```
1  ---------- MODULE EuclidEx ----------
2  EXTENDS GCD, TLAPS
3  ------------------------------------
4  CONSTANTS M, N
5  ASSUME MNPosInt ≜
6      ∧   M ∈ Nat \ {0}
7      ∧   N ∈ Nat \ {0}
   ****************************************************************
9  --algorithm Euclid{
10   variables x = M, y = N ;
11   { while ( x ≠ y ) { if ( x < y ) { y := y − x }
12                             else      { x   := x − y }
13                           } ;
14    }
15  }
   ****************************************************************
17  BEGIN TRANSLATION
18 VARIABLES x, y, pc

20 vars ≜ ⟨x, y, pc⟩

22 Init ≜  Global variables
23         ∧ x = M
24         ∧ y = N
25         ∧ pc = "Lbl_1"

27 Lbl_1 ≜  ∧ pc = "Lbl_1"
28          ∧ IF x ≠ y
29                THEN ∧ IF x < y
30                       THEN ∧ y′ = y − x
31                            ∧ x′ = x
32                       ELSE ∧ x′ = x − y
33                            ∧ y′ = y
34                     ∧ pc′ = "Lbl_1"
35                ELSE ∧ pc′ = "Done"
36                     ∧ UNCHANGED ⟨x, y⟩

38 Next ≜ Lbl_1  Allow infinite stuttering to prevent deadlock on termination.
39           ∨ (pc = "Done" ∧ UNCHANGED vars)

41 Spec ≜ Init ∧ □[Next]_vars

43 Termination ≜ ◇(pc = "Done")

45  END TRANSLATION
46 ------------------------------------
```

$PartialCorrectness \triangleq$
$(pc = \text{“Done”}) \Rightarrow (x = y) \land (x = GCD(M, N))$

$TypeOK \triangleq$
$\land \quad x \in Nat \setminus \{0\}$
$\land \quad y \in Nat \setminus \{0\}$

$Inv \triangleq$
$\land\ TypeOK$
$\land\ GCD(x, y) = GCD(M, N)$
$\land\ (pc = \text{“Done”}) \Rightarrow (x = y)$

THEOREM $Spec \Rightarrow \Box PartialCorrectness$
$\langle 1 \rangle 1.\ Init \Rightarrow Inv$
BY $MNPosInt$ DEF $Init$, Inv, $TypeOK$
$\langle 1 \rangle 2.\ Inv \land [Next]_{vars} \Rightarrow Inv'$
BY $MNPosInt$, $GCD2$, $GCD3$ DEF Inv, $TypeOK$, $Next$, Lbl_1, $vars$
$\langle 1 \rangle 3.\ Inv \Rightarrow PartialCorrectness$
BY $MNPosInt$, $GCD1$ DEF Inv, $TypeOK$, $PartialCorrectness$
$\langle 1 \rangle 4.$ QED
BY $\langle 1 \rangle 1$, $\langle 1 \rangle 2$, $\langle 1 \rangle 3$, PTL DEF $Spec$

* Modification History
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