Field and Service Robotics
Homework 4

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https://github.com/BenitoVodola/FSRHW4.git

Exercise 1

When a rigid body is submerged in a fluid under the effect of gravity, two more forces must be considered with respect to the dynamic model: the gravitational force and the buoyancy. The buoyancy is a hydrostatic effect since it is not function of the relative movement between the body and the fluid. Let $g = \begin{bmatrix} 0 & 0 & \overline{g} \end{bmatrix} \in \mathbb{R}^3$, where $\overline{g} \in \mathbb{R}$ is the gravity acceleration; $\Delta \in \mathbb{R}$, the volume of the body; $m \in \mathbb{R}$ is the mass of the body and $\rho \in \mathbb{R}$ the density of the water, then the buoyancy is $b = \rho\Delta||\overline{g}||$. The buoyancy force acts at the center of buoyancy, $r_b^b \in \mathbb{R}^3$, and it is equal to $f_b^b = -R_b^T \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}$, and it acts on the underwater body by opposing to the gravity force; this means that it tries to bring up the submerged body. If one desires to express this force into the body frame it is necessary to use the transpose of the rotation matrix between body frame and world frame. It is essential to highlight that the center of buoyancy is different from the center of the mass; furthermore if they are not aligned , a rotational torque will appear. The buoyancy effect has been neglected in the aerial robotics because the air density is much lighter than the density of the moving mechanical system, in underwater applications, instead, the density of the water is comparable with the density of the robot.

Exercise 2

2.A

False. The added mass effect **does not consider an additional load to the structure**. The added mass effect represents an effect due to a reaction force of the fluid into which underwater robot is accelerating and moving, acting on the robot. The fluid exerts a reaction force which is equal in magnitude and opposite in direction; this reaction force is the added mass contribution and it is function of the body's geometry. The added mass is not a quantity of fluid to add to the system such that it has a bigger mass. It can be taken into account in the dynamic model of a completely submerged robot with a matrix that is no more positive definite.

2.B

True. The added mass effect is considered in underwater robotics since the density of the underwater robot is comparable to the density of the water. When a rigid body is moving in a fluid, the additional inertia of the fluid surrounding the body, that is accelerated by the movement of the body, has to be considered.

2.C

True. The damping effect helps in the stability analysis. The viscosity of the fluid causes the presence of dissipative drag and lift forces on the body. The viscous effect can be considered as the sum of two forces, the drag and the lift forces. The drag is parallel to the relative velocity between the body and the fluid; the lift force is normal to the drag force. Both forces are supposed to act at the body's center of mass. It can be useful in the stability proof with the direct Lyapunov method, because of it appears as a negative term in the derivative of Lyapunov function.

2.D

False. Ocean current are mainly caused by: tidal movements, atmospheric wind system over the sea, heat exchange at the sea's surface, salinity change, Coriolis forces due to the Earth's rotation. The answer is false because Ocean current is usually considered constant and irrotational with respect to the **world frame**, not with respect to the body frame, because it would change continuously. With this assumption, ocean current can be expressed as:

$$v_c = \begin{bmatrix} v_{c,x} \\ v_{c,y} \\ v_{c,z} \\ 0 \\ 0 \\ 0 \end{bmatrix} \in \mathbb{R}^6, \dot{v}_c = 0_6.$$

Exercise 3

3.A

In order to implement the quadratic function to impose the desired ground reaction forces and desired reference for the center of mass to the robot respecting the constraints, the following line code has been added to the software program:

$$[zval, basic\_info, adv\_info] = qpSWIFT(sparse(H), g, sparse(Aeq), beq, sparse(Aineq), bineq);$$

3.B

As first examination we are going to consider the differences between the possible gaits that we can impose the quadruped robot without varying any physical parameters. The possible gaits are: trotting, bounding, pacing, galloping, trotting (running), crawling.

