Econ 722: Time Series Midterm 1

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1. Question 1

2. Question 2

3. Using the US Dollar to Euro spot exchange not seasonally adjusted and at a monthly frequency:

   (a) To optimally choose the best ARMA model that best fits the data, use the information criteria for model selection. This allows you to select the model that best fits data by running the ARMA model for different combinations of (p,q) and selecting the model that minimizes the criterion(MSE) error. In my case, I plotted the ACF and PACF to get a good sense of the maximum number of lags to choose i.e. $(p_{max}, q_{max})$. So choosing the maximum lags to be (5,5), I run an ARMA model for different combinations of p and q, compute the BIC value for each model, and then select the model with the minimum BIC which turns out to be (p,q) = (1,1). We then run an ARMA(1,1) model which is the optimal model whose results are represented in table (1).

| Parameter | Estimate | Std. Error | z value | Pr(>\|z\|) |
|---|---|---|---|---|
| $c$ | 0.00317336 | 0.00260903 | 1.2163 | 0.2239 |
| $\phi_1$ | 0.979333 | 0.013225 | 74.0516 | $< 1e{-}99$ |
| $\theta_1$ | 0.316292 | 0.0458835 | 6.89337 | $< 1e{-}11$ |

Table 1: Estimation of ARMA(1,1) model

   (b) Compute one, two, and three steps ahead forecast for the exchange rate using your model in (a).
   *Answer:*

| Step | Forecast |
|---|---|
| 1 | 0.715422 |
| 2 | 0.00269936 |
| 3 | 0.00269936 |

Table 2: Forecasts for Each Step

1

(c) Run a regression of $y_t$ on $y_{t-1}$. To an approximation, this can be considered an estimate of the largest root of $y_t$. What is your estimate? Comment.
*Answer:*
Consider the regression

$$y_t = \alpha + \beta y_{t-1} + \varepsilon_t \tag{1}$$

We obtain the results in table (**??**). We can observe that the coefficient or the root is around 0.986 which is quite close to 1. Since this estimate is not exactly 1, one can argue that our data is not a unit root process, so modeling the level data with an ARMA model seems consistent with the model's assumptions.

| Parameter | Coef. | Std. Error | t | Pr(>\|t\|) | Lower 95% | Upper 95% |
|---|---|---|---|---|---|---|
| $\alpha$ | 0.00195433 | 0.00196396 | 1.00 | 0.3205 | -0.00191029 | 0.00581896 |
| $\beta$ | 0.986909 | 0.00936303 | 105.40 | $< 1 \times 10^{-99}$ | 0.968485 | 1.00533 |

(d) Test the data for a unit root. Choose carefully which test to use and which option. Defend your choices and comment on the results.
*Answer:*
We test the data for a unit root using the Augmented Dickey-Fuller test. Although the standard Dickey-Fuller test is an option. it assumes no autocorrelation in the error terms of the regression. This can make our results biased.
Additionally, the Augment Dickey Fully test allows the inclusion of deterministic trends as the plot of the Exchange rates exhibits some trending behavior.
A much more powerful test will be the ADF-GLS test which actually detrends the series before testing for the unit root and hence improves upon the precision of the test. However, I stick with the Augmented Dickey-Fuller test since it is the standard test that is widely used and available in the JULIA programming language.

$$\mathbb{H}_0 : \rho = 1$$
$$\mathbb{H}_1 : \rho < 1$$

Table (3) represents the test for the unit root using the ADF. The p-value recorded is about 0,65 showing that we fail to reject the null. So the test shows that the Exchange rate series is actually a unit root process and so our estimating the series with an ARMA model is not the correct specification. One way to solve this problem is to model the growth of the EXCHANGE rate series.

(e) What does your result in d) and e) say about your model in a)? Comment.

| Test Summary | |
|---|---|
| Population details: | |
| Parameter of interest: | Coefficient on lagged non-differenced variable |
| Value under $H_0$: | 0 |
| Point estimate: | -0.0172752 |
| Test summary: | |
| Outcome with 95% confidence: | Fail to reject $H_0$ |
| p-value: | 0.6454 |
| Details: | |
| Sample size in regression: | 305 |
| Number of lags: | 1 |
| ADF statistic: | -1.91772 |
| Critical values at 1%, 5%, and 10%: | [-3.98857, -3.42489, -3.13551] |

Table 3: Augmented Dickey-Fuller Unit Root Test Results

Since the US-euro Exchange rate follows a unit root process, then it means that our process is not stationary which is the required assumption for the ARMA model. So by running an ARMA model, we violate the assumption of the model and the results might be spurious. One way around this is to model the growth of the exchange rate series that is we compute the first difference. This transformation converts a unit root process into a stationary one, making the series suitable for ARMA modeling or other stationary time series models.

(f) Go to FRED database and download the dollar to the euro exchange rate at a daily frequency. Estimate the best ARMA model for the daily exchange rate. How does this model compare with the model for the monthly data? Comment.

Table 4 shows the optimal ARMA model which is an ARMA(1,0) model for the daily exchange rate series. That is by the BIC criterion, the best model for the daily exchange rate series is an AR(1) model. This makes sense since the current past value contains all the information of all the past value of the prices and so that alone can explain the behavior of the series. The estimate is close to the ARMA(1,1) model of the monthly series.

| Parameter | Estimate | Std. Error | z value | Pr(>|z|) |
|---|---|---|---|---|
| $c$ | 0.000435259 | 0.000363129 | 1.19864 | 0.2307 |
| $\phi_1$ | 0.995934 | 0.00288781 | 344.876 | $< 1e-99$ |

Table 4: Mean Equation Parameters