

# CS414 Homework 1 Report

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Colab Notebook Link

<https://drive.google.com/file/d/1goXCljPsYgt85Z2S-899r4CvB5nJNmr5/view?usp=sharing>

## Question 1

In a network to study interactions between the characters of a book, nodes would represent every unique character in the book, and edges between these nodes would represent the interaction between characters, depending on how we want to create network. For instance, we may create an undirected network, where whether an edge between two individual nodes exists is determined by whether these two characters represented by that nodes are met each other or had an interaction somehow in the story. In this case, let's say if character 'x' and 'y' had met each other in some part of the book, there would be an undirected connection between the nodes representing character 'x' and 'y'. Another idea to build the network would be using weighted and directed edges. In this scenario, the direction of edges might be determined by which character was started the interaction first. Maybe if character 'x' knows about character 'y' but they did not met with each other, there would be a directed edge from 'x' to 'y'. As for the weights of edges, we may look how much a character has an influence on another character. In this way, after we construct the whole network, we might be able to determine who is the most influential character in the story. In addition, also, we can see the fractions or groups in story by looking at the clusters in the visualization of the network.

I would like to say that building such a network about the trilogy of The Lord of the Rings (I guess it is fair to consider it as a single book) would be pretty fun. Since there are tons of characters in the story, and there is no certain main character among them, the network constructed in the end will be quite rich, I believe. If I assume that we have built a directed network with weighted edges as I explained above, characters with the highest strength or influence might be:

1. Gandalf: As a wise and powerful wizard, Gandalf has high influence on other characters and one of the most important figures that shapes the story, since, he has an interaction with almost every character in story, no matter friend or foe.
2. Frodo: As being the Ring-bearer, Frodo interacts lots of characters throughout the story and plays a pivotal role in the plot. Therefore, he would probably have high rank in network.
3. Sauron: Although Sauron had no one-on-one interaction with almost anyone, if we build the network based on the influence of characters, Sauron would be one of the most influential characters in story, since he is the main antagonist and he affects the destiny of almost everyone.

## Question 2

$N = 1000, p = 10^{-3}$

- $\langle E \rangle = \text{All possible edges} * \text{connection probability} = \frac{1000*999}{2} * 10^{-3} = 499.5$

## Question 4

Let's say for a Cayley Tree:

- **k** denotes the degree
  - **d** denotes longest path on the tree
  - **n** denotes the depth of the tree
  - **S** denotes number of nodes
- $2 * n = d$  ; since the longest path would be the path between two leaves that are in totally distinct branch of the tree

so the depth of a cayley tree with longest path of 8 would be:  $n = \frac{8}{2} = 4$

- The number of nodes (S) in a cayley tree with degree k and depth n could be found with following recursive formula:

$S_0 = 1$
$S_1 = S_0 + S_0 \cdot k$
$S_2 = S_1 + (S_1 - S_0) \cdot (k - 1)$
....
$S_n = S_{(n-1)} + (S_{(n-1)} - S_{(n-2)}) \cdot (k - 1)$

- When we plug  $n = 4$  and  $k = 5$  to this recursive formula, we found that number of nodes is **426**

The implementations and calculations of the tree and the formula is on ipynb notebook.

The visualization of a cayley tree with degree  $k = 5$  and longest path  $d = 8$ :