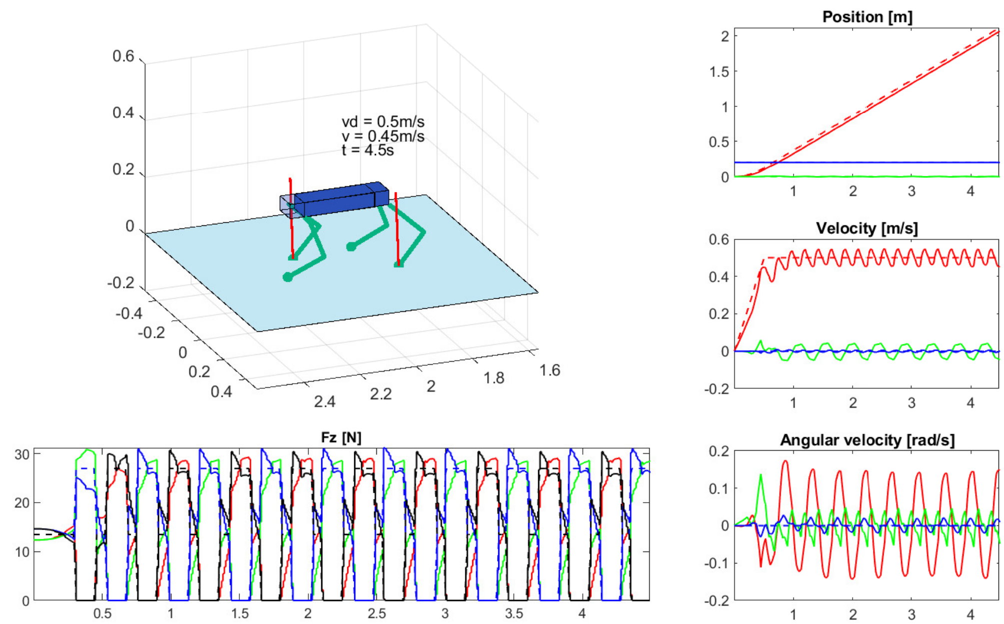

Figure 1: Gait 0: Trotting

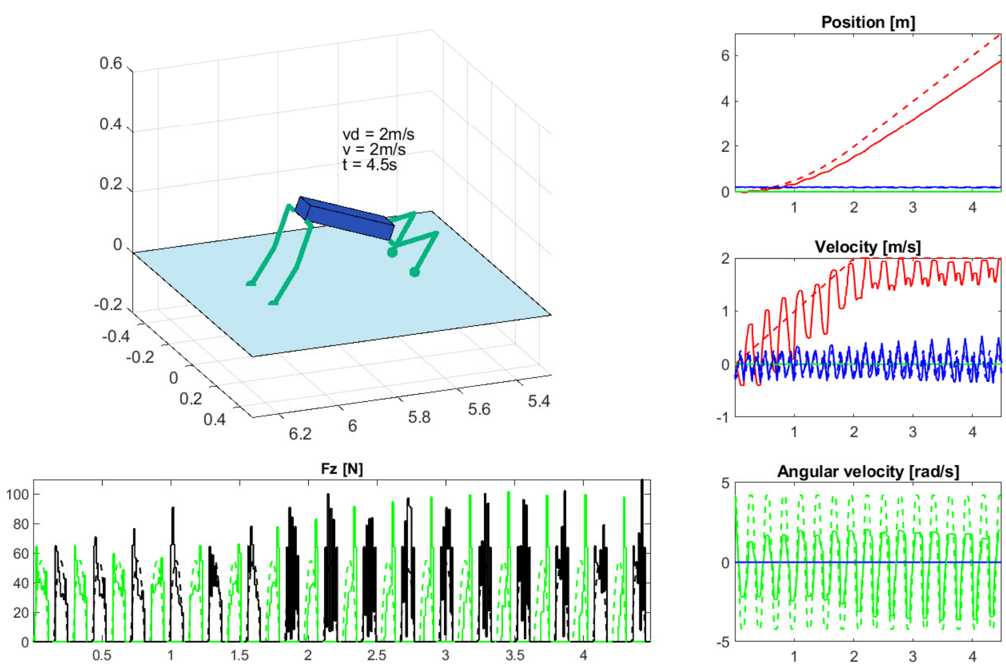

Figure 2: Gait 1: Bounding

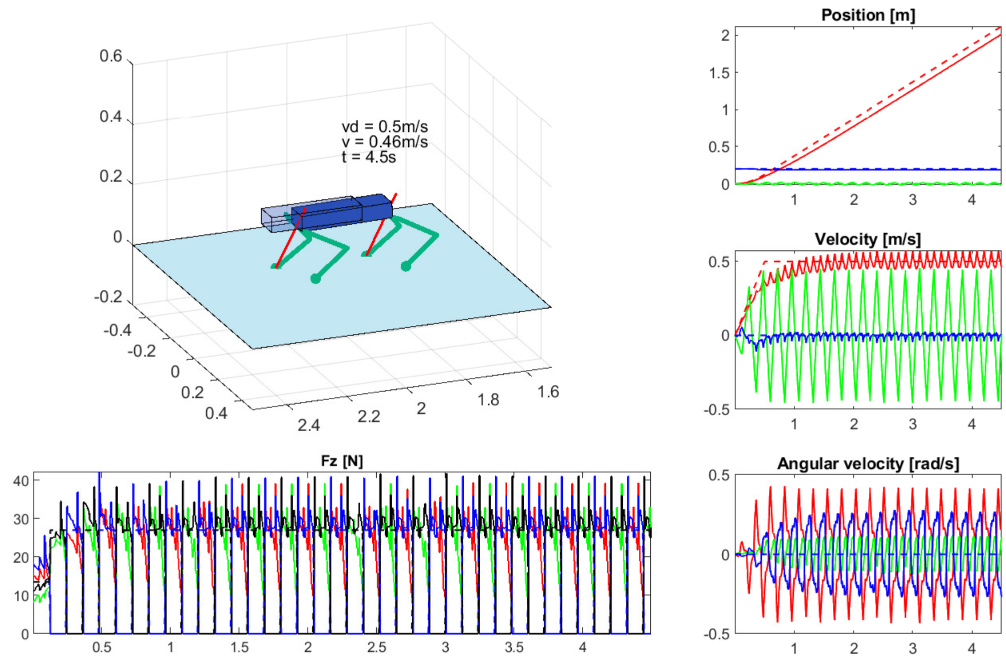

