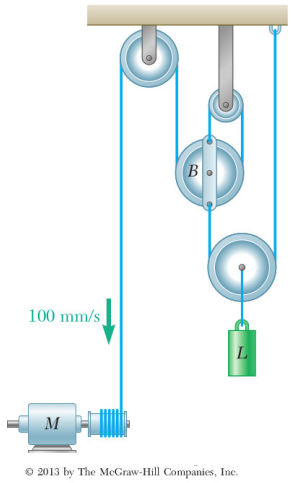
Problem Set 2

Due: 26 May 2023, 4:30 p.m.

Problem 1. The motor M reels in the cable at a constant rate of 100 mm/s. Determine (a) the velocity of load L, (b) the velocity of pulley B with respect to load L.

(1 + 1 points)



Note/Hint. In this problem, you should provide a justification (e.g., by looking at the lengths of sections of the ropes and relating positions of the pulleys), not simply use some mnemonic rules about systems of pulleys which you may have learned elsewhere.

Problem 2. A particle is launched horizontally, close to the Earth's surface, with initial speed $v(0) = v_0$. Find both the normal and the tangential components of acceleration at an instant $t > 0$. Neglect air drag.

Use a geometric argument (look at some right-angle triangles) to relate these components to the Cartesian components of velocity.

(4 points)

Problem 3. A river flows eastwards, parallel to the banks, with the speed of the flow depending on the distance $0 \le y \le L$ from the southern bank as

$$v_2(y) = v_0 \sin \frac{\pi y}{L},$$

where v_0 is a positive constant and L is the river's width. A canoe is crossing the river with constant velocity $\mathbf{v}_1$ (with respect to the water), oriented perpendicularly to the direction of the stream. Find

(a) the velocity of the canoe with respect to the bank, and its magnitude,
(b) the canoe's trajectory,
(c) and the distance the canoe has drifted down the river.

(1 + 4 + 1 points)

Problem 4. The trajectory of a particle moving in the xy–plane is given by the parametric equations

$$x(t) = bt^2, \qquad y(t) = -ct^2,$$

where b and c are positive constants (what are their units?). Find

(a) parametric equations of the trajectory in the polar coordinate system,

(b) implicit equation of the trajectory in both coordinate systems,

(c) velocity and acceleration in both systems.

(1 + 1 + 3 points)

Problem 5. Motion of a particle is given by parametric equations in polar coordinates

$$r(t) = r_0(1 - ct), \qquad \varphi(t) = \frac{ct}{1 - ct},$$

where c and $r_0 > 0$ are constants (what are their units?).

Find

(a) the implicit equations of the particle's trajectory (use a computer to plot it),

(b) velocity of the particle (transverse and radial components) and speed,

(c) as well as its acceleration (transverse and radial components).

(d) Comment on the motion in cases when $c > 0$ and $c < 0$.

(1 + 3/2 + 3/2 + 1 points)

Problem 6. The oscillation of rod OA about O is defined by the relation $\varphi = \frac{3}{\pi}\sin \pi t$, where φ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 6(1 - e^{-2t})$ where r and t are expressed in centimeters and seconds, respectively.

(a) What are the units of the numbers in the formulas?

When $t = 1$ s, determine (b) the velocity of the collar, (c) the acceleration of the collar, (d) the acceleration of the collar relative to the rod.

(1 + 3/2 + 3/2 + 1 point)