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1 |----- MODULE Consensus -----|
2 | EXTENDS Sets, TLAPS |
3 |-----|
4 | CONSTANT Value  the set of values that can be chosen
5 | VARIABLE chosen the set of values that have been chosen
6 |-----|
7 | Init   $\triangleq$  chosen = {}
8 | Next   $\triangleq$ 
9 |       $\wedge$  chosen = {}
10 |       $\wedge \exists v \in \textit{Value} : \textit{chosen}' = \{v\}$ 
11 |-----|
12 | Spec   $\triangleq$  Init  $\wedge$   $\Box[\textit{Next}]_{\textit{chosen}}$ 
13 |-----|
14 | Inv   $\triangleq$ 
15 |       $\wedge$  chosen  $\subseteq$  Value
16 |       $\wedge$  IsFiniteSet(chosen)
17 |       $\wedge$  Cardinality(chosen)  $\leq$  1
18 |-----|
19 | THEOREM Invariance   $\triangleq$  Spec  $\Rightarrow$   $\Box$ Inv
20 | <1>1. Init  $\Rightarrow$  Inv
21 |   BY CardinalityZero, SMT
22 |   DEFS Init, Inv
23 | <1>2. Inv  $\wedge$   $[\textit{Next}]_{\textit{chosen}} \Rightarrow \textit{Inv}'$ 
24 |   BY CardinalityOne, SMT
25 |   DEFS Next, Inv
26 | <1>3. QED
27 |   BY <1>1, <1>2, PTL DEF Spec
28 |-----|

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