Figure 3: Gait 2: Pacing

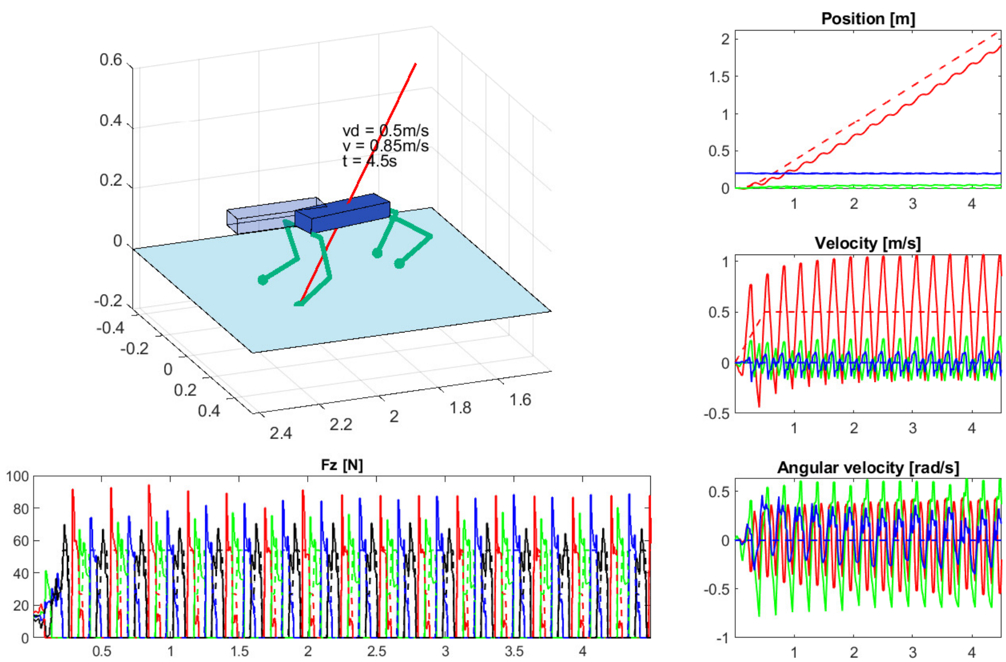

Figure 4: Gait 3: Galloping

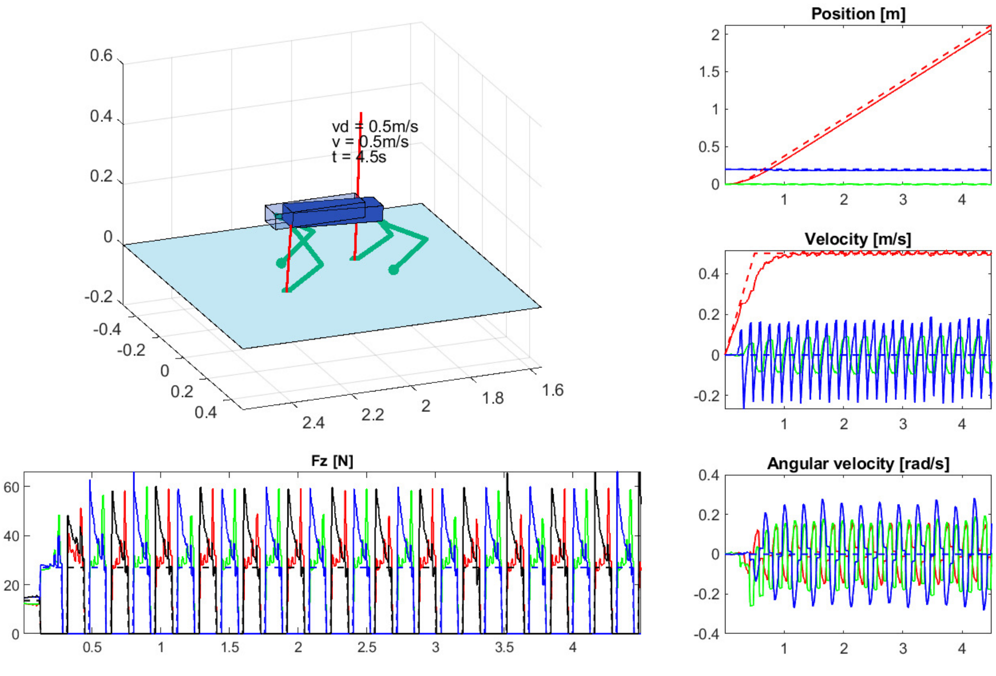

Figure 5: Gait 4: Trotting (running)

