Two Dimensional Picture Languages

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Introduction

This report provides an overview and a summary of the work of Gimmarresi and Restivo [?] on two-dimensional languages.

1 Two-dimensional Languages

Picture languages extend the concepts of formal language theory to two dimensions to possibly be applied in the domain of pattern recognition and image processing. Using symbols taken from a finite alphabet, a rectangular array describes a picture. A set of pictures (or two-dimensional strings) form a two-dimensional language.

As an example $L = \{p \mid p(1, i) = p(l_1(p), i), i = 1, \ldots, l_2(p)\}$ denotes a two-dimensional language over the alphabet $\Sigma = \{0, 1\}$ where the first row is equal to the last one. $l_1(p)$ describes the number of rows in p; $l_2(p)$ the number of columns. A picture $p \in L$ could look like this:

1	0	1	0
0	1	0	0
0	0	1	0
0	1	1	1
1	0	1	0

In order to allow scanning on pictures, any picture q of size (m, n) there is a picture $\widehat{q}$ of size $(m+2, n+2)$ that marks the borders of q with a symbol $\# \notin \Sigma$. For p, $\widehat{p}$ is:

#	#	#	#	#	#
#	1	0	1	0	#
#	0	1	0	0	#
#	0	0	1	0	#
#	0	1	1	1	#
#	1	0	1	0	#
#	#	#	#	#	#