INFERENTIAL STATISTICS CODED PROJECT

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PGP-Data Science and Business Analytics.

BATCH: PGP DSBA. O. MAY24.A

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Problem 1

A physiotherapist with a male football team is interested in studying the relationship between foot injuries and the positions at which the players play from the data collected

	Striker	Forward	Attacking Midfielder	Winger	Total
Players Injured	45	56	24	20	145
Players Not Injured	32	38	11	9	90
Total	77	94	35	29	235

Table 1: Football team data

Based on the above data, answer the following questions.

1.1 What is the probability that a randomly chosen player would suffer an injury?

Solution:

Total number of players injured = 145

Total number of players = 235

Probability that a randomly chosen player would suffer an injury

= Total Number of Players Injured/ Total Number of Players

= 145/235

= 0.62.

Probability that a randomly chosen player would suffer an injury is 0.62

1.2 What is the probability that a player is a forward or a winger?

Solution:

Number of forward players = 94

Number of winger players = 29

Probability that a player is a forward or a winger

= (Number of Forwards + Number of Wingers) / Total Number of Players

= (94+29)/235

= 0.52.

Probability that a player is a forward or a winger is 0.52

1.3 What is the probability that a randomly chosen player plays in a striker position and has a foot injury?

Solution:

Total number of players = 235

Number of injured strikers = 45

Probability that a randomly chosen player plays in a striker position and has a foot injury

= Number of Injured Strikers/ Total Number of Players

= 45/235

= 0.19.

Probability that a randomly chosen player plays in a striker position and has a foot injury is 0.19

1.4 What is the probability that a randomly chosen injured player is a striker?

Solution:

Total Number of Injured Players = 145

Number of injured strikers = 45

Probability that a randomly chosen injured player is a striker

= Number of Injured Strikers/ Total Number of Injured Players

= 45/145

= 0.31.

Probability that a randomly chosen injured player is a striker is 0.31

Problem 2

The breaking strength of gunny bags used for packaging cement is normally distributed with a mean of 5 kg per sq. centimeter and a standard deviation of 1.5 kg per sq. centimeter. The quality team of the cement company wants to know the following about the packaging material to better understand wastage or pilferage within the supply chain; Answer the questions below based on the given information; (Provide an appropriate visual representation of your answers, without which marks will be deducted)

2.1 What proportion of the gunny bags have a breaking strength of less than 3.17 kg per sq.cm?

Solution:

Given: The breaking strength of gunny bags used for packaging cement is normally distributed.

Mean (mu)= 5kg per sq.cm

Standard deviation(sigma) = 1.5 kg per sq.cm

Desired breaking strength, X=3.17 kg per sq. cm

To find: The proportion of gunny bags that have a breaking strength less than 3.17 kg per sq. cm. This involves finding the cumulative probability for the normal distribution at X = 3.17.

Step 1: Calculate the Z-score for X = 3.17

Z= X - mu/sigma

Z = 3.17 – 5 / 1.5

Z = - 1.22

Step 2: calculate the cumulative probability

P (X < 3.17) = 0.1112

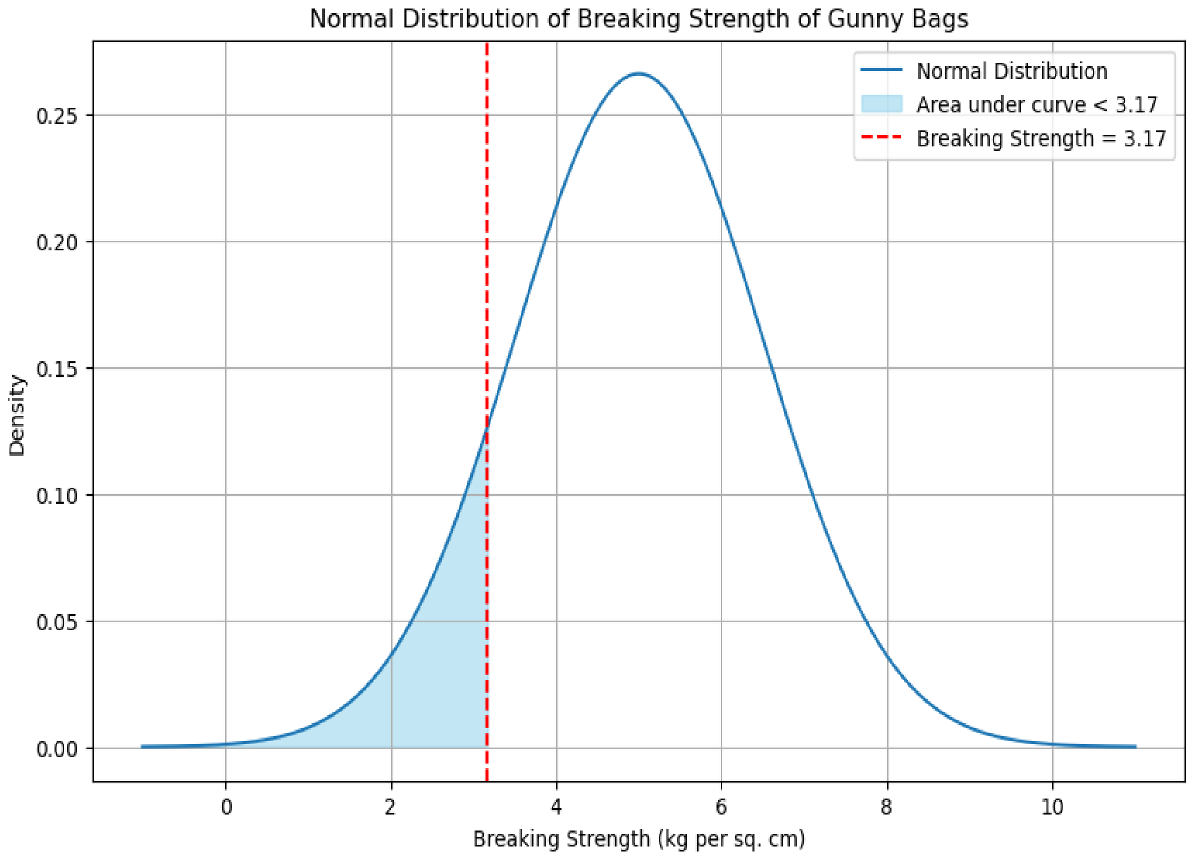

Figure 1: Gunny bags with a breaking strength of less than 3.17 kg per sq. cm

Therefore, the proportion of gunny bags with a breaking strength of less than 3.17 kg per sq. cm is approximately 0.1112, or 11.12%.

2.2 What proportion of the gunny bags have a breaking strength of at least 3.6 kg per sq. cm.?

Solution:

Given:

Mean (mu)= 5kg per sq.cm

Standard deviation(sigma) = 1.5 kg per sq.cm

Desired breaking strength, X = 3.6 kg per sq. cm

To find: The proportion of gunny bags that have a breaking strength at least 3.6 kg per sq. cm. This involves finding the cumulative probability for the normal distribution at $X \geq 3.6$

Step 1: Calculate the Z-score for X = 3.6

Z= X - mu/sigma

Z = 3.6 – 5 / 1.5

Z = - 0.933

Step 2: calculate the cumulative probability

$P(X \geq 3.6) = 1 - P(X<3.6) = 0.8247$.