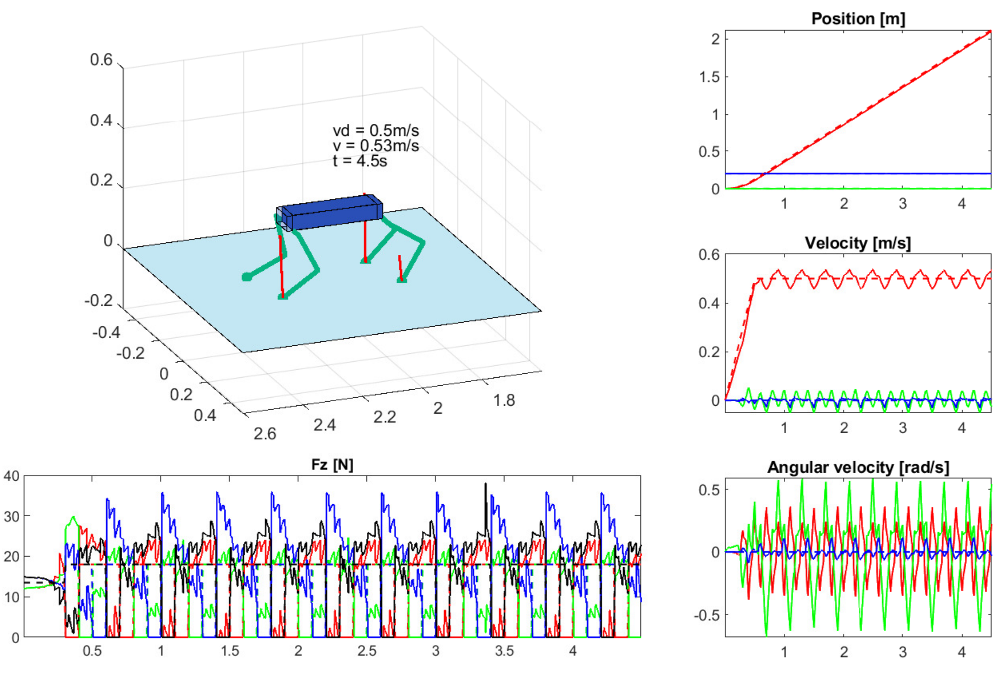

Figure 6: Gait 5: Crawling

From the results of the figures above we can state that:

- Gait 0:
  1. the tracking error about the x-component of position is pretty small and constant but it is not zero;
  2. the tracking error about the x component of velocity has a mean value that is the desired one but it exhibits an oscillatory behaviour;
  3. the four GRFs (one for each stance foot) reaches a peak value of 30 Newton.
- Gait 1:
  1. the tracking error about the x-component of position diverges;
  2. the tracking error about the x component of velocity has a mean value that is not the desired one and it exhibits an oscillatory behaviour;
  3. the four GRFs (one for each stance foot) reaches a peak value over 100 Newton.
- Gait 2:
  1. the tracking error about the x-component of position is pretty small and constant but it is not zero;
  2. the tracking error about the x component of velocity has a mean value that is the desired one but it exhibits small oscillations;
  3. the four GRFs (one for each stance foot) reaches a peak value of 40 Newton.
- Gait 3:
  1. the tracking error about the x-component of position is constant but it is not zero and exhibits small oscillations;
  2. the tracking error about the x component of velocity has a mean value that is the desired one but it exhibits huge oscillations;
  3. the four GRFs (one for each stance foot) reaches a peak value of 90 Newton.
- Gait 4:
  1. the tracking error about the x-component of position is very small and constant but it is not zero;
  2. the tracking error about the x component of velocity has a mean value that is the desired one but it exhibits very small oscillations;
  3. the four GRFs (one for each stance foot) reaches a peak value of 60 Newton.
  4. with respect to gait 0, the velocity needs less time to reach the steady state.
- Gait 5:
  1. the tracking error about the x-component of position is very small and constant but it is not zero;

2. the tracking error about the x component of velocity has a mean value that is the desired one but it exhibits oscillations;
3. the four GRFs (one for each stance foot) reaches a peak value of 35 Newton.

To sum up we can state that, gaits four and five are the best ones because they guarantee tracking error on position and velocity that is sufficiently small. Gaits zero and two exhibit bigger error than the previous gaits and they does not exhibit any oscillation on the position. Gait three not only exhibits a tracking error that is different from zero, but it exhibits oscillations on the position and very large oscillations on the velocity. Gait one is the worst case because it has a position tracking error that becomes bigger and bigger. As next step we will consider the gaits three and five and we will see what happens if we vary some physical parameters.

Gait 3

As first step, let's decrease the friction coefficient μ from 1 to 0.6. We expect that the GRFs decrease. By varying friction coefficient µ, constraints on non-sliding contact (inequality constraints) are modified.

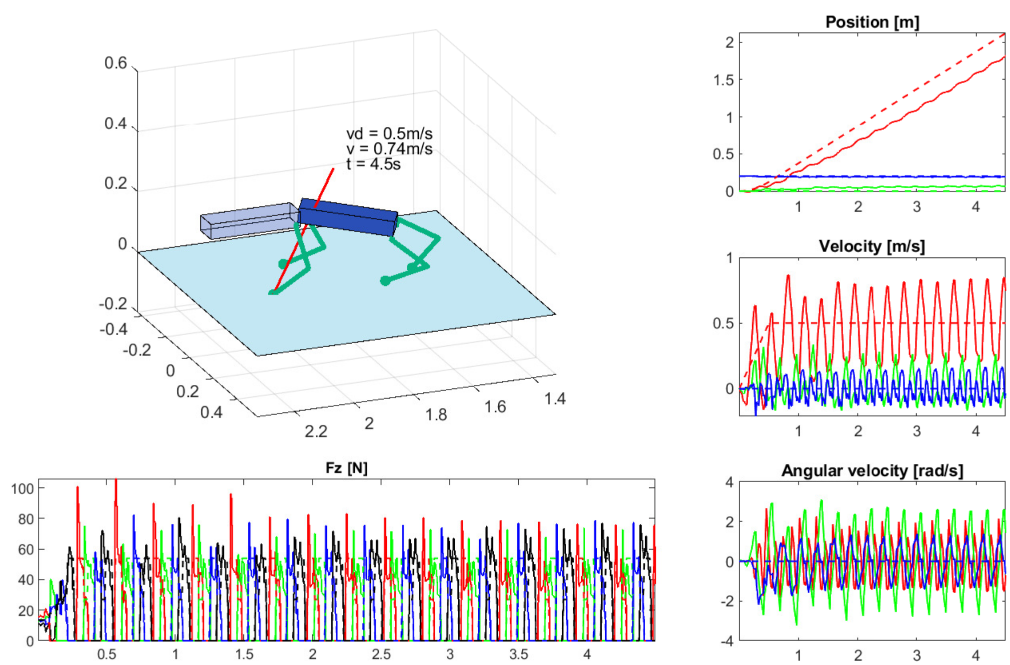

Figure 7: Gait 3 with $\mu = 0.6$

From this result we can state that:

- the GRFs at steady state exhibit smaller peak values with respect the case with $\mu = 1$;

- the tracking error of x-component of position exhibits a worse behavior than the case with $\mu = 1$;
- the x-component of the velocity exhibits smaller oscillations than the case with $\mu = 1$;

Let's analyze what happens if we decrease the mass of the quadruped robot from $5.5Kg$ to $1Kg$. We expect that the GRFs decrease. With an higher mass value ground reaction forces will be bigger, while the contrary happens for a lower mass value.

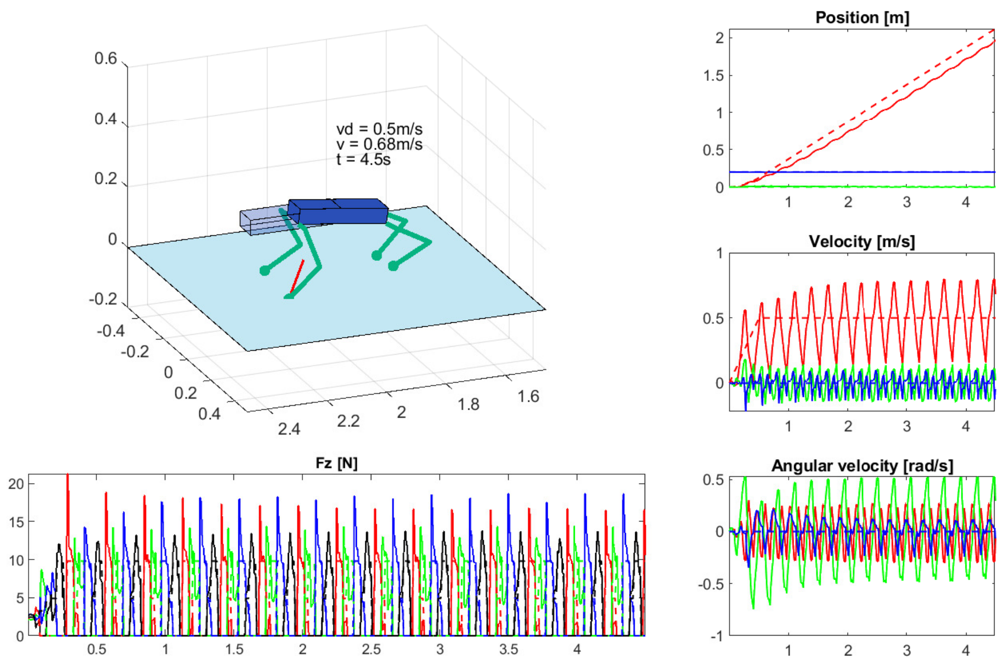

Figure 8: Gait 3 with mass $m = 1kg$

From the figure above we can state that:

- the GRFs at steady state are much smaller than the cases with $m = 5.5Kg$ and $\mu = 0.6$;
- the tracking error of x-component of position exhibits a better behavior than the case with $m = 5.5Kg$;
- the x-component of the velocity exhibits smaller oscillations than the case with $m = 5.5Kg$;

So, to improve the behavior, an option could be to build a lighter quadruped robot. As last case let's modify the desired velocity from $0.5m/s$ to $0.2m/s$.

