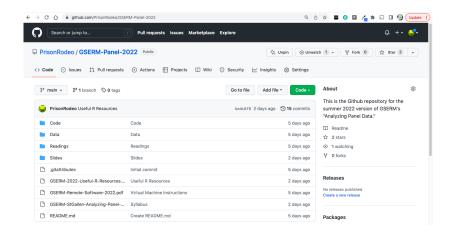
GSERM - St. Gallen 2022 Analyzing Panel Data

June 6, 2022

Preliminaries

- Instructor: Prof. Christopher Zorn (zorn@psu.edu).
- Class: June 6-10, 2022, 13:00-18:00 CET, at the University of St. Gallen (via Zoom).
- The course outline / syllabus is here.
- More important: The syllabus, slides, readings, code, data, etc. are all available on the course github repo (viewable at https://github.com/PrisonRodeo/GSERM-Panel-2022).
- Class sessions will be recorded; links to those recordings will be available on CANVAS.



Software

R

- All examples, plots, etc.
- Current version is 4.2.0
- Also be sure to get the RStudio IDE...
- The Github repo contains a bit of introductory code for people who may never have used R, and a list of R resources.
- Primary packages we'll use (see the econometrics task view for more):
 - · plm
 - \cdot lme4
 - · gee

Stata

- Current version is 17
- Mostly use the -xt- series of commands (for "cross-sectional time series")



R version 3.6.3 (2020-02-29) -- "Holding the Windsock" Copyright (C) 2020 The R Foundation for Statistical Computing Platform: x86_64-apple-darwin15.6.0 (64-bit)

R is free software and comes with ABSOLUTELY NO WARRANTY. You are welcome to redistribute it under certain conditions. Type 'license()' or 'licence()' for distribution details.

Natural language support but running in an English locale

R is a collaborative project with many contributors.

Type 'contributors()' for more information and
'citation()' on how to cite R or R packages in publications.

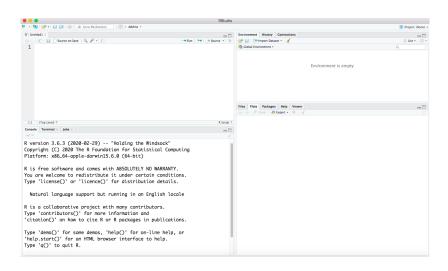
Type 'demo()' for some demos, 'help()' for on-line help, or 'help.start()' for an HTML browser interface to help. Type 'q()' to quit R.

[R.app GUI 1.70 (7735) x86_64-apple-darwin15.6.0]

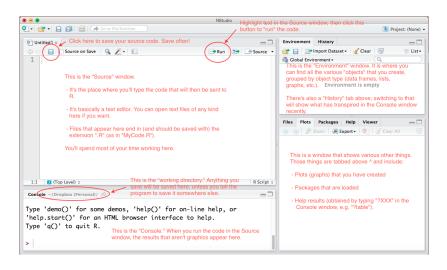
[Workspace restored from /Users/cuz10lcl/.RData] [History restored from /Users/cuz10lcl/.Rapp.history]

>

RStudio



RStudio (annotated)



Starting Points

- <u>Panel</u> data: Data comprising repeated observations over time on a set of cross-sectional units.
- Terminology:
 - "Unit" / "Units" / "Units of observation" / "Panels" = Things we observe repeatedly
 - "Observations" = Each (one) measurement of a unit
 - "Time points" = When each observation on a unit is made
 - $i \in \{1...N\}$ indexes units
 - $t \in \{1...T\}$ or $\{1...T_i\}$ indexes observations / time points
 - If $T_i = T \ \forall i$ then we have "balanced" panels / units
 - Balanced panels also imply $N_t = N \ \forall t$
 - NT = Total number of observations (if balanced)
- "Panel" ≠ "Time Series"
- "Panel" ≠ "Multilevel" / "Hierarchical" / etc.

More Terminology

 $N >> T \rightarrow$ "panel" data...