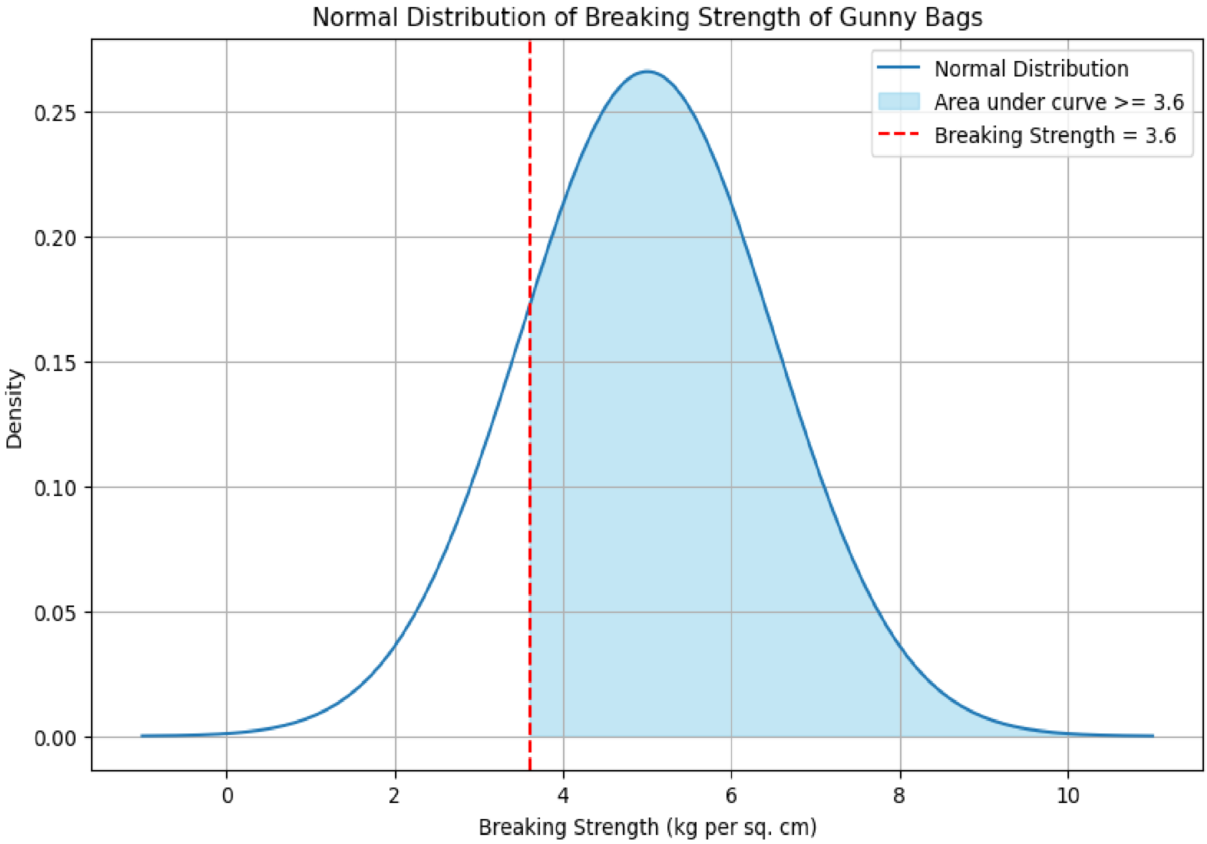

Figure 2: The gunny bags have a breaking strength of at least 3.6 kg per sq.cm.

Therefore, the proportion of gunny bags with a breaking strength of at least 3.6 kg per sq. cm is approximately 0.8247, or 82.47%.

2.3 What proportion of the gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.?

Solution:

Given:

Mean (mu)= 5kg per sq.cm

Standard deviation(sigma) = 1.5 kg per sq.cm

Desired breaking strength,

X1 = 5 kg per sq. cm

X2 = 5.5 kg per sq. cm

To find: The proportion of gunny bags that have a breaking strength between 5 and 5.5 kg per sq. cm. This involves finding the cumulative probability for the normal distribution at P (5< X < 5.5)

Step 1: Calculate the Z-score for X = 3.6

Z= X - mu/sigma

Z1 = (5 – 5)/ 1.5

Z 1= 0

Z2= (5.5 – 5) / 1.5

Z2 = 0.33

Step 2: calculate the cumulative probability

P (5< X <5.5) = P (X<5.5) – P (X<5) = 0.1306

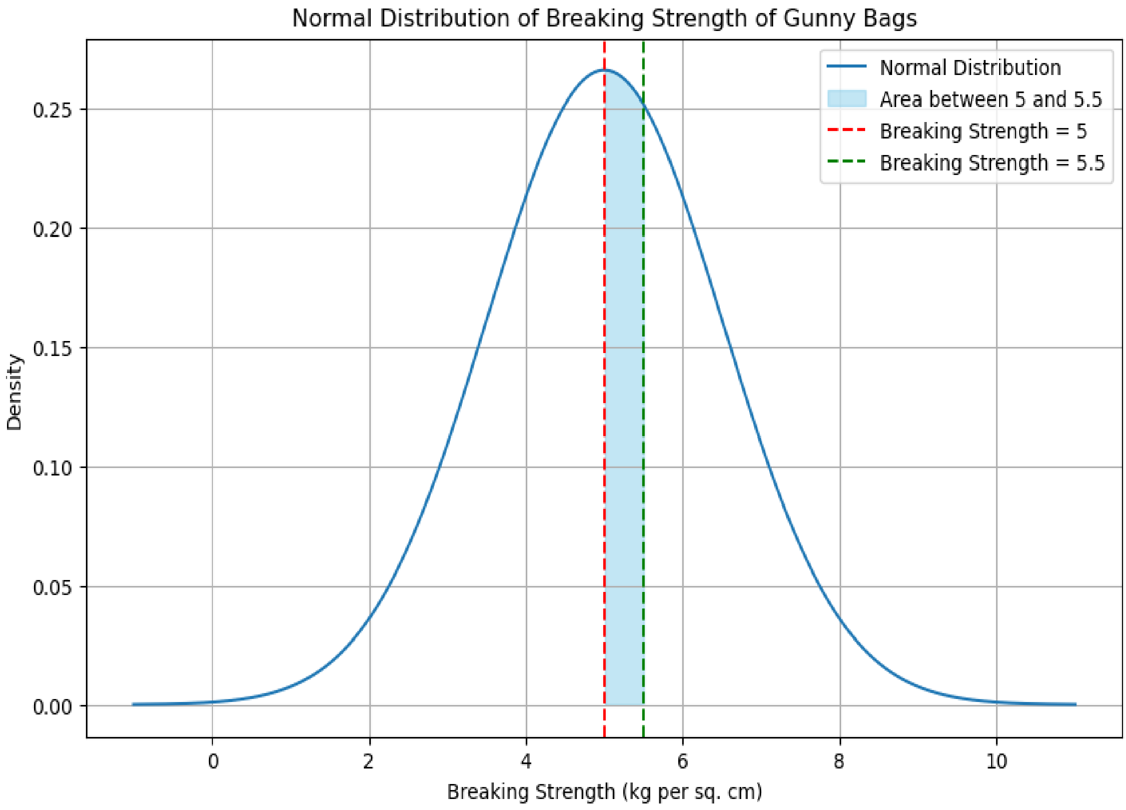

Figure 3: The gunny bags have a breaking strength between 5 and 5.5 kg per sq. cm.

Therefore, the proportion of gunny bags with a breaking strength between 5 and 5.5 kg per sq. cm is approximately 0.1306, or 13.06%.

2.4 What proportion of the gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq cm.?

Solution:

Given:

Mean (mu)= 5kg per sq.cm

Standard deviation(sigma) = 1.5 kg per sq.cm

Desired breaking strength,

X1 = 3 kg per sq. cm

X2 = 7.5 kg per sq. cm

To find: The proportion of gunny bags that have a breaking strength NOT between 3 and 7.5 kg per sq. cm. This involves finding the cumulative probability for the normal distribution at P (3< X > 7.5)

Step 1: Calculate the Z-score for X = 3.6

Z= X - mu/sigma

Z1 = (3 – 5)/ 1.5

Z 1= -1.33

Z2= (7.5 – 5) / 1.5

Z2 = 1.66

Step 2: calculate the cumulative probability

P (X < 3 and X > 7.5) = P (X<3) + 1 - P (X<7.5) = 0.1390 ≈ 0.14

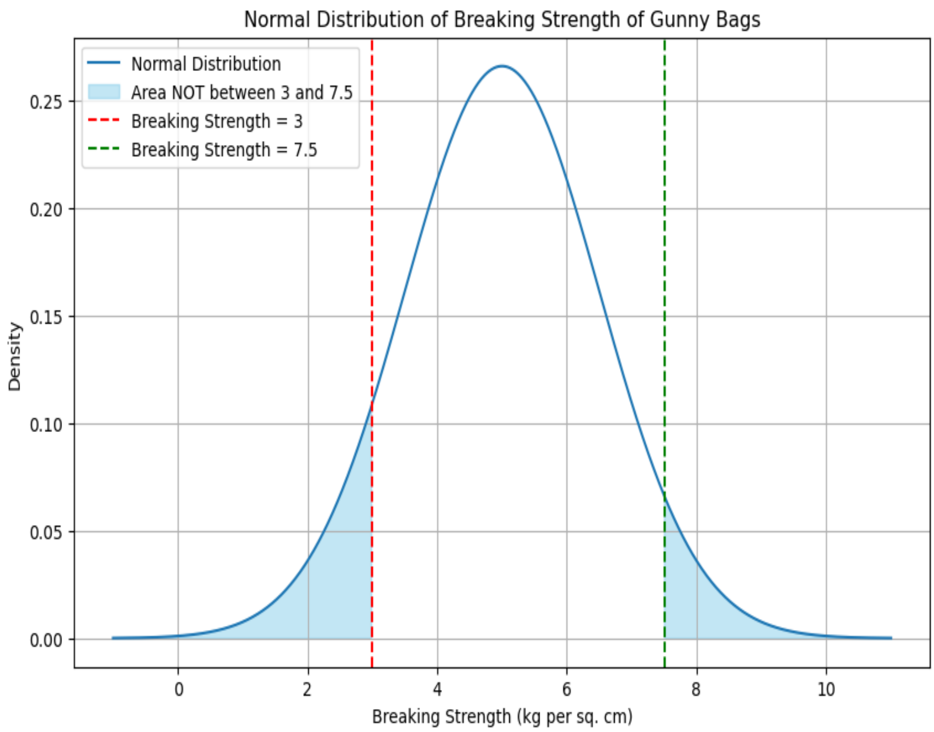

Figure 4: The gunny bags have a breaking strength NOT between 3 and 7.5 kg per sq. cm.