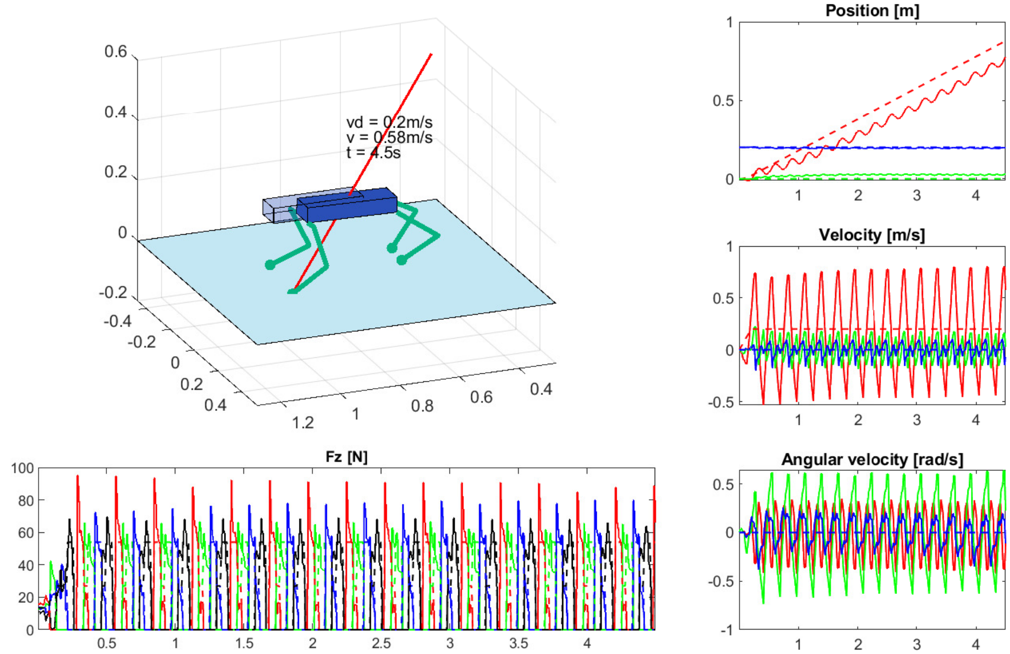

Figure 9: Gait 3 with desired velocity $v_d = 0.2m/s$

From the picture above we can state that:

- the tracking error of x-component of position exhibits a more oscillatory behavior;
- GRFs are identical to the untouched case;
- the mean value of the x-component of velocity is equivalent to the desired velocity but we still have huge oscillations.

To sum up, for the gallop gait:

- decreasing the friction coefficient does not produce any improvements of the gait, this means that we should avoid wet surfaces and we should prefer some materials (for the ground and for the quadruped's feet) that produce a high value of friction coefficient;
- the mass of the robot drastically affects the behavior of the gait, we should avoid heavy materials;
- varying the velocity does not produce any improvements of the gait.

Gait 5

As the previous analysis, we are going to study how changing the physical parameters affects the gait. As first step, let's decrease the friction coefficient μ from 1 to 0.6. We expect that the GRFs decrease. By varying friction coefficient µ, constraints on non-sliding contact (inequality constraints) are modified.

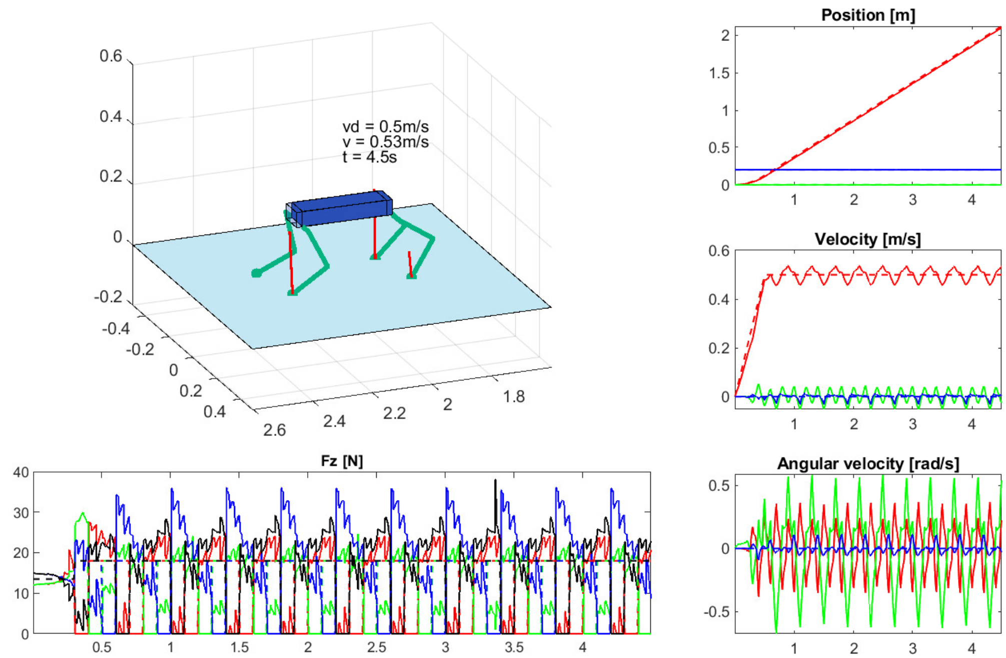

Figure 10: Gait 5 with $\mu = 0.6$

From this result we can state that:

- the behavior of GRFs is similar to the case with $\mu = 1$;
- the tracking error of x-component of position exhibits the same behavior than the case with $\mu = 1$, it is still very small and constant;
- the x-component of the velocity exhibits the same oscillations than the case with $\mu = 1$;

As before, let's analyze what happens if we decrease the mass of the quadruped robot from $5.5Kg$ to $1Kg$. We expect that the GRFs decrease. With an higher mass value ground reaction forces will be bigger, while the contrary happens for a lower mass value.

