

## Problem Set #2

1. Suppose that

$$y_t = x_t + e_t$$

$$x_t = \alpha x_{t-1} + u_t$$

where  $e_t$  and  $u_t$  are mutually independent zero mean i.i.d. processes. Show that  $y_t$  is an  $ARMA(1, 1)$  process.

2. Take the model

$$\alpha(L)y_t = u_t$$

$$\beta(L)u_t = e_t$$

where  $\alpha(L)$  and  $\beta(L)$  are polynomials of order  $p$  and  $q$ , respectively. Show that this implies  $\gamma(L)y_t = e_t$  for some polynomial  $\gamma(L)$ . What is the order of  $\gamma(L)$ ?

3. Take the quarterly series pnfix (non residential real private fixed investment).
- (a) Transform the series into quarterly growth rates
  - (b) Estimate an AR(4) model. Report using heteroskedastic-consistent standard errors.
  - (c) Repeat using the Newey-West standard errors, using  $M = 5$
  - (d) Comment on the magnitude and interpretation of the coefficients
  - (e) Calculate, numerically, the impulse response function for  $j = 1, 2, \dots, 10$ .
4. Bonus question (optional). This question is hard. Hint: look on page 102 of Hamilton but try on your own first. Suppose that

$$y_t = u_t + e_t$$

$$u_t = v_t + \theta v_{t-1}$$

where  $u_t$  and  $e_t$  are mutually independent i.i.d. zero mean processes. Show that  $y_t$  is a  $MA(1)$  process  $y_t = \eta_t + \psi \eta_{t-1}$  for an i.i.d. error  $\eta_t$ . Find an expression for  $\psi$