- (American) National Election Study panel studies ($N \approx 2000, T = 3$)
- Swiss Household Panel (FORS) (N = large, T = 21)
- Often:
 - · Cross-sectional units are a sample from a population
 - · T is (relatively) fixed

T>>N or $T \approx N \rightarrow$ "time-series cross-sectional" ("TSCS") data

- National OECD data (N=20 original members, $T\approx70$)
- Often:
 - · N is an entire population, and is (relatively) fixed
 - · Asymptotics are in T

 $N=1 \rightarrow$ "time series" data

 $T=1 \rightarrow$ "cross-sectional" data

${\sf Panel\ Data\ Structure}\ +\ {\sf Organization}$

id	t	Y	X_1	
1	1	250	3.4	
1	2	290	3.3	
:	:	:	:	
2	1	160	4.7	
2	2	150	4.9	
:	:	:	:	
_ •	•	•	•	•••

Introduction to Panel Variation: A Tiny (Fake) Example

		ID	Year	Female	Pres	GOP	Approve
1		1	2014	1	obama	0	4
2		1	2016	1	obama	0	5
3		1	2018	1	trump	0	2
4		1	2020	1	trump	0	1
5		2	2014	0	obama	1	2
6		2	2016	0	obama	1	1
7		2	2018	0	trump	1	4
8		2	2020	0	trump	1	3
9		3	2014	0	obama	1	2
1	0	3	2016	0	obama	1	2
1	1	3	2018	0	trump	1	4
1	2	3	2020	0	trump	0	1

Aggregation (means)

Cross-Sectional:

	ID	Year	${\tt Female}$	Pres	GOP	Approve
1	1	2017	1	?	0.00	3.00
2	2	2017	0	?	1.00	2.50
3	3	2017	0	?	0.75	2.25

Temporal:

	Year	${\tt Female}$	Pres	GOP	Approve
1	2014	0.333	obama	0.667	2.67
2	2016	0.333	obama	0.667	2.67
3	2018	0.333	trump	0.667	3.33
4	2020	0.333	trump	0.333	1.67

The Point

Aggregation:

- Always loses information
- Sometimes distorts relationships
- Occasionally forces arbitrary decisions

If you have variation in multiple dimensions, use it.

Two-Way Variation

Two "dimensions" of variation:

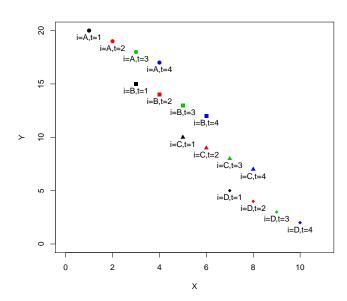
- <u>Unit-Level</u> Variation (how each unit is on average different from other units on average) – a/k/a **between-unit** variation
- <u>Time-Level</u> Variation (how each measurement / time point is on average different from other time points on average) a/k/a/within-unit variation

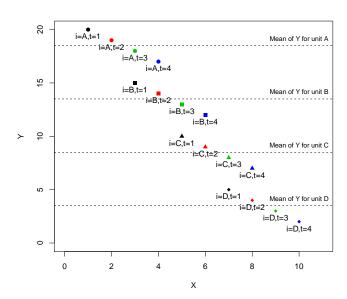
A random variable may:

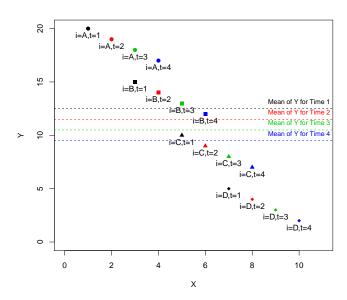
- Have only between-unit variation (i.e., lack temporal variation)
- Have only within-unit variation (i.e., lack cross-sectional variation)
- Have both within- and between-unit variation

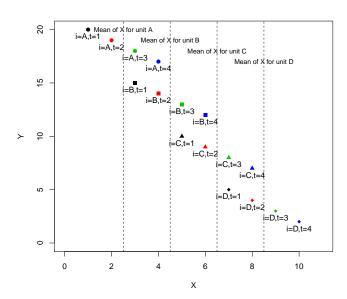
For Y_{it} , a variable that varies over both units and time:

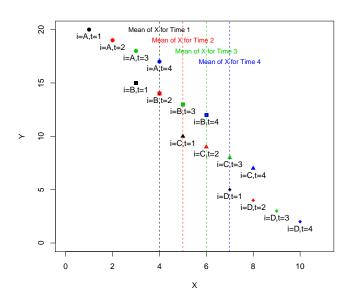
- $\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T} Y_{it}$ is the over-time mean of Y for unit i,
- $\bar{Y}_t = \frac{1}{N_t} \sum_{i=1}^N Y_{it}$ is the across-unit mean of Y at time t, and
- $\bar{Y} = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{it}$ is the grand mean of Y.











