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1  ┌────────────────────────── MODULE PaxosTuple ───────────────────┐
2  EXTENDS Integers, Sets
3  └──────────────────────────┐
4  CONSTANT Value, Acceptor, Quorum

6  ASSUME QuorumAssumption  $\triangleq$   $\wedge \forall Q \in \text{Quorum} : Q \subseteq \text{Acceptor}$ 
7                                      $\wedge \forall Q1, Q2 \in \text{Quorum} : Q1 \cap Q2 \neq \{\}$ 
8  Ballot  $\triangleq$  Nat
9  None  $\triangleq$  CHOOSE  $v : v \notin \text{Ballot}$ 
10 └──────────────────────────┐
11 Message  $\triangleq$ 
12   {“1a”}  $\times$  Ballot
13    $\cup$  {“1b”}  $\times$  Acceptor  $\times$  Ballot  $\times$  (Ballot  $\cup$  {−1})  $\times$  (Value  $\cup$  {None})
14    $\cup$  {“2a”}  $\times$  Ballot  $\times$  Value
15    $\cup$  {“2b”}  $\times$  Acceptor  $\times$  Ballot  $\times$  Value
16 └──────────────────────────┐
17 VARIABLE maxBal,
18           maxVBal,  $\langle \text{maxVBal}[a], \text{maxVal}[a] \rangle$ : the vote with the largest ballot number cast by a;
19           maxVal,   it is  $\langle -1, \text{None} \rangle$  if a has not cast any vote.
20           msgs

22 Send(m)  $\triangleq$  msgs' = msgs  $\cup$  {m}

24 vars  $\triangleq$   $\langle \text{maxBal}, \text{maxVBal}, \text{maxVal}, \text{msgs} \rangle$ 

26 TypeOK  $\triangleq$   $\wedge \text{maxBal} \in [\text{Acceptor} \rightarrow \text{Ballot} \cup \{-1\}]$ 
27              $\wedge \text{maxVBal} \in [\text{Acceptor} \rightarrow \text{Ballot} \cup \{-1\}]$ 
28              $\wedge \text{maxVal} \in [\text{Acceptor} \rightarrow \text{Value} \cup \{\text{None}\}]$ 
29              $\wedge \text{msgs} \subseteq \text{Message}$ 
30 └──────────────────────────┐
31 Init  $\triangleq$   $\wedge \text{maxBal} = [a \in \text{Acceptor} \mapsto -1]$ 
32            $\wedge \text{maxVBal} = [a \in \text{Acceptor} \mapsto -1]$ 
33            $\wedge \text{maxVal} = [a \in \text{Acceptor} \mapsto \text{None}]$ 
34            $\wedge \text{msgs} = \{\}$ 

36 Phase1a(b)  $\triangleq$   $\wedge \text{Send}(\langle \text{“1a”}, b \rangle)$ 
37                $\wedge \text{UNCHANGED } \langle \text{maxBal}, \text{maxVBal}, \text{maxVal} \rangle$ 

39 Phase1b(a)  $\triangleq$   $\wedge \exists m \in \text{msgs} :$ 
40                $\wedge m[1] = \text{“1a”}$ 
41                $\wedge m[2] > \text{maxBal}[a]$ 
42                $\wedge \text{maxBal}' = [\text{maxBal} \text{ EXCEPT } ![a] = m[2]]$ 
43                $\wedge \text{Send}(\langle \text{“1b”}, a, m[2], \text{maxVBal}[a], \text{maxVal}[a] \rangle)$ 
44                $\wedge \text{UNCHANGED } \langle \text{maxVBal}, \text{maxVal} \rangle$ 

46 Phase2a(b, v)  $\triangleq$ 
47    $\wedge \neg \exists m \in \text{msgs} : m[1] = \text{“2a”} \wedge m[3] = b$ 

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48    $\wedge \exists Q \in Quorum :$ 
49     LET  $Q1b \triangleq \{m \in msgs : \wedge m[1] = \text{"1b"} \wedge m[2] \in Q \wedge m[3] = b\}$ 
50
51      $Q1bv \triangleq \{m \in Q1b : m[4] \geq 0\}$ 
52
53     IN    $\wedge \forall a \in Q : \exists m \in Q1b : m[2] = a$ 
54          $\wedge \vee Q1bv = \{\}$ 
55          $\vee \exists m \in Q1bv :$ 
56              $\wedge m[5] = v$ 
57              $\wedge \forall mm \in Q1bv : m[4] \geq mm[4]$ 
58          $\wedge Send(\langle \text{"2a"}, b, v \rangle)$ 
59          $\wedge \text{UNCHANGED } \langle maxBal, maxVBal, maxVal \rangle$ 
60
61    $Phase2b(a) \triangleq \exists m \in msgs : \wedge m[1] = \text{"2a"} \wedge m[2] \geq maxBal[a]$ 
62
63        $\wedge maxBal' = [maxBal \text{ EXCEPT } ![a] = m[2]]$ 
64        $\wedge maxVBal' = [maxVBal \text{ EXCEPT } ![a] = m[2]]$ 
65        $\wedge maxVal' = [maxVal \text{ EXCEPT } ![a] = m[3]]$ 
66        $\wedge Send(\langle \text{"2b"}, a, m[2], m[3] \rangle)$ 
67
68    $Next \triangleq \vee \exists b \in Ballot : \vee Phase1a(b)$ 
69
70        $\vee \exists v \in Value : Phase2a(b, v)$ 
71
72        $\vee \exists a \in Acceptor : Phase1b(a) \vee Phase2b(a)$ 
73
74    $Spec \triangleq Init \wedge \Box [Next]_{vars}$ 
75
76    $votes \triangleq [a \in Acceptor \mapsto \{\langle m[3], m[4] \rangle : m \in \{mm \in msgs : \wedge mm[1] = \text{"2b"} \wedge mm[2] = a\}\}]$ 
77
78    $V \triangleq \text{INSTANCE } Voting$ 
79
80   THEOREM  $Spec \Rightarrow V!Spec$ 

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