## MATH10073: Linear Programming, Modelling and Solution (LPMS)

# Case study: EPower

## The problem

As presented by the client: `EPower`

### Overview

`EPower` is provides electricity for a small country from six power sources: gas, coal, nuclear, wind, hydro and an interconnect. The company needs your advice on how to plan for a reduction in carbon dioxide emissions. The objective is to maximize profit.

### Terminology

Power output is measured in Megawatts (MW). Electricity (energy) generated is power output over a period of time: it is equal to "power" × "time" and is measured in Megawatt hours (MWh). So, for example, a power station operating at 5,000MW for four hours generates 20,000MWh of energy.

### Power source parameters

The model of each power source has three parameters

- The maximum power output (MW)

  This is an upper bound on the power output

- The running cost (£/MWh)

  This is the cost of energy per MWh. For example, if a particular power source costs £30/MWh and operates at 5,000MW for four hours then the energy it generates [20,000MWh] costs £600,000= £30/MWh×20,000MWh

- The increase cost (£/MW)

  There is a cost for increasing the output of a power source. When fuel is used to produce heat, power cannot just be "switched on": extra fuel must be used before the output can be increased. When power is bought from another country, there is a penalty for increasing the amount bought. For renewable energy sources such as wind and hydro, there is a low cost of increasing power output. Hydro power can be switched on and, although wind power output increases immediately if the wind blows harder, this clearly cannot be controlled!

  The cost of increasing power output cannot be modelled by multiplying the difference in power outputs by the cost in £/MW. If the difference is negative, this would allow money to be made by reducing power output! Thus the increase is modelled as a non-negative decision which must be at least the difference in power output.

Electricity is bought from EPower at a cost of £32/MWh

### Emissions

Burning fossil fuels creates emissions which damage the environment, the principal ones being carbon dioxide ($CO_2$) and sulphur (as oxides $SO_2$ and $SO_3$). Sulphur is associated with burning coal and causes "acid rain" which has harmful effects on plants and aquatic animals. Carbon dioxide is the principal cause of global warming. `EPower` must not exceed daily limits of these two pollutants, whose emissions are linear functions of the amount of electricity generated from coal and gas fired power stations.

### Demand

Power demand varies through the day and the total (net) output from the power sources must equal demand. Electricity itself cannot be stored on a large scale, although pump-storage schemes allow (potential) energy to be stored. Since there is a cost associated with increasing power output, the optimal power generation strategy uses sources with low "start-up" cost to satisfy "spikes" in demand. Assuming that the demand pattern each day is the same, the power outputs in the final period of the day should be set with a view to demand "rolling over" at the end of the period to the level of the first period of the day.

### Wind power

Generating electricity from the power of the wind is the most popular form of "renewable" energy. `EPower` has an installed capacity of 12,000MW but, on a given day, the wind will only blow with a particular power and the output may be very much less than this.

### Pumped-storage hydro

Natural flow of water due to rain falling in higher areas flowing to the sea via water systems is a source of energy used in many countries with high mountains. The potential energy of water falling as rain in mountains is converted into kinetic energy of flowing water which can be used to driver generating turbines. This is usually achieved by collecting water in a reservoir and releasing it when power is required. For `EPower`, the average natural flow into its reservoirs corresponds to a power output of 400MW. If this is collected over 24 hours, the reservoir contains 9600MWh of energy which can be released at a maximum output of 2000MW. This is a cheap energy source [once the expensive reservoir has been built] and can be switched on almost instantaneously. As such, it is ideal for satisfying spikes in demand. However, with the exception of countries like Norway, the nature of the geography is such that relatively little total power demand can be satisfied from pure hydro systems.
In countries whose upland areas are relatively small, reservoirs can be used to store significantly more potential energy than would accumulate naturally by using power generated at times of low demand to pumping water up to them. [Examples in the UK are Ffestiniog (360MW) and Dinorwig (1,728MW) in Wales and Cruachan (440MW) in Scotland.] At times of high demand, the water is released and the pumps operate as turbines to generate electricity. Not all the energy used to pump water up is recovered: the efficiency is about 80%. The store of energy in the reservoir is a natural inventory problem: in each period there is a flow in and out, and the amount stored must lie between zero and the maximum reserve. This store of energy must be the same at the end of a day as it was at the beginning of the day: the standard inventory "wrap-around". [The energy loss due to the imperfect efficiency of the system means that it should never be optimal to pump and release in the same period.]