Within- and Between-Unit Variation

The within-unit mean of Y is:

$$\bar{Y}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Y_{it}$$

That means that we can write:

$$Y_{it} = \bar{Y}_i + (Y_{it} - \bar{Y}_i).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The between-unit variation in the \bar{Y}_i s, and
- The within-unit variation around \bar{Y}_i (that is, $Y_{it} \bar{Y}_i$).

Within- and Between-Unit Variation (continued)

Note that (while unusual) one could do a similar decomposition vis-à-vis time:

$$Y_{it} = \bar{Y}_t + (Y_{it} - \bar{Y}_t).$$

That is, the *total* variation in Y_{it} can be decomposed into:

- The temporal variation in the \bar{Y}_t s, and
- The within-time-point variation around \bar{Y}_t (that is, $Y_{it} \bar{Y}_t$).

In a similar fashion, we can also calculate the within- and between-unit variability (e.g., the standard deviations) of the constituent variables \bar{Y}_i and $(Y_{it} - \bar{Y}_i)$...

Variation ("Toy" Data from Above)

"Total" Variation:

```
> with(toy, describe(Y))
   vars n mean sd median trimmed mad min max range skew kurtosis se
X1   1 16   11 5.9   11   11 7.4   2   20   18   0   -1.5 1.5
```

"Between" Variation:

"Within" Variation:

Regression!

Model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

...makes all the usual OLS assumptions, plus

- $\beta_{0i} = \beta_0 \forall is$
- $\beta_{1i}=\beta_1 \ \forall \ \textit{is}$

For the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

...the same is true.

Variable Intercepts

Unit-specific intercepts:

$$Y_{it} = \beta_{0i} + \beta_1 X_{it} + u_{it} \tag{1}$$

Time-point-specific intercepts:

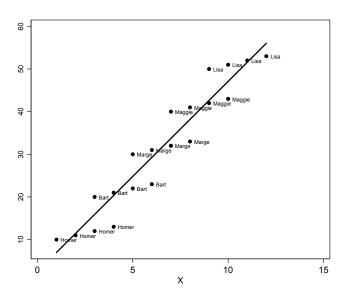
$$Y_{it} = \beta_{0t} + \beta_1 X_{it} + u_{it} \tag{2}$$

Both:

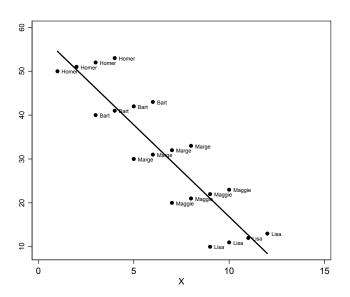
$$Y_{it} = \beta_{0it} + \beta_1 X_{it} + u_{it} \tag{3}$$

Note: Equation 3 is not identified (as written)!

Varying Intercepts



Varying Intercepts



Varying Slopes (+ Intercepts)

Unit-specific slopes:

$$Y_{it} = \beta_0 + \beta_{1i} X_{it} + u_{it} \tag{4}$$

(...one can also have time-point specific slopes, or both – again, the last of those is not identified as written.)

Unit-specific slopes + intercepts:

$$Y_{it} = \beta_{0i} + \beta_{1i} X_{it} + u_{it} \tag{5}$$

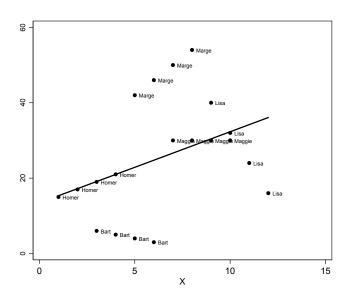
Time-point-specific-specific slopes + intercepts:

$$Y_{it} = \beta_{0t} + \beta_{1t} X_{it} + u_{it} \tag{6}$$

Both...:

$$Y_{it} = \beta_{0it} + \beta_{1it} X_{it} + u_{it} \tag{7}$$

${\sf Varying\ Slopes}\,+\,{\sf Intercepts}$



The Error Term...

