Name (block capital): May 2015

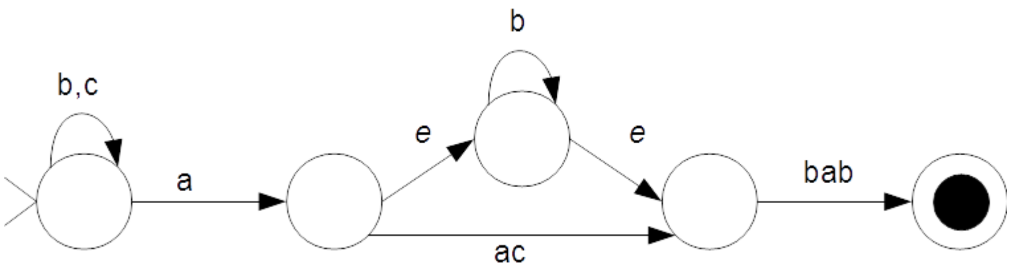
The theory of computation

1. Give RE R with L(R) = {w∈{a, b}*| #a mod 3 = 2}! (3p)

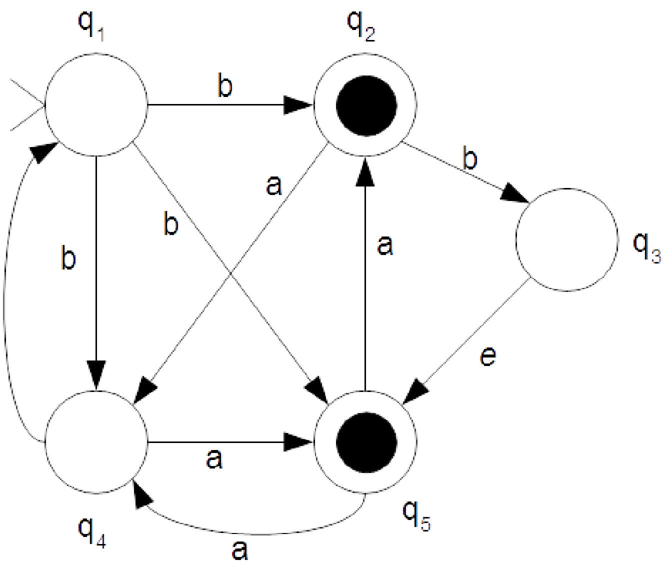
b*ab*ab*(ab*ab*ab*)*

2. Give NFA M with equivalent with the following RE: (3p)

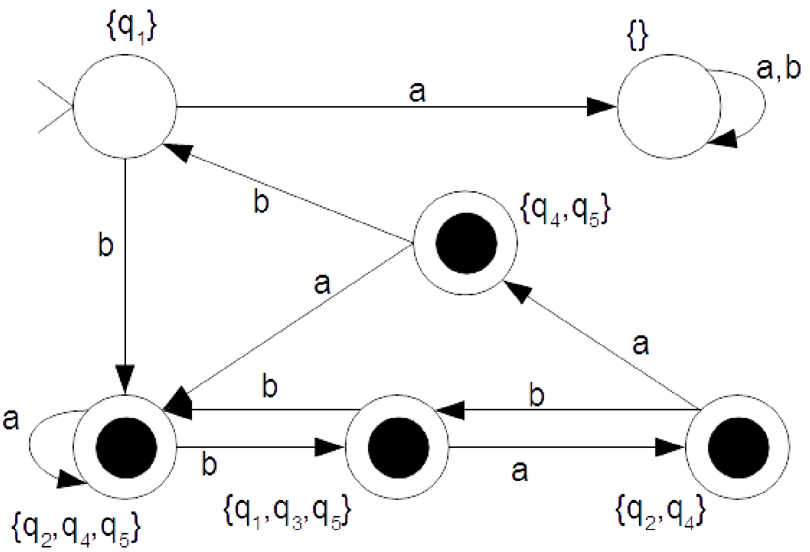
(bUc)*a(b*Uac)bab



3. Give DFA M' such that L(M') = L(M)! (6p)



$E(q_3)=\{q_3,q_5\}$



4. Give CFG G such that $L(G) = \{a^{2n}(bc)^n ddca^m d^{p+1} abc^m \mid m, n, p \geq 0\}$? (3p)

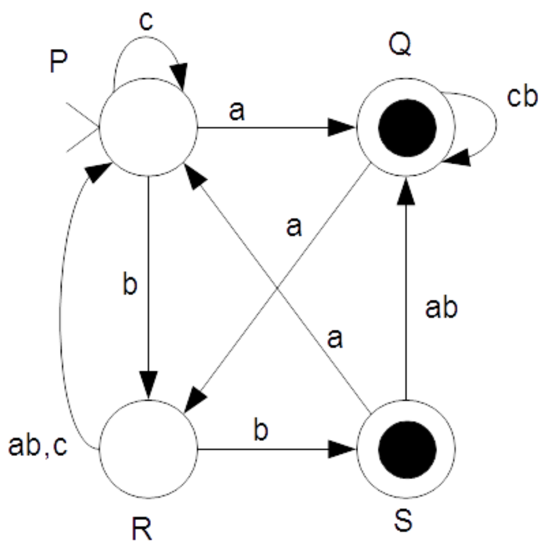
$V=\{a, b, c, d, S, N, M, P\}$

$\Sigma =\{a, b, c, d\}$

$R=\{S \rightarrow NddcM,\ N \rightarrow aaNbc|e,\ M \rightarrow aMc|Pdab,\ P \rightarrow dP|e\}$

$S=S$

5. Construct such RG G which is equivalent with the given NFA! (3p)



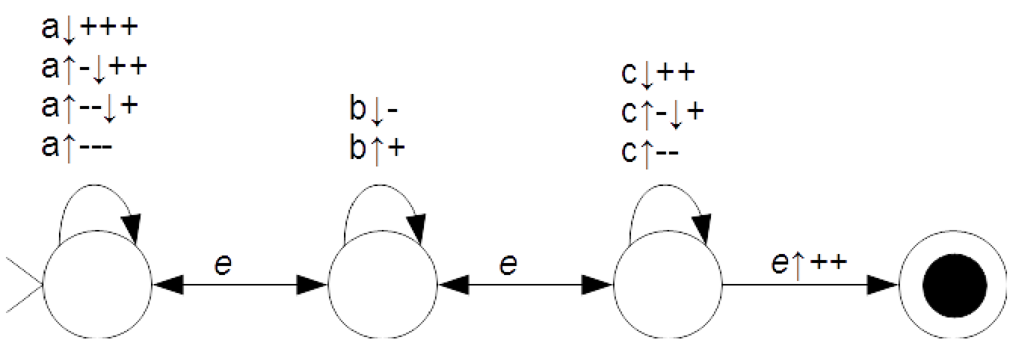
V={a, b, c, P, Q, R,S}

Σ ={a, b, c}

R={P→aQ|cP|bR, Q→cbQ|aR|*e*, R→abP|cP|bS, S→abQ|aP|*e*}

S=P

6. Give the state diagram of PDA M for which L(M) = {w∈{a, b, c}* | 3#a+2#c=#b+2}! (5p)



7. Give the definition of PDA! (3p)

PDA M is an ordered six-tuple $(K, \Sigma, \Gamma, \Delta, s, F)$, where:
K is the set of states
Σ is the alphabet of the input
Γ is the alphabet of the stack