### Interconnect

Another source of electricity is to buy it from another power system. For example, there is a cable connecting the British and French systems, allowing power to be transmitted between them. For `EPower` it is convenient to view this as a (relatively expensive) power source.

### Data

The file `EPower.dat` contains all the data required to model the company's system for planning purposes.

## The consultancy process

As presented by a senior colleague of LPMS consulting.

**Overview**

- Develop a "**base case**" model which replicates current decision-making. This builds confidence with the client, demonstrating understanding of status quo and generating faith in future recommendations.
- Use the model to investigate what `EPower` has asked (see below for details). Gives the client what was asked for
- Identify a few further specific ideas for the client to consider. This shows we do more than the minimum
- Suggest what further consulting might consider. Hopefully this will lead to us getting a further fee!

**Stage 1 (Week 9)**

Stage 1 of the case study is to develop a model the company and use it to identify how generating policy changes as $CO_2$ emissions are reduced by 50%. At this stage, assume an constant maximum wind power output of 6,000MW.

Identify the **Decision variables:**

- The primary decisions in each time period are the power output for each source and the power demand for pumping water.
- Other decisions in each time period are the increase in power output for each source (which is not simply the difference in output levels) and the reserve in hydro reservoir. The latter is implied by the flows in/out but, like manufacturing inventory, it is easier to model with excess variables related by equations.

Identify the **Objective:**

- The income from the electricity produced to satisfy demand, less the costs due to electricity generation and increase in power output.

Identify the **Constraints:**

- Generator power output in each period must not exceed the maximum available
- Total power output in each period must equal the demand to be supplied plus the demand for pumping to the hydro reservoir
- Emissions limits must not be exceeded
- Reservoir inventory must be maintained

The necessary data are in `EPower.dat`. Note that simplified (interim) models which can be used for development are

- Model with data for `Times: [1]` corresponding to average demand over the day.
- Model with a full seven-period day, but without pump storage hydro

`EPower` understands that the limits on carbon dioxide emissions are to be reduced by up to 50%. Use your model to identify the change in generating policy as this limit is reduced from the current value to half of that value.

Note that in order to develop the model and carry out investigations, it will be necessary to set up "special case" constraints etc for specific power sources. The aim of developing wholly generic models driven which are blind to the content of index sets is an ideal which cannot always be achieved.

### Stage 2 (Week 10)

To be announced

### Stage 3 (Week 11)

Each group will deliver its findings as an audio-visual presentation and written report.

## Deliverables

Each group should give a 10-minute presentation, a report with a cover page plus no more than 4 sides of A4, one Mosel file and one data file. The breakdown of marks to be assigned is as follows

| Marks | For |
|---:|---|
| 20 | Results |
| 20 | Mosel skills |
| 20 | Report content |
| 10 | Slide content |
| 10 | Report-writing skills |
| 10 | Slide presentation skills |
| 10 | Bonus |

The bonus is for observations/conclusions/recommendations and unprompted investigations

### Presentation

The audience for the presentation is the manager and his assistant who understand electricity supply and business, but not linear programming. Everyone is to speak for 2-3 minutes and the presentation should introduce the group members, the context and base case results, present the results for the scenario investigated and suggest ideas and scope for further investigation. You should be graphically creative, but not too much!

### Report

**Contents**

The report should introduce the aspect of the company to be investigated, placing the investigation in context. It should not include large tables of data (refer to an appendix if necessary) or "equations". It must state the "base case" results clearly, using tables and charts (sparingly), showing the client that you are starting from a point well-understood. Consider the points to be investigated systematically, using tables and charts (sparingly), and don't just give results, but also interpretation and analysis. Think what else you could investigate with your model and offer general observations, conclusions and recommendations.

**Appearance**
Aim to produce a neat and tidy document with right-left justified text, reported values quoted to a sensible number of significant figures and with tables and charts a sensible size. Don't spend **hours** beautifying your document since this can raise suspicions that the content might be weak. Something of **you** should come across!

**Model**

Write the model to be readable by a Mosel user, ensuring that the base case and (at least) your scenario can be solved using your (modified) `EPower.dat`. Submit the file `EPower.dat` corresponding to your scenario. Use good Mosel style.