Usual OLS assumption:

$$u_{it} \sim \text{i.i.d.} N(0, \sigma^2) \ \forall \ i, t$$

or, equivalently:

$$\mathbf{u}\mathbf{u}'\sim\sigma^2\mathbf{I}$$

implies:

$$Var(u_{it}) = Var(u_{jt}) \ \forall \ i \neq j \ (i.e., no cross-unit heteroscedasticity)$$

 $Var(u_{it}) = Var(u_{is}) \ \forall \ t \neq s \ (i.e., no temporal heteroscedasticity)$
 $Cov(u_{it}, u_{is}) = 0 \ \forall \ i \neq j, \ \forall \ t \neq s \ (i.e., no auto- or spatial correlation)$

Pooling

Pooling pros:

- Adds data (→ precision)
- Enhances generalizability

BUT: fitting the model:

$$Y_{it} = \beta_0 + \beta_1 X_{it} + u_{it}$$

Implies

- that the process governing the relationship between *X* and *Y* is exactly the same for each *i*,
- that the process governing the relationship between X and Y is the same for all t.
- that the process governing the us is the same $\forall i$ and t as well.

Q: When can we "pool" data on different units?

"Partial" Pooling (Bartels 1996)

Two regimes:

$$Y_A = \beta_A' \mathbf{X}_A + u_A$$

$$Y_B = \beta_B' \mathbf{X}_B + u_B$$

with $\sigma_A^2 = \sigma_B^2$, and $Cov(u_A, u_B) = 0$.

Estimators:

$$\hat{eta}_{A,B} = (\mathbf{X}_{A,B}^{\prime}\mathbf{X}_{A,B})^{-1}\mathbf{X}_{A,B}^{\prime}Y_{A,B}$$

and

$$\widehat{\mathsf{Var}(eta_{A,B})} = \hat{\sigma}_{A,B}^2(\mathbf{X}_{A,B}'\mathbf{X}_{A,B})^{-1},$$

A Pooled Estimator

$$\hat{\beta}_{P} = (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}(\mathbf{X}'_{A}Y_{A} + \mathbf{X}'_{B}Y_{B})
= (\mathbf{X}'_{A}\mathbf{X}_{A} + \mathbf{X}'_{B}\mathbf{X}_{B})^{-1}[\beta_{A}(\mathbf{X}'_{A}\mathbf{X}_{A}) + \beta_{B}(\mathbf{X}'_{B}\mathbf{X}_{B})],$$

$$E(\hat{\beta}_P) = \beta_A + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_B'\mathbf{X}_B(\beta_B - \beta_A)$$
$$= \beta_B + (\mathbf{X}_A'\mathbf{X}_A + \mathbf{X}_B'\mathbf{X}_B)^{-1}\mathbf{X}_A'\mathbf{X}_A(\beta_A - \beta_B)$$

Pooling: A Test

$$F = \frac{\frac{\hat{\mathbf{u}}_{P}'\hat{\mathbf{u}}_{P} - (\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{K}}{\frac{(\hat{\mathbf{u}}_{A}'\hat{\mathbf{u}}_{A} + \hat{\mathbf{u}}_{B}'\hat{\mathbf{u}}_{B})}{(N_{A} + N_{B} - 2K)}} \sim F_{[K,(N_{A} + N_{B} - 2K)]}$$

Fractional Pooling

Bartels suggests:

$$\hat{\beta}_{\lambda} = (\lambda^2 \mathbf{X}_A' \mathbf{X}_A + \mathbf{X}_B' \mathbf{X}_B)^{-1} (\lambda^2 \mathbf{X}_A' Y_A + \mathbf{X}_B' Y_B)$$

with $\lambda \in [0,1]$:

- $\lambda=0$ \to separate estimators for \hat{eta}_A and \hat{eta}_B ,
- $\lambda = 1 \rightarrow$ "fully pooled" estimator $\hat{\beta}_P$,
- $0 < \lambda < 1 \rightarrow$ a regression where data in regime A are given some "partial" weighting in their contribution towards an estimate of β .

