Assignment 1

Student Info

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Problem

If the first 6 terms of Taylor expansion are used to calculate

$$\int_0^1 e^{-x^2}\,dx$$

find its error, including truncation error and roundoff error.

Answer

Let $f(x) = e^{-x^2}$ and expand the original equation with Taylor Series at point 0.

$$\begin{aligned}
\int_0^1 f(x) &= \int_0^1 \Sigma_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}(x)^n \\
&= \int_0^1 \Sigma_{m=0}^{\infty} \frac{(-1)^m x^{2m}}{m!} \\
&= \Sigma_{m=0}^{m\to\infty} \frac{(-1)^m}{(2m+1)\cdot m!}
\end{aligned}$$

And its first 6 non-zero terms are:

$$\begin{aligned}
\int_0^1 f(x) &\approx \int_0^1 (1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!})|_0^1 \\
&\approx x - \frac{x^3}{3} + \frac{x^5}{5\cdot 2!} - \frac{x^7}{7\cdot 3!} + \frac{x^9}{9\cdot 4!} - \frac{x^{11}}{11\cdot 5!}|_{x=0}^{x=1} \\
&\approx \Sigma_{m=0}^{m=5} \frac{(-1)^m}{(2m+1)\cdot m!}
\end{aligned}$$

Result

Definition of Variables

Exact Value	x
Estimated Value	x^*
Truncation Error	R_n
Roundoff Error	E

Truncation Error

$$R_n = x - x^* = \Sigma_{n=6}^{n \to \infty} \frac{(-1)^n}{(2n+1) \cdot n!}$$

Roundoff Error:

The python code suggests that the $x^* = 0.7467291967291968$. Suppose that only preseve 4 digits after decimal point.

Round-by-chop

$$x = 0.7467 \; E = x^* - x = -0.0000291967291968$$

Round-to-nearest

$$x = 0.7467 \; E = x^* - x = -0.0000291967291968$$