Therefore, the proportion of gunny bags with a breaking strength NOT between 3 and 7.5 kg per sq. cm is approximately 0.1390, or 14 %.

Problem 3

Zingaro stone printing is a company that specializes in printing images or patterns on polished or unpolished stones. However, for the optimum level of printing of the image, the stone surface has to have a Brinell's hardness index of at least 150. Recently, Zingaro has received a batch of polished and unpolished stones from its clients. Use the data provided to answer the following (assuming a 5% significance level);

UNDERSTANDING THE STRUCTURE OF DATA:

Dataset Name: Zingaro_Company

This data dictionary provides a clear and concise reference for understanding the dataset's structure and the meaning of each column, ensuring accurate and consistent data handling in analysis.

Some important technical information about the dataset:

- The data set contains of 75 rows & 2 columns.
- The data contains 2 float 64 data type columns.

SI.NO	Column name	Description	Datatype	Example values
1	unpolished	Number of unpolished stone received from the client	Float 64	164.345,132.567
2	Treated and Polished	Number of treated and polished stone received from the client	Float 64	133.209,123.567

Table 2: Data Dictionary

STATISTICAL ANALYSIS:

The statistical analysis provides a summary of the key metrics for the numerical columns in the dataset. This includes measures such as the count, mean, standard deviation, minimum, 25th percentile (Q1), median (50th percentile, Q2), 75th percentile (Q3), and maximum values for each column.

	Unpolished	Treated and Polished
count	75.000000	75.000000
mean	134.110527	147.788117
std	33.041804	15.587355
min	48.406838	107.524167
25%	115.329753	138.268300
50%	135.597121	145.721322
75%	158.215098	157.373318
max	200.161313	192.272856

Figure 5. Statistical summary

Observation:

- Mean of unpolished stone is 134.11
- Mean of treated and polished stone is 147.78
- Standard deviation of unpolished stone is 33.04
- Standard deviation of treated and polished stone is 15.58
- There are no missing values and no duplicate values.
- Both the data are Normally distributed across the samples.

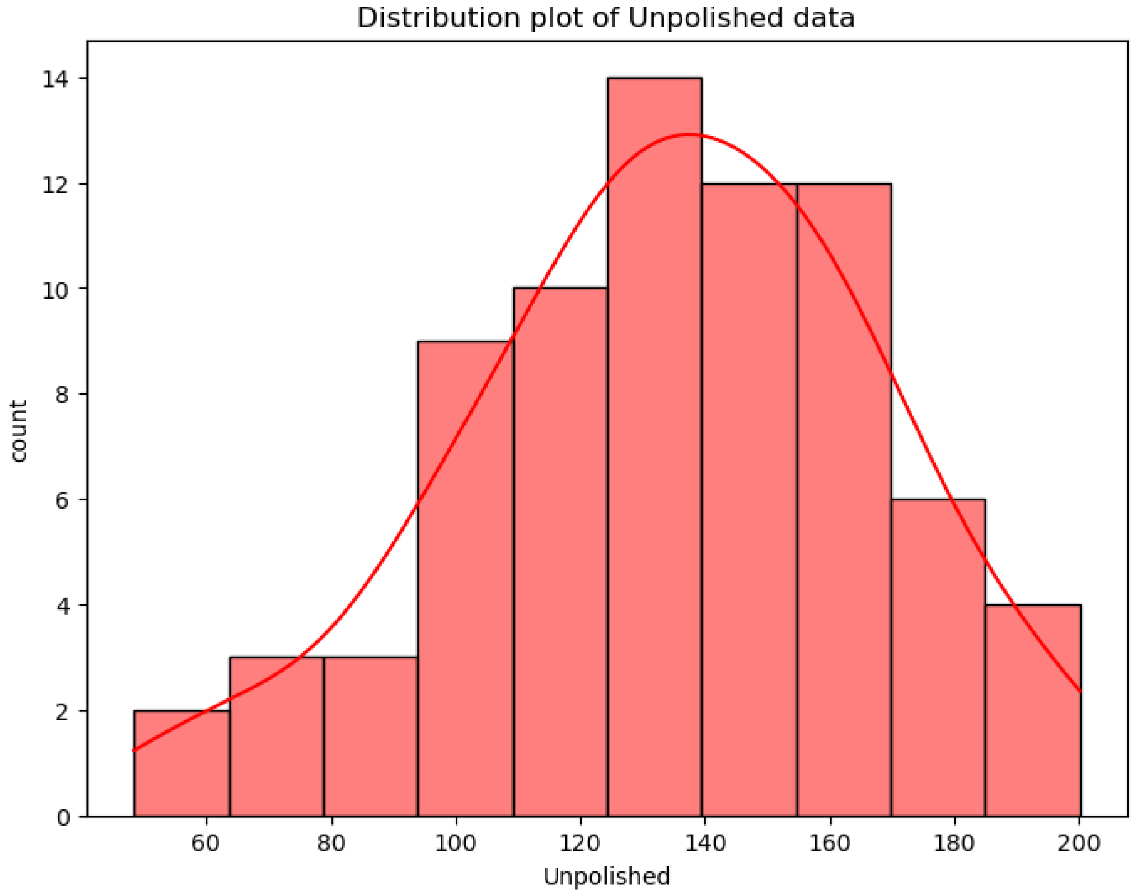

Figure 6. Distribution plot – Unpolished data

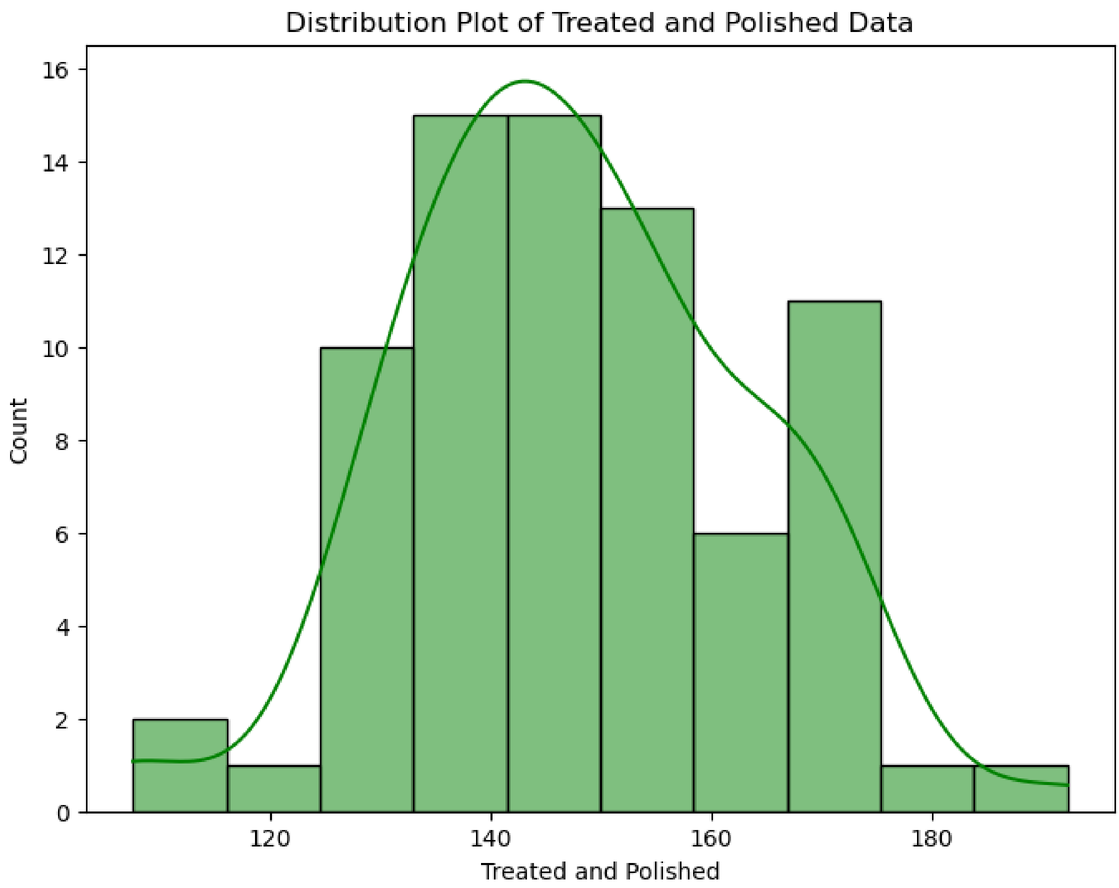

Figure 7. Distribution plot – Treated and Polished data

3.1 Zingaro has reason to believe that the unpolished stones may not be suitable for printing. Do you think Zingaro is justified in thinking so?