Pooling, Summarized

"(R)oughly speaking, it makes sense to pool disparate observations if the underlying parameters governing those observations are sufficiently similar, but not otherwise."

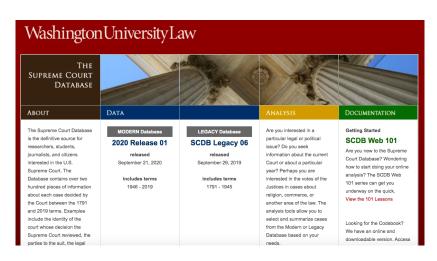
- Bartels (1996)

Exploring Variation: A Running Example

The U.S. Supreme Court, 1946-2020

- Court has nine "justices" at any time
- Appointed by the President, confirmed by the Senate (simple majority vote)
- Serve for life / good behavior
- One is appointed as the "Chief Justice" (a sitting justice may be elevated to that position)
- Sit in annual "terms" (October through June/July); decide 80-150 cases per term
- Cases are appealed from lower federal and state supreme courts
- Simple majority decision rule ("five")
- Nearly all decisions have a ideological (left / right) valence ("liberal" vs. "conservative")

The Supreme Court Database



http://scdb.wustl.edu/

Supreme Court Panel Data

Structure: One observation per justice (i) per term (t)

Important variables:

- justice: A justice (unit) ID variable [range: 78-116]
- term: A term (time) variable [range: 1946-2019]
- LiberalPct: The percentage of cases in that term in which that justice voted in a politically left / "liberal" direction
- MajPct: The percentage of cases in that term in which that justice voted with the majority in a case
- Ideology: A variable measuring the justice's (pre-confirmation) political ideology [range: 0 (most conservative) - 1 (most liberal)]*
- Qualifications: A measure of the justice's qualifications prior to his/her appointment [range: 0 (least qualified) 1 (most qualified)]*
- President: The name of the president who appointed that justice*
- YearApptd: The year that justice was appointed*
- NCases: The number of cases the Court decided during that term**
- ChiefJustice: The identity of the Chief Justice during that term**

^{*} indicates variables that are non-time-varying (that is, that have only between-unit variation)

^{**} indicates variables that are non-unit-varying (that is, that have only within-unit variation)

Summary Statistics

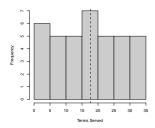
> summary(SCData	a)			
term	justice	justiceName	LiberalPct	MajPct
Min. :1946	Min. : 78.0	Length:672	Min. :16.7	Min. : 46.7
1st Qu.:1964	1st Qu.: 91.0	Class :character	1st Qu.:38.4	1st Qu.: 76.2
Median :1982	Median:100.0	Mode :character	Median:49.5	Median: 82.8
Mean :1982	Mean : 98.1		Mean :51.9	Mean : 81.7
3rd Qu.:2001	3rd Qu.:106.0		3rd Qu.:65.7	3rd Qu.: 88.0
Max. :2019	Max. :116.0		Max. :87.7	Max. :100.0
Order	Nominee	SenateVote	Ideology	qualifications
Min. : 1.0	Length: 672	Length:672	Min. :0.0	000 Min. :0.125
1st Qu.:15.0	Class : character	Class :charact	er 1st Qu.:0.1	.60 1st Qu.:0.750
Median:27.0	Mode :character	Mode :charact	er Median:0.4	88 Median :0.885
Mean :24.5			Mean :0.4	188 Mean :0.802
3rd Qu.:36.0			3rd Qu.:0.7	750 3rd Qu.:0.978
Max. :47.0			Max. :1.0	000 Max. :1.000
President	YearApptd	NCases	ChiefJustice	
Length:672	Min. :1937	Min. : 76	Length:672	
Class :characte	er 1st Qu.:1955	1st Qu.: 96	Class :character	
Mode :characte	er Median :1970	Median:141	Mode :character	•
	Mean :1970	Mean :142		
	3rd Qu.:1988	3 3rd Qu.:182		
	Max. :2018	3 Max. :258		

Some Basics

How many justices are in the data?

> length(unique(SCData\$justice))
[1] 38

How many terms do justices typically serve?

