Econ 722: Time Series Assignment 2

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1. Suppose that

$$\begin{aligned} y_t &= x_t + e_t \\ x_t &= \alpha x_{t-1} + u_t \end{aligned}$$

where e_t and u_t are mutually independent zero mean iid process. Show that y_t is an $ARMA(1,1)$ process.

Answer
Given

$$\begin{aligned} y_t &= x_t + e_t \\ x_t &= \alpha x_{t-1} + u_t \\ y_t &= \alpha\left(y_{t-1} - e_{t-1}\right) + u_t + e_t \\ y_t &= \alpha y_{t-1} + e_t - \alpha e_{t-1} + u_t \\ (1-\alpha L)y_t &= (1-\alpha L)e_t + u_t \end{aligned}$$

Hence $y_t \sim ARMA(1,1)$ with a stochastic trend u_t.

2. Take the Model;

$$\begin{aligned} \alpha(L)y_t &= u_t \\ \beta(L)u_t &= e_t \end{aligned}$$

where $\alpha(L)$ and $\beta(L)$ are polynomials of order p and q respectively. Show that this implies $\gamma(L)y_t = e_t$ for some polynomial $\gamma(L)$. What is the order of $\gamma(L)$?

Answer:

$$\begin{aligned} \alpha(L)y_t &= u_t \\ \beta(L)u_t &= e_t \\ \implies \beta(L)\alpha(L)y_t &= e_t \\ \gamma(L)y_t &= e_t \end{aligned}$$

Hence $\gamma(L) = \beta(L) \times \alpha(L)$ with order $p+q$.

3. Take the quarterly series private (non-residential real private fixed investment).

(a) Transform the series into quarterly growth rates.

We transform the series by taking the log difference of the series. Figure **??** shows a graph of the series after the transformation. One can observe some seasonality patterns of this series occurring on April of each year. There seems to be a high growth rate at each April of the year and a low growth rate in January of each year. Additionally, one can also observe the unusual dip in the growth rate of this series, occurring in early 2009, as a reaction to the financial crisis.

We also compute the Autocorrelation function of this series in Figure **??** and one can see that the autocorrelation depicts a gradual slow decay. The decay is not visible after about the 40th lag. This shows that we can model this series with an AR process and past lags have a lasting effect on the private investment.

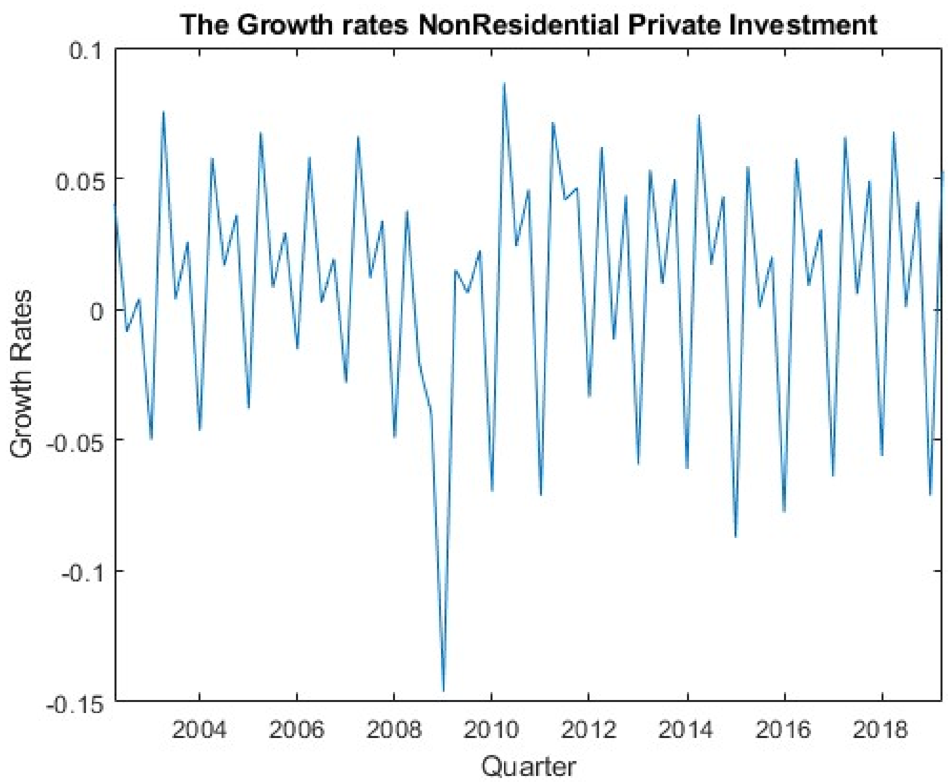

Figure 1: The Growth rates of Non-residential private investment from 2004 ro 2018.

(b) Estimate an AR(4) model. Report using heteroskedastic- consistent standard errors.

Answer:

We estimate the AR(4) model using fitlm in MATLAB which basically estimate the

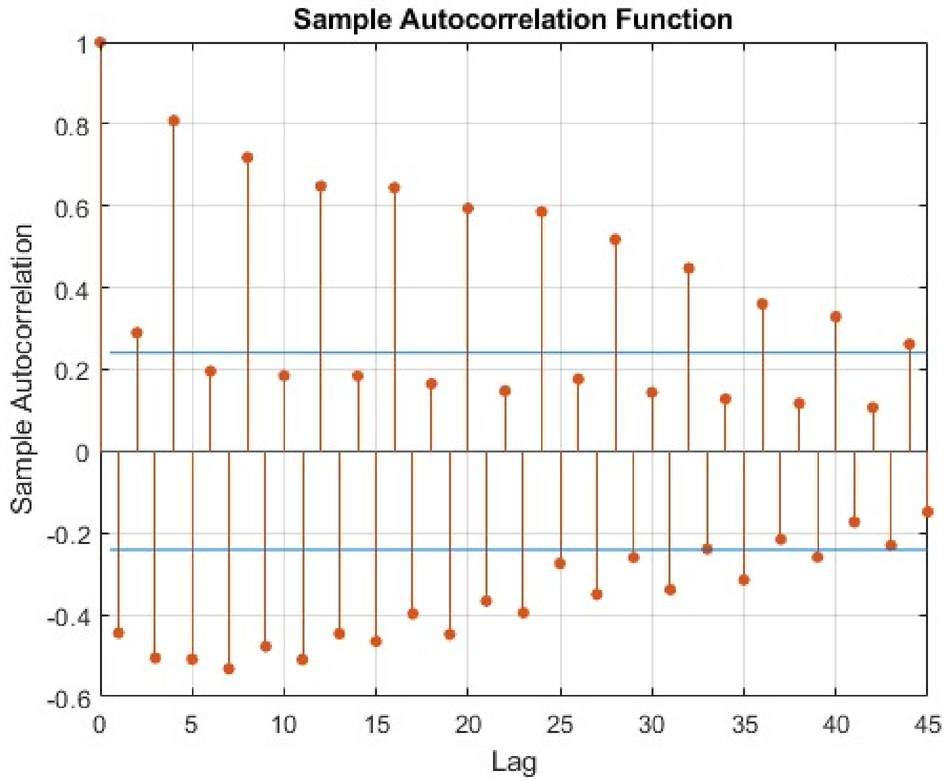

Figure 2: The Autocorrelation plot.

AR model using least squares method and we report the robust standard errors as seen in table 1.

	Estimate	SE	tStat	pValue
(Intercept)	0.0107	0.0031	3.501	0.0009
x1	-0.11122	0.066	-1.6993	0.0944
x2	-0.0821	0.0634	-1.2951	0.2003
x3	-0.2480	0.0633	-3.9156	0.0002
x4	0.6953	0.0666	10.442	4.0524e-15

Table 1: Regression Results

(c) Repeat using the Newey-West standard errors, using M = 5.

Answer:

We again estimate the model using the Newey west standard errors with a bandwidth of 5. We use the function hac in MATLAB. Table 2 represents the estimated coefficients and the standard errors using with the Newey west methodology.

	Coefficient	**Standard Error**
Intercept	0.0107	0.0088
x1	-0.1112	0.1814
x2	-0.0821	0.1129
x3	-0.2480	0.0639
x4	0.6953	0.1207

Table 2: Regression Coefficients and Standard Errors with Newey west.

(d) Comment on the magnitude and interpretation of the coefficients.

Comparing the estimated coefficients of the robust model and the Newey west model in table 2, one can observe that the estimated coefficients are the same which makes sense because the estimators are consistent. However the standard errors of the Newey west model is relatively larger as compared to the robust standard errors of the least squares estimated.

(e) Calculate, numerically, the impulse response function for j = 1,2,...10.

Theimpulse response function in Figure 3 shows the effect of a shock to the future private fixed investment in quarters 1 to 10. The behavior of the IRF depicts the business cycle in which the shock causes the investment to fall in the first 3 quarters and rises the following year.

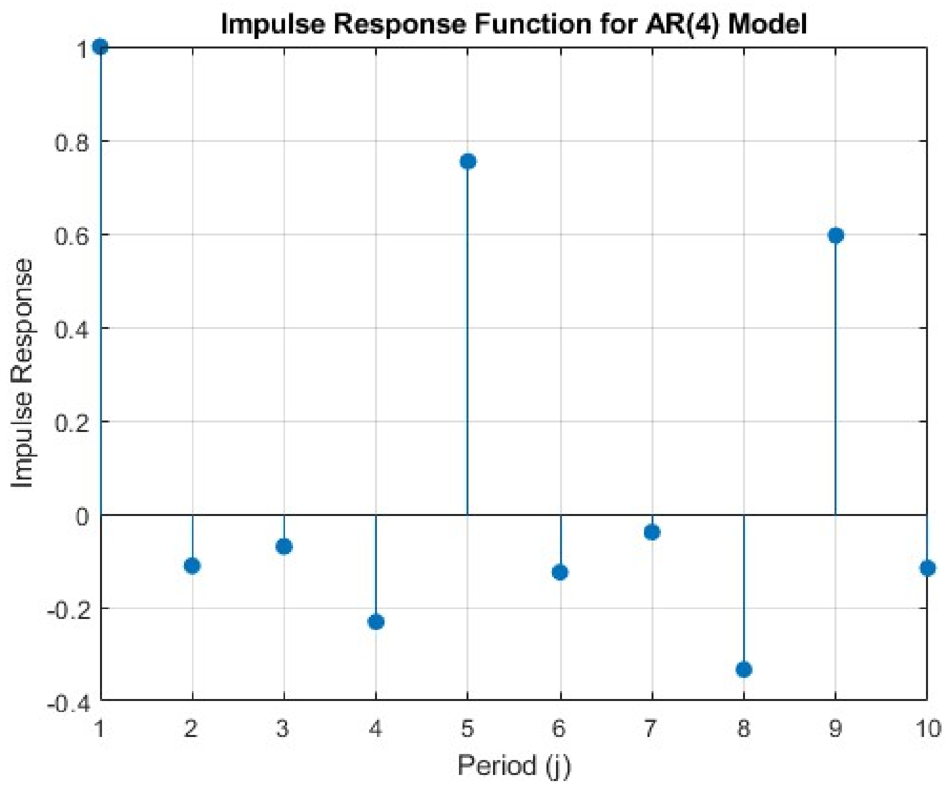

Figure 3: The Impulse response function of for j = 1,2 ...,10