To determine if the unpolished stones are suitable for printing, we need to test whether the mean hardness index of unpolished stones is less than 150. This will be a one-sample one-tailed t-test.

<u>Steps to Perform the Tests:</u>

Step 1: State the null and alternative hypotheses

H0=Mean Brinell's hardness Index of unpolished stone surface is greater than or equal to 150.

Ha= Mean Brinell's hardness Index of unpolished stone surface is less than 150.

OR

$H0$: μ => 150

HA: $\mu < 150$

Step 2: Decide the significance level Here we select $\alpha = 0.05$.

Step 3: Identify the test statistics:

The sample size for this problem is 75.

We do not know the population standard deviation although the sample size is more than 30 still, we use the t distribution and the *tSTAT* test statistic.

Step 4: Calculate the p - value and test statistics:

scipy.stats.ttest_1samp calculates the t test for the mean of one sample given the sample observations and the expected value in the null hypothesis. This function returns t statistic and the two-tailed p value. Here we will need answer in one tailed p value. So, we are dividing p value by 2 to get one tailed p value.

By performing one sample t test, we can say that the,

T - statistics of unpolished stone is -4.1646296

P - value of unpolished stone is 4.17128700e-05

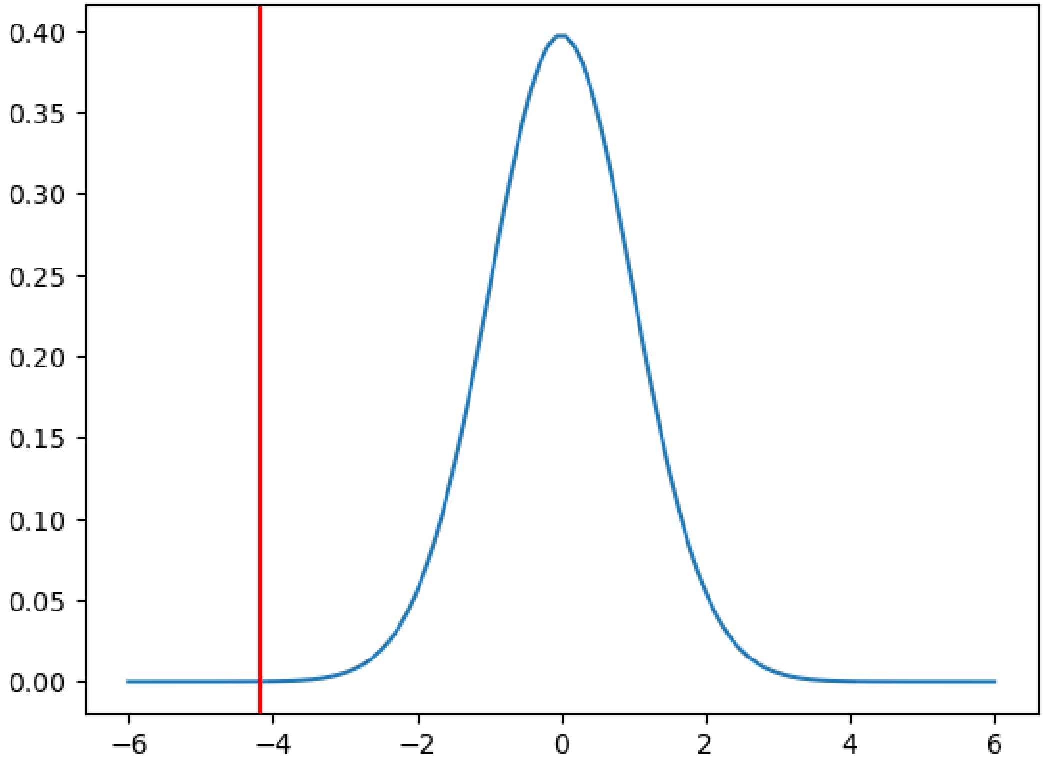

Figure 8: T-statistics of unpolished stone

Step 5: Decide to reject or accept null hypothesis for the Level of significance: 0.05:

p value < Level of significance

So, the statistical decision is we reject the null hypothesis at 5% level of significance.

It means that there is sufficient evidence for Zingaro stone printing company to believe that unpolished stones are not suitable for printing, that is they have Brinell's hardness index of less than 150.

3.2 Is the mean hardness of the polished and unpolished stones the same?

To determine if the mean hardness of polished and unpolished stones is the same, we will perform a two-sample t-test.

Steps to Perform the Tests:

Step 1: State the null and alternate hypotheses

Null Hypothesis (H0): The mean hardness of polished and unpolished stones are same.

Alternative Hypothesis (Ha): The mean hardness of polished and unpolished stones are different. OR

$H0: \mu 1 = \mu 2$

$Ha: \mu 1 \neq \mu 2$

Step 2: Decide the significance level Here we select $\alpha = 0.05$.

Step 3: Identify the test statistics:

The two-sample t-test compares the means of two independent groups to determine if there is a statistically significant difference between them.

It is suitable for comparing the means of two independent groups, especially when the population standard deviations are unknown.

After Performing two-sample t-test for polished and unpolished stones We can say that the,

T - statistics of unpolished stone is 3.2422320501414053

P - value of unpolished stone is 0.0014655150194628353

Step 5: Decide to reject or accept null hypothesis for the Level of significance: 0.05:

p value < Level of significance

So, the statistical decision is we reject the null hypothesis at 5% level of significance.

It means that, there is sufficient evidence for the mean hardness of polished and unpolished stones are different.

Recommendations:

- Here the mean hardness of unpolished stones is less than 150, Zingaro should either reject the batch or seek stones with higher hardness for optimal printing quality.
- There is a significant difference between the hardness of polished and unpolished stones, Zingaro should consider using only polished stones for printing to ensure consistent quality.

Problem 4

Dental implant data: The hardness of metal implants in dental cavities depends on multiple factors, such as the method of implant, the temperature at which the metal is treated, the alloy used as well as the dentists who may favor one method above another and may work better in his/her favourite method. The response is the variable of interest.

UNDERSTANDING THE STRUCTURE OF DATA:

Dataset Name: Dental+Hardness+data

This data dictionary provides a clear and concise reference for understanding the dataset's structure and the meaning of each column, ensuring accurate and consistent data handling in analysis.

Some important technical information about the dataset:

- The data set contains of 90 rows & 5columns.