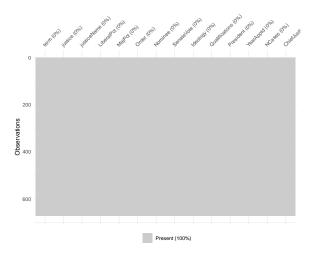


So we have:

- N = 38 units (justices)
- $\bar{T}=17.7$ time periods (terms) of data per justice, on average [range: 2-35], for a total of
- NT = 672 justice-terms in the data

Missing Data

- > library(naniar)
- > vis_miss(SCData)



Variation: LiberalPct

```
> # Total variation:
> with(SCData, describe(LiberalPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 51.9 16.2 49.5 51.5 19 16.7 87.7 71 0.19
> # Between-Justice variation:
>
> LibMeans <- ddply(SCData,.(justice),summarise,
                MeanLibPct=mean(LiberalPct))
> with(LibMeans, describe(MeanLibPct))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 38 52.5 14.8 48.3 52.2 16.5 29.9 77.2 47.2 0.31 -1.29 2.39
> # Within-Justice variation:
>
> SCData <- ddply(SCData,.(justice), mutate,
             LibMean=mean(LiberalPct))
> SCData$LibWithin <- with(SCData, LiberalPct-LibMean)
> with(SCData, describe(LibWithin))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 0 7.36 -0.16 -0.02 7.05 -30.8 32.8 63.6 0.04
```

Variation: Ideology

```
> # Total variation:
> with(SCData, describe(Ideology))
         n mean sd median trimmed mad min max range skew kurtosis
X1
  1 672 0.49 0.32 0.49 0.48 0.39 0 1 1 0.09
> # Between-Justice variation:
>
> IdeoMeans <- ddply(SCData,.(justice),summarise,
                  MeanIdeo=mean(Ideology))
> with(IdeoMeans, describe(MeanIdeo))
  vars n mean sd median trimmed mad min max range skew kurtosis
X1 1 38 0.54 0.33 0.58 0.54 0.43 0
                                          1
                                              1 -0.11 -1.46 0.05
> # Within-Justice variation (hint - there is none):
>
> SCData <- ddply(SCData,.(justice), mutate,
                IdeoMean=mean(Ideology))
> SCData$IdeoWithin <- with(SCData, Ideology-IdeoMean)
> with(SCData, describe(IdeoWithin))
         n mean sd median trimmed mad min max range skew kurtosis se
X 1
     1 672
             0 0
                       0
                              0
                                  0
                                      0
                                          0
                                               0 NaN
                                                           NaN 0
```

Variation: NCases

```
> # Total variation:
> with(SCData, describe(NCases))
         n mean sd median trimmed mad min max range skew kurtosis
X1 1 672 142 44.7 141 141 60.8 76 258 182 0.18 -1.06 1.73
> # Between-Term variation:
>
> NCMeans <- ddply(SCData,.(term),summarise,
                   MeanNCases=mean(NCases))
> with(NCMeans, describe(MeanNCases))
  vars n mean sd median trimmed mad min max range skew kurtosis
                             141 60.8 76 258 182 0.17
X1 1 74 142 45.1 142
> # Within-Term variation (none):
>
> SCData <- ddply(SCData,.(term), mutate,
                NCMean=mean(NCases))
> SCData$NCWithin <- with(SCData, NCases-NCMean)
> with(SCData, describe(NCWithin))
         n mean sd median trimmed mad min max range skew kurtosis se
X1 1 672
             0 0
                       0
                              0
                                  0
                                      0
                                          0
                                               0 NaN
                                                           NaN 0
```

Visualization: ExPanDaR

An interactive tool for exploring panel data...

- Creator: Joachim Gassen (Department of Accounting, Humboldt-Universität zu Berlin)
- Built upon / consistent with ggplot / tidyverse
- Requires installing the ExPanDaR package
- Calling
 - > ExPanD()

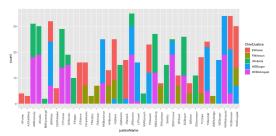
...opens the Shiny app, and asks for a (pre-formatted) data frame (typically in CSV format)

 More information is here: https://joachim-gassen.github.io/ExPanDaR/

Some examples...