$\Delta \subseteq (K\times\Sigma^*\times\Gamma^*)\times(K\times\Gamma^*)$ is the state transition relation
$S \notin K$ is the starting state
$F \subset K$ is the set of final states

8. Prove the theorem: let $M(K, \Sigma, \delta, s, F)$ be a DFA, long enough words in $L(M)$ ($|w| \geq |K|$) has a form, $w = xyz$, $y \neq e$, such that $xy^nz \in L(M)$, $\forall\ n \geq 0$. (Pumping theorem) (4p)

- let $w \in L(M)$ such that $|w| = k \geq |K|$
  - w exists since L is infinite
  - $w = \sigma_1\sigma_2\ldots\sigma_k$
- $(q_0, \sigma_1\sigma_2\ldots\sigma_k)$ |- $(q_1, \sigma_2\ldots\sigma_k)$ |- ... |- (q_{k-1}, σ_k) |- (q_k, e)
  - $q_0 = s$, $q_k \in F$
  - the number of yield in one step is k
- since $k \geq |K|$, $\exists\ q_i, q_j$, such that $q_i = q_j$, $i \neq j$, $(i<j)$
- $\sigma_{i+1}\sigma_{i+2}\ldots\sigma_j$ string moves M from state q_i to state q_j
- $\sigma_{i+1}\sigma_{i+2}\ldots\sigma_j$ can be removed or repeated without affecting the acceptance of w
- $\sigma_1\sigma_2\ldots\sigma_i(\sigma_{i+1}\sigma_{i+2}\ldots\sigma_j)^n\sigma_{j+1}\ldots\sigma_k \in L(M)$ for $n \geq 0$
  - $x = \sigma_1\sigma_2\ldots\sigma_i$
  - $y = \sigma_{i+1}\sigma_{i+2}\ldots\sigma_j$
  - $z = \sigma_{j+1}\ldots\sigma_k$