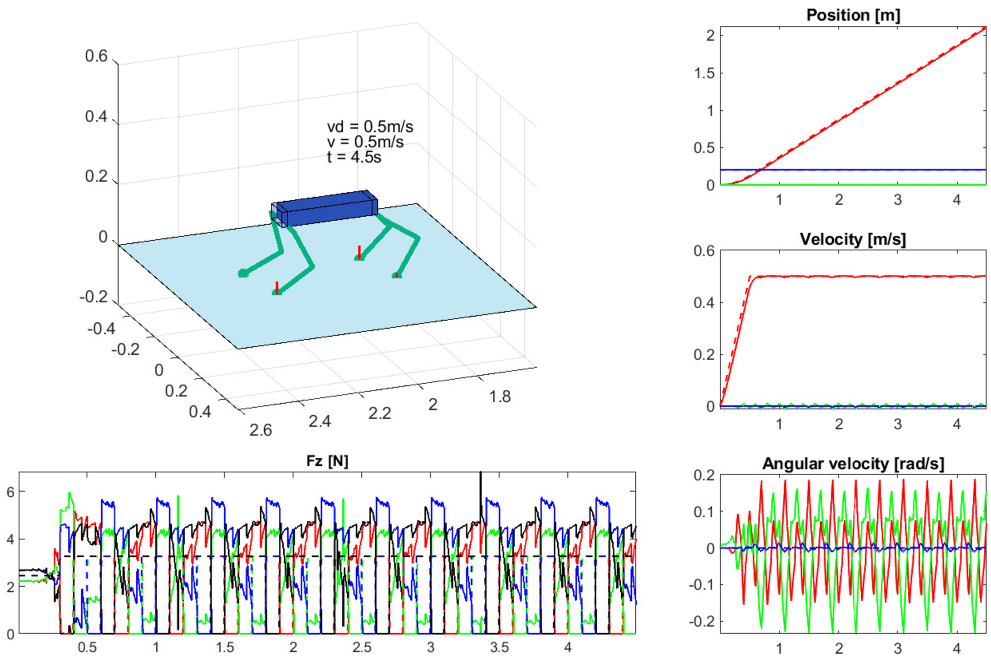

Figure 11: Gait 5 with mass $m = 1kg$

From the figure above we can state that:

- the GRFs at steady state are much smaller than the cases with $m = 5.5Kg$ and $\mu = 0.6$;
- the tracking error of x-component of position exhibits a better behavior than the case with $m = 5.5Kg$;
- the x-component of the velocity exhibits no oscillations with respect the case with $m = 5.5Kg$;

So, to improve the behavior, an option could be to build a lighter quadruped robot. As last case let's modify the desired velocity from $0.5m/s$ to $0.2m/s$.

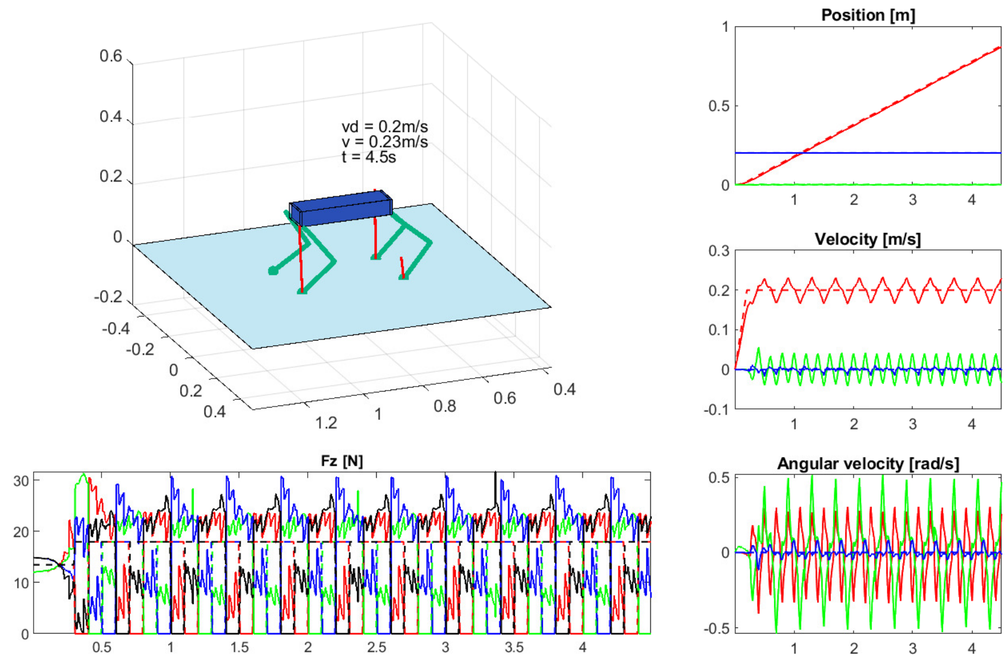

Figure 12: Gait 5 with desired velocity $v_d = 0.2m/s$

From the picture above we can state that:

- the tracking error of x-component of position is sufficiently small and constant;
- GRFs are smaller than the GRFs of the case with $\mu = 0.6$ and $v_d = 0.5m/s$;
- the mean value of the x-component of velocity is equivalent to the desired velocity but we still have oscillations.

To sum up, for the crawl gait:

- decreasing the friction coefficient does not produce any improvements of the gait, this means that we should avoid wet surfaces and we should prefer some materials (for the ground and for the quadruped's feet) that produce a high value of friction coefficient;
- the mass of the robot drastically affects the behavior of the gait, we should avoid heavy materials;
- varying the velocity produces small improvements of the gait.

Further analyses have been computed for the other gaits by varying the mass from five kilograms and half to one kilograms. From these experiments it has been obtained that:

- gait 0: by decreasing the mass, the performances about position error do not improve to much, but the velocity oscillations are drastically reduced, and, obviously, the GRFs are reduced;

- gait 1: by decreasing the mass causes an improvement of position tracking error, but more time is needed to reach the reference value. The velocity oscillations are still present and the GRFs are drastically reduced;
- gait 2: by decreasing the mass causes a large improvement of position and velocity tracking error, furthermore, the oscillations on velocity behavior are drastically reduced. As obvious, the GRFs, are smaller;
- by decreasing the mass, the performances about position and velocity error are improved; the transient of velocity is faster than case with $m = 5.5Kg$; the oscillation are smaller and the GRFs are smaller.