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 90 entries, 0 to 89
Data columns (total 5 columns):
 #   Column    Non-Null Count  Dtype
---  ------    --------------  -----
 0   Dentist   90 non-null     int64
 1   Method    90 non-null     int64
 2   Alloy     90 non-null     int64
 3   Temp      90 non-null     int64
 4   Response  90 non-null     int64
dtypes: int64(5)
memory usage: 3.6 KB
```

Figure 9: Data Dictionary

- Dentist: The identifier for the dentist performing the implant.
- Method: The method used for the implant.
- Alloy: The type of alloy used.
- Temp: The temperature at which the metal is treated.
- Response: The hardness of the metal implant.

STATISTICAL ANALYSIS:

	count	mean	std	min	25%	50%	75%	max
Dentist	90.0	3.000000	1.422136	1.0	2.0	3.0	4.0	5.0
Method	90.0	2.000000	0.821071	1.0	1.0	2.0	3.0	3.0
Alloy	90.0	1.500000	0.502801	1.0	1.0	1.5	2.0	2.0
Temp	90.0	1600.000000	82.107083	1500.0	1500.0	1600.0	1700.0	1700.0
Response	90.0	741.777778	145.767845	289.0	698.0	767.0	824.0	1115.0

Table 3: Statistical Analysis

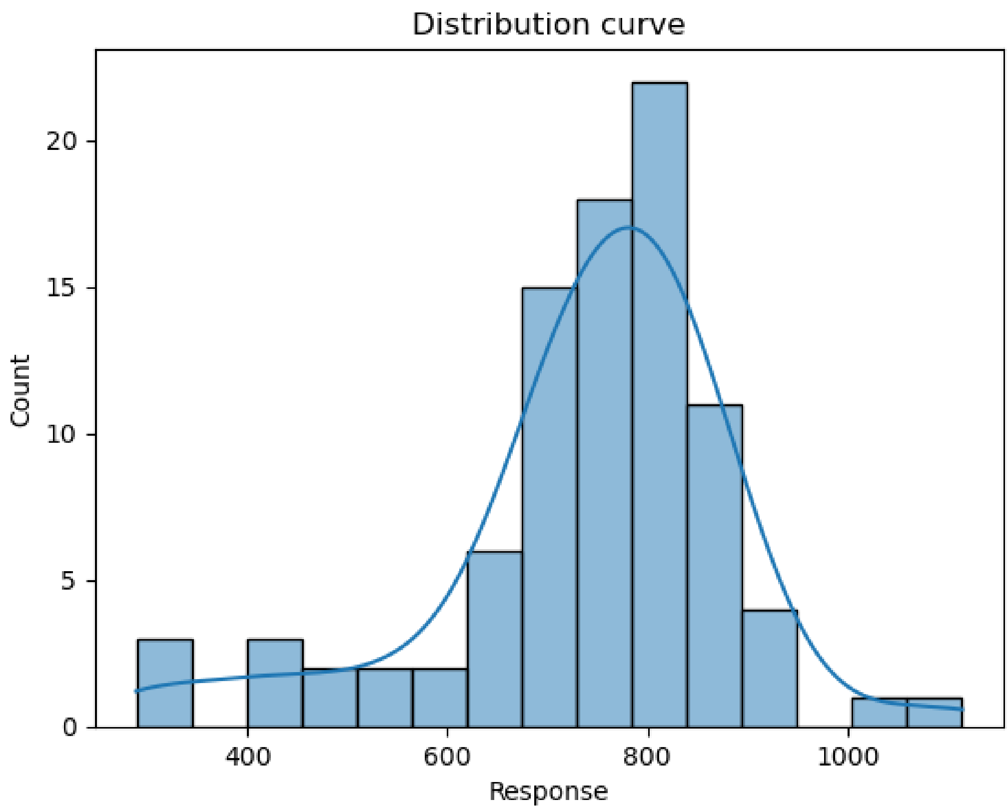

Figure10: distribution curve of response

- Response is almost normally distributed with some skewness.

4.1 How does the hardness of implants vary depending on dentists?

Hypothesis Testing

For Alloy 1:

Step 1: Define null and alternative hypotheses for One-Way ANOVA

Null Hypothesis ($H0$): The mean hardness of dental implants is the same across all dentists.

$H0: \mu1=\mu2=\mu3=\ldots=\mu k$

Alternate Hypothesis (Ha): At least one dentist's mean hardness of dental implants is different.

Ha: At least one μi is different.

$Ha: \mu1\neq\mu2\neq\mu3\neq\ldots\neq\mu k$

Step 2: Select Appropriate test

This is a problem, concerning more than two factors. One-way ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

One-way ANOVA test:

- In a one-way ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.
- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene test is applied to the response variable.

Shapiro-Wilk's test:

H0: The Dentist for alloy 1 follows a normal distribution.

Ha: The Dentist for alloy 1 does not follow a normal distribution.

```
Shapiro-Wilk Test for Alloy 1 (A1) - Normality:
Test Statistic: 0.8304629921913147
P-value: 1.1945070582441986e-05
```

Figure11: Shapiro wilk test – Alloy 1

Observation:
Since p-value of the test is lesser than the 5% significance level, we reject the null hypothesis that the response does not follow the normal distribution.

Levene's test

We will test the null hypothesis against the alternative hypothesis

H0: All the population variances are equal

Ha: At least one variance is different from the rest

```
Levene's Test for Alloy 1 (A1) - Equality of Variances:
Test Statistic: 1.3847146992797106
P-value: 0.2565537418543795
```

Figure12: Levene Test for alloy 1

Observation:

Since the p-value is large than the 5% significance level, we fail to reject the null hypothesis of homogeneity of variances.

Step 3: Decide the significance level

As given in the problem statement, we select alpha = 0.05

Step 4: Calculate the p-value

- We will use the f_oneway() function from the scipy.stats library to perform a one-way ANOVA test.
- The f_oneway() function takes the sample observations from the different groups and returns the test statistic and the p-value for the test.

Step 5: Compare the p-value with alpha= 0.05

As the p-value 0.11656712140267628 is greater than the level of significance, we fail to reject the null hypothesis.

Step 6: Draw inference

Since the p-value is greater than the level of significance (5%), we fail to reject the null hypothesis. Hence, we have enough statistical evidence to say that the mean hardness of dental implants is the same across all dentists for alloy 1.

For Alloy 2:

Step 1: Define null and alternative hypotheses for One-Way ANOVA

Null Hypothesis ($H0$): The mean hardness of dental implants is the same across all dentists.

$H0$: $\mu1=\mu2=\mu3=\ldots=\mu k$

Alternate Hypothesis (Ha): At least one dentist's mean hardness of dental implants are different.

Ha: At least one μi is different.

Ha: $\mu1\neq\mu2\neq\mu3\neq\ldots\neq\mu k$

Step 2: Select Appropriate test

This is a problem, concerning more than two factors. One-way ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

One-way ANOVA test

- In a one-way ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.
- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene test is applied to the response variable.

Assumption 1: Normality

Use the shapiro function for the scipy.stats library for this test

Shapiro-Wilk's test:

We will test the null hypothesis against the alternative hypothesis

H0: The Dentist for alloy 2 follows a normal distribution.

Ha: The Dentist for alloy 2 does not follow a normal distribution.

```
Shapiro-Wilk Test for Alloy 2 (A2) - Normality:
Test Statistic: 0.887769341468811
P-value: 0.00040293222991749644
```

Figure13: Shapiro wilk test – Alloy 2

Since p-value of the test is lesser than the 5% significance level, we reject the null hypothesis that the response does not follows the normal distribution.

Assumption 2:
Levene's test

We will test the null hypothesis against the alternative hypothesis

H0: All the population variances are equal

Ha: At least one variance is different from the rest