ExPanDaR: Summaries

Counts by factors:

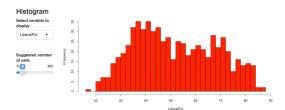


Summary statistics:

iariable peralPct	N 672 672	Mean 351.475 51.856	Std. dev. 208.836 16.213	Min. 1.000 16.667	25 % 168.750 38.446	Median 336.500	75 % 538.250	Max. 707.000
	672							
		51.856	16.213	16.667	20 446			
iPet					J3.440	49.473	65.713	87.662
apr or	672	81.714	8.827	46.667	76.193	82.828	87.964	100.000
der	672	24.531	12.784	1.000	15.000	27.000	36.000	47.000
ology	672	0.488	0.320	0.000	0.160	0.487	0.750	1.000
alifications	672	0.802	0.245	0.125	0.750	0.885	0.978	1.000
arApptd	672	1,970.324	20.975	1,937.000	1,955.000	1,970.000	1,988.000	2,018.000
	ology alifications	ology 672 alifications 672	ology 672 0.488 allifications 672 0.802	ology 672 0.488 0.320 allifications 672 0.802 0.245	ology 672 0.488 0.320 0.000 allifications 672 0.802 0.245 0.125	ology 672 0.488 0.320 0.000 0.160 allifications 672 0.802 0.245 0.125 0.750	ology 672 0.488 0.320 0.000 0.160 0.487 allfications 672 0.802 0.245 0.125 0.750 0.885	ology 672 0.488 0.320 0.000 0.160 0.487 0.750 allfications 672 0.802 0.245 0.125 0.750 0.885 0.978

ExPanDaR: Distributions

Histograms:

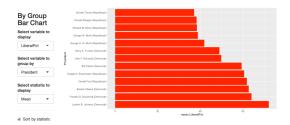


Outlier Detection:

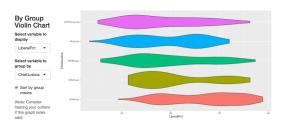
Extreme	justice	term	Liber	ralPct
Observations	81	1958		87.7
Select variable to	81	1956		87.0
sort data by	81	1971		84.7
Liberair ct +	90	1963		84.1
Select period to subset to	81	1955		83.7
All -				
	102	1979		21.3
	108	2003		21.3
	102	1998		20.2
	108	1998		20.2
	115	2016		16.7

ExPanDaR: More Distributions

Bar Charts of Means (by factors):

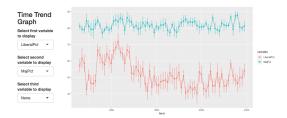


Violin Plots:

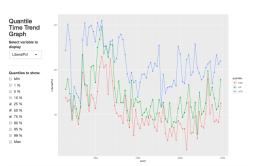


ExPanDaR: Trends

General Trends + Variation:

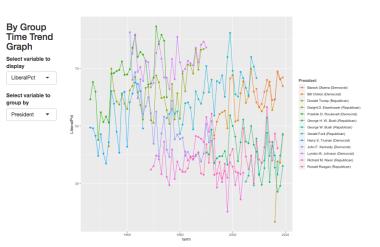


Trends in Quantiles:



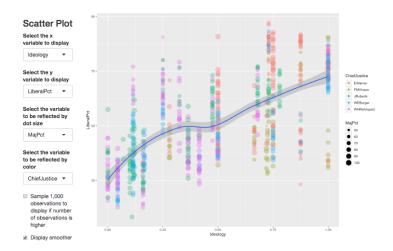
ExPanDaR: More Trends

Trends By Group:



ExPanDaR: Scatterplots

Fancy Scatterplots:

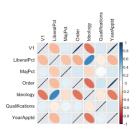


ExPanDaR: Correlations / Regression

Bivariate Correlations:



sample correlations (Pearson above, Spearman below diagonal). Reports correlations for all continuous variables. Hover over ellipse to get rho, P-Value and n.



Regression (OLS) Analysis:



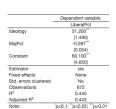
Select the
dependent variable
LiberalPct

Select independent
variable(s)

Ideology
MajPct

Select a categorial variable as the first fixed effect





The Plan

- Tuesday, 7 June: One- and Two-Way "Unit Effects"
 Models (fixed, "random," etc.)
- Wednesday, 8 June: Dynamics in Panel Data
- Thursday, 9 June: Panel Data and Causal Inference
- Friday, 10 June: Models for Discrete Responses