Exercise 4

4.A

Assuming that the robot's foot and leg are massless, this system can be modelled as an inverted pendulum without any elastic or damping terms: $\ddot{\theta} = -g \cdot l \cdot sin(\theta)$. The point $\theta = \frac{\pi}{2}$ is a well known unstable equilibrium point for this system, therefore if we add a small perturbation ε, without an actuator at the point P is unstable at point $\theta = \frac{\pi}{2} + \varepsilon$.

4.B

The zero moment point (ZMP) can be computed as:

$$ZMP = p_c^{x,y} - \frac{p_c^z}{\ddot{p}_c^z - g_0^z}(\ddot{p}_c^{x,y} - g_0^{x,y}) + \frac{1}{m(\ddot{p}_c^z - g_0^z)} S\dot{L}^{x,y} \in \mathbb{R}^3$$

Where the matrix S is not the skew symmetric matrix operator, but: $S = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Assuming as positive directions for the Z and X the reference frame in the figure below, our goal is to computed a parametric expression of the zero moment point as function of the angle θ (positive in counterclockwise and zero when it is aligned to the flat floor): Since the body of the robot (that is a point mass) can rotate only around the y-axis, the y-component of the zero moment point will be null, therefore we can compute the x-component as follows:

$$ZMP = p_c^x - \frac{p_c^z}{\ddot{p}_c^z - g_0^z}(\ddot{p}_c^x - g_0^x) + \frac{1}{m(\ddot{p}_c^z - g_0^z)} S\dot{L}^y \in \mathbb{R}^3$$

Since we have chosen our reference frame as in picture above we can state that:

- $p_c^x = l \cdot cos\theta$;
- $\ddot{p}_c^x = -l \cdot \ddot{\theta} \cdot sin\theta - l \cdot \dot{\theta}^2 \cdot cos\theta$;
- $p_c^z = h + l \cdot sin\theta$;
- $\ddot{p}_c^z = l \cdot \ddot{\theta} \cdot cos\theta - l \cdot \dot{\theta}^2 \cdot sin\theta$;

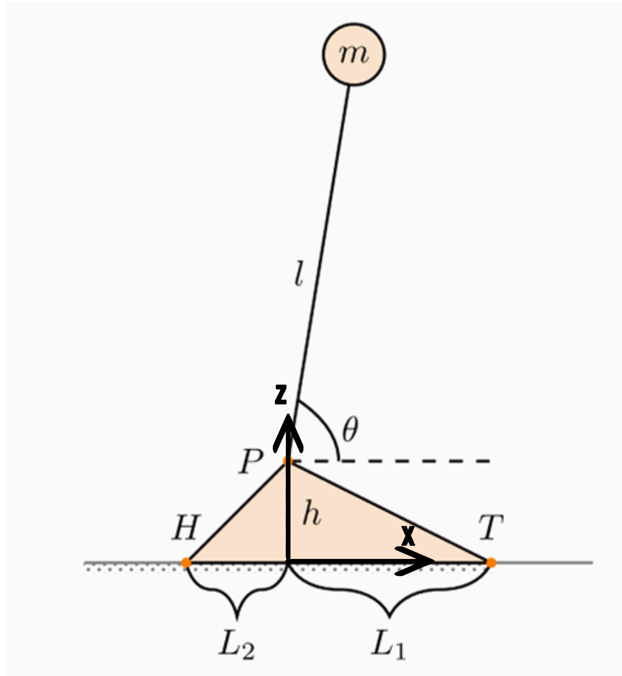

Figure 13: Reference Frame

- $g_0^x = 0$, this because we have a flat horizontal surface;
- $g_0^z = -g$, this because we have a flat horizontal surface;
- $L^y = m \cdot l^2 \cdot \dot{\theta}$;
- $\dot{L}^y = m \cdot l^2 \cdot \ddot{\theta}$;

From these results we can compute the x-component of zero moment point by substituting in the expression above and we obtain:

$$ZMP = l{\cdot}cos\theta - \frac{l \cdot sin\theta + h}{(l \cdot \ddot{\theta} \cdot cos\theta - l \cdot \dot{\theta}^2 \cdot sin\theta) + 9.81} + \frac{1}{m(l \cdot \ddot{\theta} \cdot cos\theta - l \cdot \dot{\theta}^2 \cdot sin\theta + 9.81)}(-m{\cdot}l^2{\cdot}\ddot{\theta})$$

4.C

Supposing to have an actuator at the ankle capable of perfectly cancelling the torque around the point P due to the gravity (i.e., $\ddot{\theta}$=0, $\dot{\theta}$=0), the stability condition to achieve for the robot is that the mass m projected on the ground should never be outside the support polygon, that in our case is equal to the line $L = L_1 + L_2$. Then, the robot will stay stable until the following condition is satisfied:

$$-L_2 \leq l \cdot cos\theta \leq L_1 \quad \implies arccos(\frac{L_1}{l}) \leq \theta \leq arccos(-\frac{L_2}{l})$$