```
Levene's Test for Alloy 2 (A2) - Equality of Variances:
Test Statistic: 1.4456166464566966
P-value: 0.24
```

Since the p-value is large than the 5% significance level, we fail to reject the null hypothesis of homogeneity of variances.

Figure14: Levene Test for alloy 2

Step 3: Decide the significance level

As given in the problem statement, we select alpha = 0.05

Step 4: Calculate the p-value using f_oneway

Step 5: Compare the p-value with alpha= 0.05

As the p-value 0.7180309510793431 is greater than the level of significance, we fail to reject the null hypothesis.

Step 6: Draw inference

P-value is 0.72which is very greater than alpha i.e., 0.05. Hence, we Fail to reject null hypothesis and consider there is no difference in means among the dentists in terms of implant hardness for Alloy 2

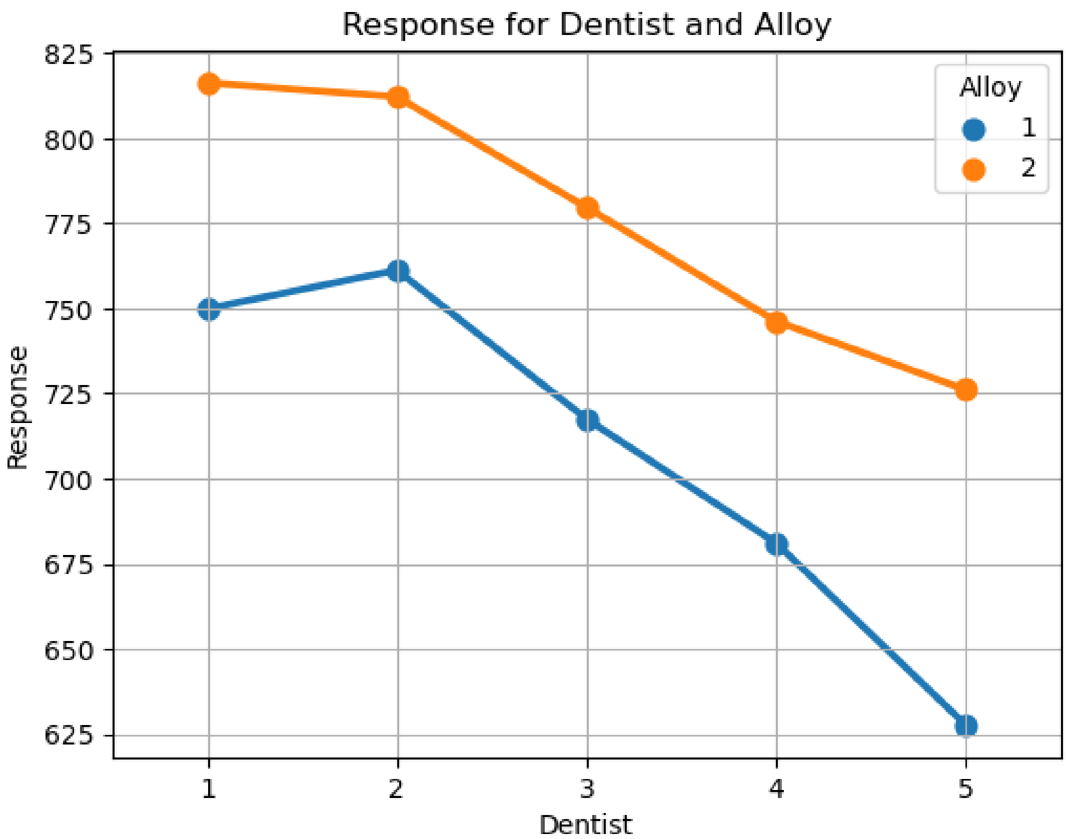

Figure15: Plot for dentist vs response for alloy 1 and 2

4.2 How does the hardness of implants vary depending on methods?

For Alloy 1:

Step 1. State the Null and Alternate Hypotheses

Null Hypothesis ($H0$): The mean hardness of dental implants is the same across all methods.

$H0$: $\mu1=\mu2=\mu3=\ldots=\mu k$

Alternate Hypothesis (Ha): At least one method's mean hardness of dental implants is different.

Ha: At least one μi is different.

Ha: $\mu 1 \neq \mu 2 \neq \mu 3 \neq \ldots \neq \mu k$

Step 2: Select Appropriate test

This is a problem, concerning more than two factors. One-way ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

One-way ANOVA test

- In a one-way ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.
- For testing of normality, Shapiro-Wilk's test is applied to the response variable.
- For equality of variance, Levene test is applied to the response variable.

Assumption 1: Normality
Use the shapiro function for the scipy.stats library for this test

Shapiro-Wilk's test

We will test the null hypothesis against the alternative hypothesis

H0: The Method for alloy 1 follows a normal distribution.
Ha: The Method for alloy 1 does not follow a normal distribution.

```
Shapiro-Wilk Test for Alloy 1 (A1) - Normality:
Test Statistic: 0.8304629921913147
P-value: 1.1945070582441986e-05
```

Figure16: Shapiro – wilk test – alloy 1

Since p-value of the test is lesser than the 5% significance level, we reject the null hypothesis that the response does not follows the normal distribution.

Assumption 2: Homogeneity of Variance

```
Levene's Test for Alloy 1 (A1) - Equality of Variances:
Test Statistic: 6.52140454403598
P-value: 0.0034160381460233975
```

Figure17: Levene test – alloy 1

Since the p-value is lesser than the 5% significance level, we reject the null hypothesis of homogeneity of variances.

Step 3: Decide the significance level

As given in the problem statement, we select alpha = 0.05

Step 4: Calculate the p-value

```
ANOVA Results for Alloy 1:
                  sum_sq    df         F    PR(>F)
C(Method)  148472.177778   2.0  6.263327  0.004163
Residual   497805.066667  42.0       NaN       NaN
```

Table 4: Method Variation – alloy 1

Step 5: Compare the p-value with alpha= 0.05

p-value is 0.004163 which is less than the level of significance, we reject the null hypothesis.

Step 6: Draw inference

Since the p-value is lesser than the level of significance (5%), we reject the null hypothesis. Hence, we have enough statistical evidence to say that the mean hardness of dental implants is different across all method for alloy 1.

For alloy 2

Step 1. State the Null and Alternate Hypotheses

Null Hypothesis ($H0$): The mean hardness of dental implants is the same across all methods.

$H0$: $\mu1=\mu2=\mu3=\ldots=\mu k$

Alternate Hypothesis (Ha): At least one method's mean hardness of dental implants is different.

Ha: At least one μi is different.

Ha: $\mu1\neq\mu2\neq\mu3\neq\ldots\neq\mu k$

Step 2: Select Appropriate test

This is a problem, concerning more than two factors. One-way ANOVA is an appropriate test here provided normality and equality of variance assumptions are verified.

One-way ANOVA test

- In a one-way ANOVA test, we compare the means from several populations to test if there is any significance difference between them. The results from an ANOVA test are most reliable when the assumptions of normality and equality of variances are satisfied.
- For testing of normality, Shapiro-Wilk's test is applied to the response variable.

- For equality of variance, Levene test is applied to the response variable.

Assumption 1: Normality

Use the shapiro function for the scipy.stats library for this test

<u>Shapiro-Wilk's test</u>

We will test the null hypothesis against the alternative hypothesis

H0: The Method for alloy 2 follows a normal distribution.

Ha: The Method for alloy 2 does not follow a normal distribution.

```
Shapiro-Wilk Test for Alloy 2 (A2) - Normality:
Test Statistic: 0.953279435634613
P-value: 0.25722309947013855
```

Figure18: Shapiro – wilk test for alloy 2

Since p-value of the test is larger than the 5% significance level, we fail to reject the null hypothesis that the response follows the normal distribution.

Assumption 2: Homogeneity of Variance

```
Levene's Test for Alloy 2 (A1) - Equality of Variances:
Test Statistic: 1.1314570586352275
P-value: 0.33917195814890017
```

Figure19: Levene test for alloy 2

Since p-value of the test is larger than the 5% significance level, we fail to reject the null hypothesis that the response follows the Homogeneity of Variance

Step 3: Decide the significance level

As given in the problem statement, we select alpha = 0.05

Step 4: Calculate the p-value

