## The Minimum Surface Area Bounded by Two Coaxial Rings

The minimum surface area bounded by two coaxial rings perpendicular to the axis will, by symmetry, be symmetrical about the axis. Thus it is sensible to consider a sequence of varied surfaces that are symmetric about the axis and are bounded by the two rings (Fig. AIII.1), when applying the Euler–Lagrange equation. The axis of symmetry is taken as the $x$-axis. In the $x$–$y$ plane an element of the length of the surface is $ds$ (Fig. AIII.1). The area of surface contained between two planes perpendicular to the axis at $x$ and $x+dx$ is

$$dA = 2\pi y ds. \tag{AIII.1}$$

Now expressing $ds$ in Cartesian coordinates,

$$ds = (1+y_x^2)^{1/2}\, dx, \tag{AIII.2}$$

where

$$y_x = \frac{dy}{dx}. \tag{AIII.3}$$

Therefore

$$dA = 2\pi y(1+y_x^2)^{1/2} dx. \tag{AIII.4}$$

If the rings are at positions $x_1$ and $x_2$, the total area of the surface is

$$A = \int_{x_1}^{x_2} 2\pi y(1+y_x^2)^{1/2}\, dx. \tag{AIII.5}$$

Thus $A$ is in the appropriate form for the application of the Euler–Lagrange equation. The Euler–Lagrange function, $f$, Eq. (AI.16), is given by

$$f = 2\pi y(1+y_x^2)^{1/2}. \tag{AIII.6}$$