Problem Set 4

Due: 9 June 2023, 4:30 p.m.

Problem 1. A ball with mass m is thrown vertically upwards with initial speed v_0. Assuming linear air drag write down and solve the equation of motion for the ball, i.e. find the time dependence of its position. Find the position of the highest point of the trajectory as well the time needed to reach it.

(*5 points*)

Problem 2. We have seen in class that the general solution of the equation of motion of a simple (neither driven, nor damped) harmonic oscillator can be expressed in the form $x(t) = A\cos(\omega_0 t + \phi)$, where A is the amplitude and ϕ is the phase shift of the oscillations to be determined from the initial conditions.

From the two tasks given below, please choose and complete one.

(a) The linear combination $x(t) = B\cos\omega_0 t + C\sin\omega_0 t$, where B and C are real constants, is another possible representation of the general solution. Check that these two forms are equivalent, i.e. express B and C in terms of A and ϕ or *vice–versa*.

(b) Show that given the initial conditions $x(0) = x_0$ and $v(0) = v_0$ the amplitude and the phase shift can be found as

$$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}, \qquad \phi = \arctan\left(-\frac{v_0}{\omega_0 x_0}\right),$$

respectively.

(*2 points*)

Problem 3. Consider a cylinder of weight w and cross-sectional area S that is floating upright, partially submerged, in a liquid. If we push it a little deeper it will start to oscillate. Neglecting fluid viscosity, show that the cylinder follows simple harmonic motion. Find the density of the liquid, if the period of oscillations is T. Acceleration due to gravity g is given.

Hint. Archimedes' principle (which we will discuss in detail later in the course).

(*5 points*)

Problem 4. A horizontal platform oscillates in the vertical direction with amplitude A. Find the maximum angular frequency of oscillations at which a block placed on the platform is still in contact with the surface of the platform.

Clearly indicate the frame of reference you are solving the problem in. The acceleration due to gravity is g.

(*3 points*)

Problem 5. For a critically damped harmonic oscillator show that the oscillating mass may pass through the equilibrium position at most once, regardless of initial conditions.

(*2 points*)

Problem 6. Consider a harmonic oscillator with linear drag, driven by an external force $F_0 \cos \omega_{\mathrm{dr}} t$.

(a) Check that the driving frequency for which the amplitude of steady-state oscillations is maximum, *i.e.* the resonance frequency, is $\omega_{\mathrm{res}} = \sqrt{\omega_0^2 - b^2/2m^2}$.

(b) Suppose $b/m = 4\ \mathrm{s}^{-1}$. At what frequency should the force be driving the system, to make the phase of steady-state oscillations lag by $\pi/4$ behind that of the driving force?

(c) For two different driving angular frequencies Ω_1 and Ω_2 the amplitudes of steady-state oscillations are equal. Find the natural angular frequency of this oscillator.

(*3/2 + 3/2 + 3 points*)

Problem 7. A heavy tractor has driven over a dirt road leaving tracks in the form of parallel dents separated by $l = 30$ cm from each other. A children's pram, with weight of magnitude $W = 100$ N and fitted with two parallel shock absorbers, each in the form of a coil spring, was then pushed on that road. Each of the springs compresses by $d_0 = 2$ cm under the force $F_0 = 10$ N.

What was the constant speed of the pram if it went into resonance?

(*3 points*)