```
ANOVA Results for Alloy 2:
                sum_sq    df          F    PR(>F)
C(Method)     499640.4   2.0   16.4108  0.000005
Residual      639362.4  42.0        NaN       NaN
```

Table 5: Method Variation – alloy 2

Step 5: Compare the p-value with alpha= 0.05

p value (0.000005) less than the level of significance, we reject the null hypothesis.

Step 6: Draw inference

Since the p-value is lesser than the level of significance (5%), we reject the null hypothesis. Hence, we have enough statistical evidence to say that the mean hardness of dental implants is different across all method for alloy 2.

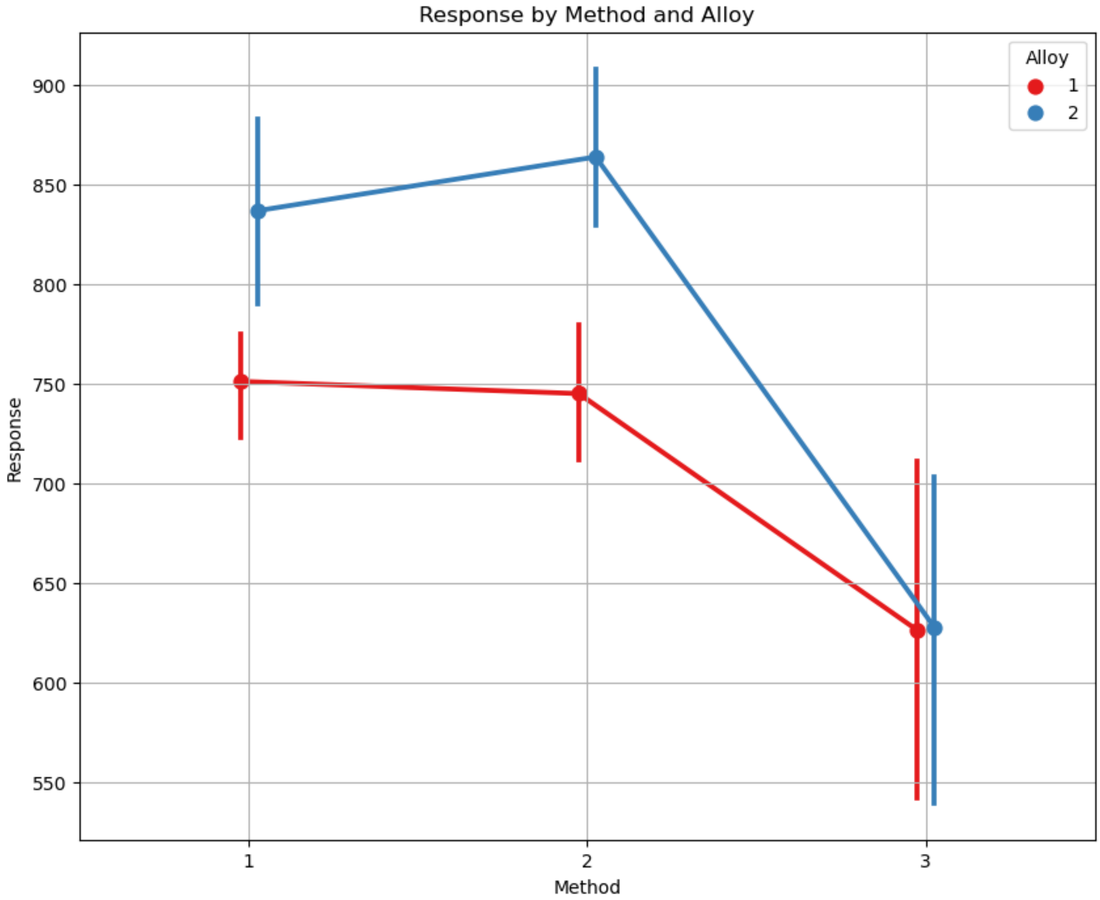

Figure 20: Plot for Response vs Method

4.3 What is the interaction effect between the dentist and method on the hardness of dental implants for each type of alloy?

Step 1: Define null and alternative hypotheses for Two-Way ANOVA

The null and alternative hypotheses can be formulated as:

($H0$): The effect of dentist on the hardness of dental implants for alloy 1 does not depend on the effect of the method variable (a.k.a. no interaction effect)

(Ha): There is an interaction effect between dentist and method on alloy.

Step 2: Select Appropriate test

This is a problem, concerning the effect of two independent variables on a dependent variable. Two-way ANOVA test is an appropriate test here.

Following are the assumptions of the Two-way ANOVA test:

- The populations from which the samples are obtained must be normally distributed.
- Sampling is done correctly. Observations for within and between groups must be independent.
- The variances among populations must be equal (homoscedastic).
- The dependent data must be measured at an interval scale.

Step 3: Decide the significance level

As given in the problem statement, we select alpha =0.05

Step 4: Check for Interaction Effect

We will now analyse the effect of both the dentist and method on the Alloy 1.

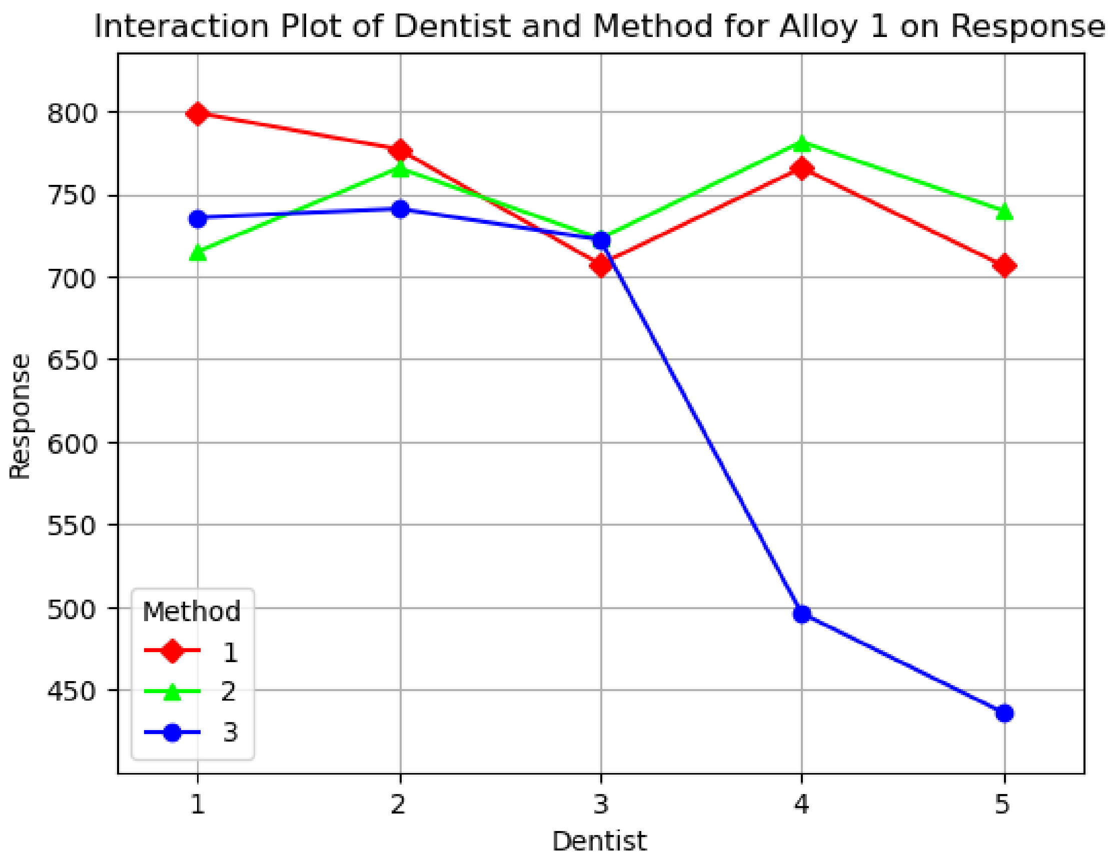

Figure 21: Interaction plot of Dentist and Method for alloy 1 on Response

We can see that there is an interaction between the Dentist and Method for alloy 1 on Response.

Step 5: Calculate the p-value

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist):C(Method)	14.0	441097.244444	31506.946032	4.606728	0.000221
Residual	30.0	205180.000000	6839.333333	NaN	NaN

Table 6: Dentist and Method Interaction

Step 6: Compare the p-value with alpha

p value is less than the level of significance, we reject the null hypothesis

Step 7: Draw inference

P-value is lesser than alpha i.e., 0.05. Hence, we have enough evidence to reject null hypothesis and consider there is a difference in means among the Interaction effect between Dentist and Method levels in terms of implant hardness for Alloy 1.

For Alloy 2:

Step 1: Define null and alternative hypotheses for Two-Way ANOVA

The null and alternative hypotheses can be formulated as:

($H0$): The effect of dentist on the hardness of dental implants for alloy 2 does not depend on the effect of the method variable (a.k.a. no interaction effect)

(Ha): There is an interaction effect between dentist and method on alloy 2.

Step 2: Select Appropriate test

This is a problem, concerning the effect of two independent variables on a dependent variable. Two-way ANOVA test is an appropriate test here.

Following are the assumptions of the Two-way ANOVA test:

- The populations from which the samples are obtained must be normally distributed.
- Sampling is done correctly. Observations for within and between groups must be independent.
- The variances among populations must be equal (homoscedastic).
- The dependent data must be measured at an interval scale.

Step 3: Decide the significance level

As given in the problem statement, we select alpha =0.05

Step 4: Check for Interaction Effect

We will now analyse the effect of both the dentist and method on the Alloy 2.

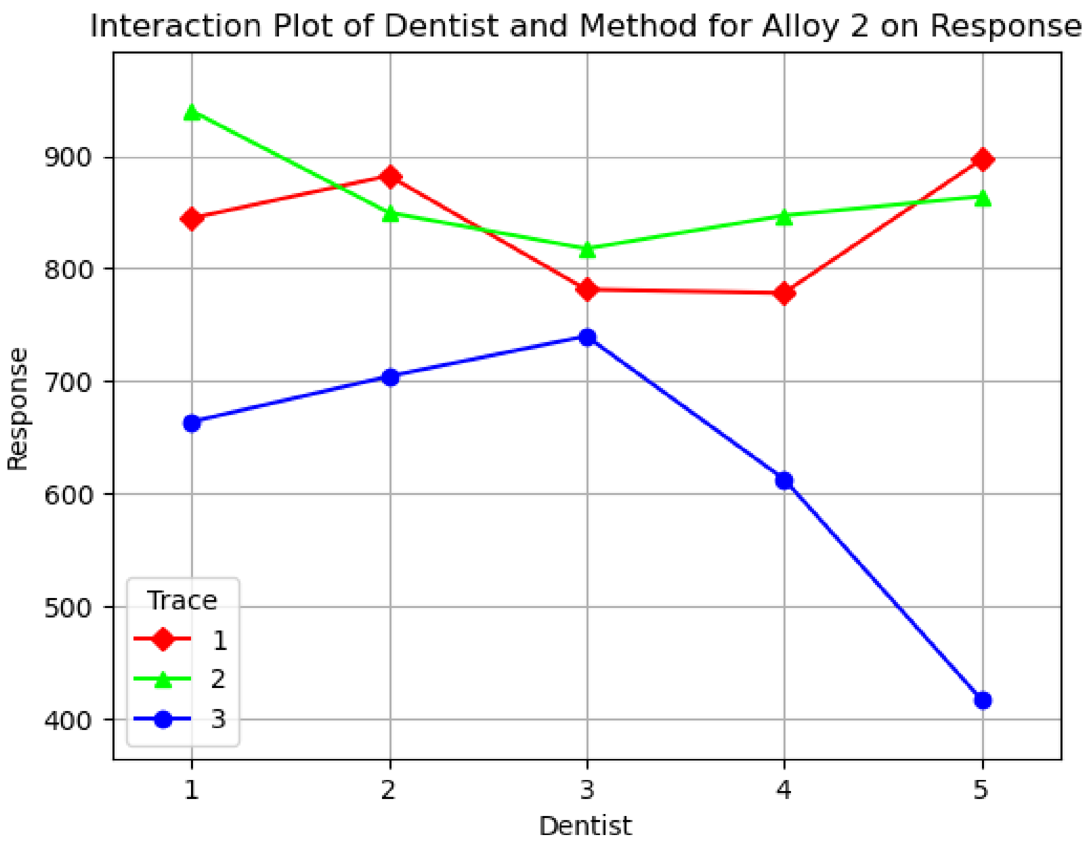

Figure 22: Interaction plot of Dentist and Method for alloy 2 on Response

- There is some sort of interaction between method and dentist.

Step 5: Calculate the p-value

	df	sum_sq	mean_sq	F	PR(>F)
C(Dentist):C(Method)	14.0	753898.133333	53849.866667	4.194953	0.000482
Residual	30.0	385104.666667	12836.822222	NaN	NaN

Table 7: Dentist and Method Interaction for alloy 2

Step 6: Compare the p-value with alpha

p-value is less than the level of significance, we reject the null hypothesis.

Step 7: Draw inference

P-value is lesser than alpha i.e., 0.05. Hence, we have enough evidence to reject null hypothesis and consider there is difference in means among the Interaction effect between Dentist and Method levels in terms of implant hardness for Alloy 2. That is, there is an interaction upon response.

4.4 How does the hardness of implants vary depending on dentists and methods together?

<u>Hypothesis Test for Alloy 1</u>

Step 1: Define null and alternative hypotheses for Two-Way ANOVA

The null and alternative hypotheses can be formulated as:

($H0$): The effect of dentist on the hardness of dental implants for alloy 1 does not depend on the effect of the method variable (a.k.a. no interaction effect)

(Ha): There is an interaction effect between dentist and method on alloy 1.

Step 2: Select Appropriate test

This is a problem, concerning the effect of two independent variables on a dependent variable. Two-way ANOVA test is an appropriate test here.

Following are the assumptions of the Two-way ANOVA test:

- The populations from which the samples are obtained must be normally distributed.
- Sampling is done correctly. Observations for within and between groups must be independent.
- The variances among populations must be equal (homoscedastic).
- The dependent data must be measured at an interval scale.

Step 3: Decide the significance level

As given in the problem statement, we select alpha =0.05

Step 4: Calculate the p-value

