PHYSICS I

S

Problem Set 4

Xu Zifei 徐子飞
522370910189

2023.6.5

22.7

Problem 1.

Let upwards be positive.

$F = W + F_{drag} = mg + kv$

$a = -g - \frac{k}{m}v = \frac{dv}{dt}$

$\frac{dv}{g + \frac{k}{m}v} = -dt \Rightarrow \frac{dv}{v + \frac{mg}{k}} = -\frac{k}{m}dt \Rightarrow \ln\frac{v(t) + \frac{mg}{k}}{v_0 + \frac{mg}{k}} = -\frac{k}{m}t$

$v(t) = (v_0 + \frac{mg}{k})e^{-\frac{k}{m}t} - \frac{mg}{k}$

$x(t) = (\frac{m}{k}v_0 + \frac{m^2}{k^2}g)(1 - e^{-\frac{k}{m}t}) - \frac{mg}{k}t.$

when it reaches highest, $v(t) = 0$, $t_m = \frac{m}{k}\ln\frac{kv_0 + mg}{mg}$

$x(t_m) = \frac{mv_0}{k} - \frac{m^2 g}{k^2}\ln\frac{kv_0 + mg}{mg}$

Problem 2.

(a). $x(t) = A\cos(\omega_0 t + \varphi) = A\cos\varphi \cdot \cos\omega_0 t - A\sin\varphi \cdot \sin\omega_0 t$

$= B\cos\omega_0 t + C\sin\omega_0 t$

$B = A\cos\varphi$, $C = -A\sin\varphi$

Problem 3.

$F = \rho \cdot S \cdot \Delta h \cdot g$, $\omega_0^2 = \frac{F}{\Delta h \cdot m} = \frac{\rho \cdot S \cdot g^2}{W}$, $T = \frac{2\pi}{\omega_0}$

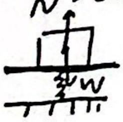
SHO?

$\rho = \frac{\omega_0^2 W}{S \cdot g^2} = \frac{4\pi^2 W}{S \cdot T^2 \cdot g^2}$

-2

Problem 4.

Select the ground as FOR. Let upwards be positive.



$ma = N - W$

$a = \frac{N}{m} - g$

$x(t) = A\cos\omega_0 t$

$a = \ddot{x}(t) = -\omega_0^2 A\cos\omega_0 t$

$\omega_0^2 A\cos\omega_0 t = g - \frac{N}{m}$, $N \geq 0$

$\Rightarrow \omega_0^2 A\cos\omega_0 t \leq g$, at any t.

$\Rightarrow \omega_0^2 A \leq g$, $\omega_0 \leq \sqrt{\frac{g}{A}}$

The maximum angular frequency is $\sqrt{\frac{g}{A}}$.

oblem 5.

For critical damping, $x(t) = (D_1 + D_2 t)\cdot e^{-\frac{b}{2m}t}$

When $x(t)=0$, it passes through equilibrium position.

$(D_1+D_2t)\cdot e^{-\frac{b}{2m}t} = 0$, $e^{-\frac{b}{2m}t} > 0$

$\Rightarrow D_1 + D_2 t = 0$, $t = -\frac{D_1}{D_2}$, Only one solution.

$\Rightarrow$ It may pass through equilibrium position at most once.

Problem 6.

(a). $A(\omega_{dr}) = \dfrac{F_0}{m\sqrt{(\omega_0^2-\omega_{dr}^2)^2 + (\frac{b\omega_{dr}}{m})^2}} = \dfrac{F_0}{m\sqrt{\omega_{dr}^4 - (2\omega_0^2 - \frac{b^2}{m^2})\omega_{dr}^2 + \omega_0^4}}$

$= \dfrac{F_0}{m\sqrt{[\omega_{dr}^2 - (\omega_0^2 - \frac{b^2}{2m^2})]^2 + \omega_0^4 - (\omega_0^2 - \frac{b^2}{2m^2})^2}}$

When $\omega_{dr}^2 = \omega_0^2 - \frac{b^2}{2m^2}$, $\omega_{dr} = \omega_{res} = \sqrt{\omega_0^2 - \frac{b^2}{2m^2}}$,

A is maximum.

(b). $\varphi = \frac{-\pi}{4}$. $\tan\varphi = \dfrac{b\omega_{dr}}{m(\omega_{dr}^2 - \omega_0^2)} = -1$.

$-\frac{b}{m} = \dfrac{\omega_{dr}^2 - \omega_0^2}{\omega_{dr}}$, $\omega_{dr}^2 + 4\omega_{dr} - \omega_0^2 = 0$, $\omega_{dr} \geq 0$

$\omega_{dr} = -2 + \sqrt{4+\omega_0^2}$

(c). $A_1 = A_2$, $[\Omega_1^2 - (\omega_0^2 - \frac{b^2}{2m^2})]^2 = [\Omega_2^2 - (\omega_0^2 - \frac{b^2}{2m^2})]^2$, $\Omega_1 \neq \Omega_2$

$\dfrac{\Omega_1^2 + \Omega_2^2}{2} = \omega_0^2 - \frac{b^2}{2m^2}$

$\omega_0 = \sqrt{\frac{\Omega_1^2+\Omega_2^2}{2} + \frac{b^2}{2m^2}}$.

Problem 7.

$L = 0.3m$. $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{kg}{w}} = \sqrt{\dfrac{\frac{10}{2\times10^{-3}}\times 9.8}{100}} = 22.14$ rad/s.

-0.25

$T = \frac{2\pi}{\omega_0} = 0.28s$

$V = \frac{L}{T} = \dfrac{0.3}{\frac{2\pi}{\sqrt{\frac{\frac{10}{2\times10^{-3}}\times 9.8}{100}}}} = 1.06 m/s$

-1