

let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(A) = (a-b) + (b-c)x + (c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ The image of T is the set of all polynomials of degree at most 2, denoted as P_2

Rank of T:

The rank of T is the dimension of its image since P_2 has a dimension of 3 (coefficients for x^0, x^1, and x^2) the rank of T is 3.

The Null space of symmetric matrix

$$T(A) = 0$$

this leads to the system of equations

$$a - b = 0 \qquad b - c = 0 \qquad c - a = 0$$

$\therefore$ $a = b = c$

$\therefore$ T is the set of symmetric matrices of the form.

$\begin{bmatrix} t & t \\ t & t \end{bmatrix}$ where t is any scaler.

$\therefore$ Dimension = 1
(using only 't')

$\therefore$ rank T is 3
the nullity of T is 1

6) $T: P_2 \to P_2$ is linear transformation.

$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$

$\rightarrow$ $T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$ is a linear transformation, we ~~we~~ need to check two properties.

1) Additivity. $T(u+v) = T(u)+T(v)$

2) homogeneity of degree 1:
$T(ku) = kT(u)$ for all u in the domain of T and all scalars k.

1) $T(u+v) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$
$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$
$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$
$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$T(a_1+b_1x+c_1) + T(a_2+b_2x+c_2)$

So function is additive.

$\therefore$ Homogeneity of Degree 1:

$T(Kv) = T(K(a+bx+c))$

$= T(Ka+Kbx+Kc) = (Ka+1)+(Kb+1)x+(Kc+1)$

$= K(a+1)+K(b+1)x+K(c+1)x^2$

$= KT(a+bx+c)$

So, the function is homogeneous of degree 1.

$\therefore$ it indeed. linear transformed

7) $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determine sub space - spanned by S.

$\rightarrow$ $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$

can be arranged as a matrix

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

Now, let's perform row reduction to obtained the Echelon form:

$$\begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \leftarrow R_2 - 2R_1$ and $R_3 \leftarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \to R_3 + \frac{9R_2}{5} \quad \begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

∴ Third row of zeros indicates that the vectors in S are linearly dependent.

for basis of the subspace spanned by S.

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

(1,3,2) and (0,−5,5) these vectors form a basis for the subspace spanned by S.

∴ Dimension of subspace spanned by S = 2.

∴ Set S is not a basis of R^3 because the row-reduced form has a row of zeros.

∴ the basis for the subspace spanned by S is {(1,3,−2), (0,−5,5)}

∴ The dimension of the subspace is 2

8) using Jocobis method (perform 3 iterations) solve.

$3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$,

with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

→ $3x - 6y + 2z = 23$

$-4x + y - z = -15$

$x - 3y + 7z = 16$

with initial values $x = 1, y = 1, z = 1$

Iteration-1.

$$x^{(1)} = \frac{23+6y^{(0)} - 2(z)^{(0)}}{3} \approx 9.0$$

$$y^{(1)} = \frac{-15+4(x^{(1)})+z^{(0)}}{1} \approx -9.0$$

$$z^{(1)} = \frac{16 - x^{(1)} - 3(y^{(1)})}{7} = \cancel{\frac{16-9-3(2)}{7}} \; 2.0$$

Iteration-2

$$x^{(2)} = \frac{23+6y^{(1)} - 2(z^{(1)})}{3} = \cancel{\frac{23+6(2)-2(-8)}{3}} \cancel{= \frac{165}{3} = 55}$$

$$= 5.0$$

$$y^{(2)} = \frac{-15+4(x)^{(1)}+z^{(1)}}{1} = -5.0$$

$$z^{(2)} = \frac{16 - x^{(1)} + 3y^{(1)}}{7} \approx 3.0.$$

Iteration-3

$$x^{(3)} = \frac{23+6y^{(2)} - 2z^{(2)}}{3} = 6.0$$

$$y^{(3)} = \frac{-15+4x^{(2)}+z^{(2)}}{1} \approx -6.0$$

$$z^{(3)} = \frac{16 - x^{(2)} + 3y^{(2)}}{7} \approx 2.0$$