```
ANOVA Table for Alloy 1:
                              sum_sq    df          F    PR(>F)
C(Dentist)             106683.688889   4.0   3.899638  0.011484
C(Method)              148472.177778   2.0  10.854287  0.000284
C(Dentist):C(Method)   185941.377778   8.0   3.398383  0.006793
Residual               205180.000000  30.0        NaN       NaN

P-value for Interaction (Alloy 1): 0.006792747204237292
```

Table 8: Dentist and Method Interaction effect for alloy 1

Step 5: Compare the p-value with alpha

P value for Method is 0.000284, p value for Dentist is 0.011484 and Interaction variable - Dentist: Method is 0.006793. So, we have enough evidence to reject null hypothesis stating that at least one of the pair means is different for Alloy 1. Therefore, there is an interaction effect between dentist and method on alloy 1

Step 6: Draw inference

P-value is lesser than alpha i.e., 0.05. Hence, we have enough evidence to reject null hypothesis and consider there is a difference in means among the Interaction effect between Dentist and Method levels in terms of implant hardness for Alloy 1.

TUKEY HSD TEST FOR ALLOY 1:

```
Tukey's HSD Test Results:
   Multiple Comparison of Means - Tukey HSD, FWER=0.05
==========================================================
group1 group2  meandiff p-adj    lower     upper   reject
----------------------------------------------------------
   1:1    1:2     -84.0 0.9933 -332.8283  164.8283  False
   1:1    1:3  -63.3333 0.9996 -312.1617   185.495  False
   1:1    2:1     -22.0    1.0 -270.8283  226.8283  False
   1:1    2:2  -33.3333    1.0 -282.1617   215.495  False
   1:1    2:3     -58.0 0.9999 -306.8283  190.8283  False
   1:1    3:1  -91.6667 0.9853  -340.495  157.1617  False
   1:1    3:2     -76.0 0.9975 -324.8283  172.8283  False
   1:1    3:3  -76.6667 0.9972  -325.495  172.1617  False
   1:1    4:1  -33.3333    1.0 -282.1617   215.495  False
   1:1    4:2  -17.6667    1.0  -266.495  231.1617  False
   1:1    4:3 -302.6667  0.007  -551.495  -53.8383   True
   1:1    5:1  -92.3333 0.9844 -341.1617   156.495  False
   1:1    5:2     -59.0 0.9998 -307.8283  189.8283  False
```

Table 9: Tukey's HSD Test Results for alloy 1

<u>Hypothesis Test for Alloy 2</u>

Step 1: Define null and alternative hypotheses for Two-Way ANOVA

The null and alternative hypotheses can be formulated as:

($H0$): The effect of dentist on the hardness of dental implants for alloy 2 does not depend on the effect of the method variable (a.k.a. no interaction effect)

(Ha): There is an interaction effect between dentist and method on alloy 2.

Step 2: Select Appropriate test

This is a problem, concerning the effect of two independent variables on a dependent variable. Two-way ANOVA test is an appropriate test here.

Following are the assumptions of the Two-way ANOVA test:

- The populations from which the samples are obtained must be normally distributed.
- Sampling is done correctly. Observations for within and between groups must be independent.
- The variances among populations must be equal (homoscedastic).
- The dependent data must be measured at an interval scale.

Step 3: Decide the significance level

As given in the problem statement, we select alpha =0.05

Step 4: Calculate the p-value

```
ANOVA Table for Alloy 2:
                          sum_sq    df          F    PR(>F)
C(Dentist)            56797.911111   4.0   1.106152  0.371833
C(Method)            499640.400000   2.0  19.461218  0.000004
C(Dentist):C(Method) 197459.822222   8.0   1.922787  0.093234
Residual             385104.666667  30.0        NaN       NaN

P-value for Interaction (Alloy 2): 0.09323403966149585
```

Table 10: Dentist and Method Interaction effect for alloy 2

Step 5: Compare the p-value with alpha = 0.05

P value for Method is 0.000004, which is only less than 0.05 when compared to Dentist (0.371833) and Interaction variable - Dentist: Method (0.093234). So, we have enough evidence to reject null hypothesis stating that at least one of the pair means is different for Alloy 2.

Step 6: Draw inference

P-value is lesser than alpha i.e., 0.05. Hence, we have enough evidence to reject null hypothesis and consider there is a difference in means among the Interaction effect between Dentist and Method levels in terms of implant hardness for Alloy 2.

TUKEY HSD TEST FOR ALLOY 2

```
P-value for Interaction (Alloy 2): 0.09323403966149585

Interaction is not significant, so Tukey's HSD test is not performed.
```

Table 11: Tukey’s HSD Test Results for alloy 2