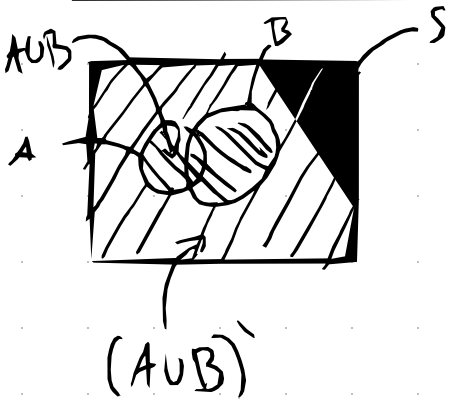
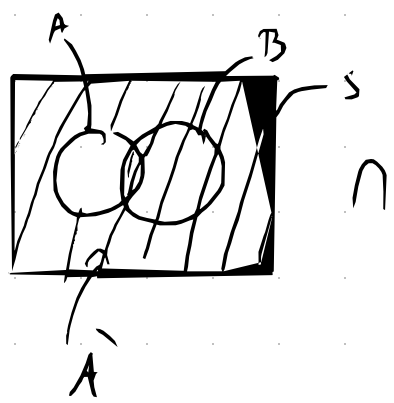
Innlevering 1

oppgave 7





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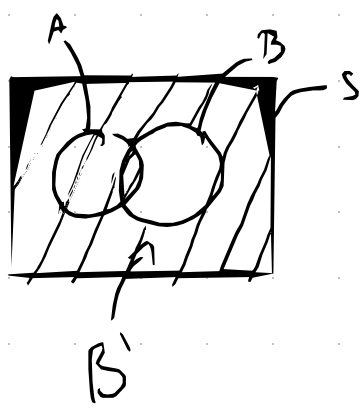

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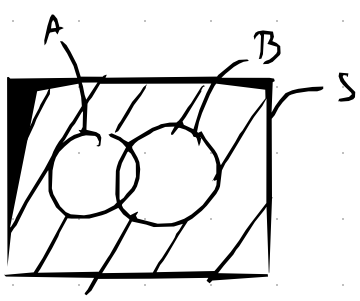

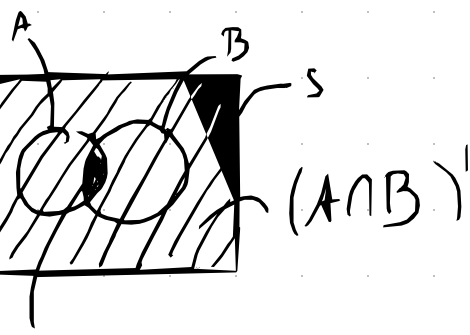

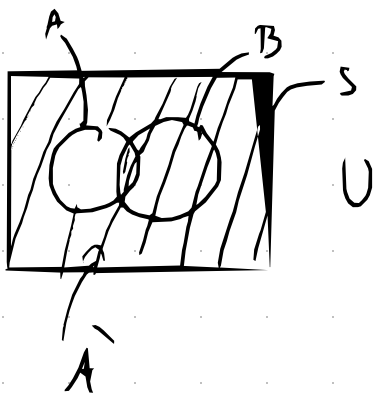

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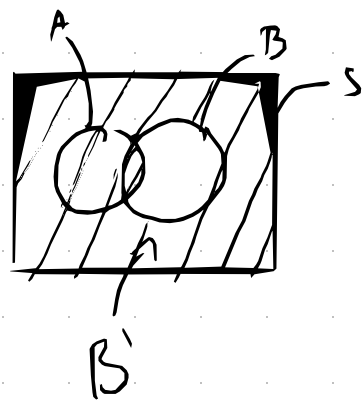

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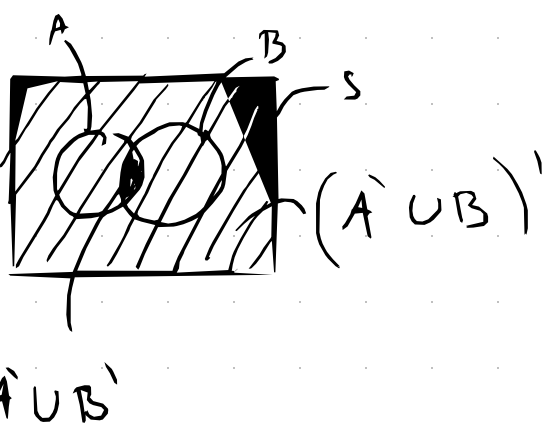

Oppgave 2

Sannsynligheten for å trekke 7 røde kuler er

$$\left(\frac{7}{54}\right)\left(\frac{6}{53}\right)\left(\frac{5}{32}\right)\left(\frac{4}{31}\right)\left(\frac{3}{30}\right)\left(\frac{2}{29}\right)\left(\frac{1}{28}\right)$$

$$= \frac{7!}{34!/27!} = \frac{7! \cdot 27!}{34!} = \underline{1.85 \cdot 10^{-7}}$$

Alternativt:

antall mulige kombinasjoner av 7 kuler av 34 er

$$_{34}C_7 = \binom{34}{7} = 5374616$$

hvor 7 røde bare er en av de

$$\Rightarrow \frac{g}{m} = 1/5374616 = \underline{1.85\,10^{-7}}$$

$$\frac{\binom{27}{3}\binom{7}{4}}{\binom{34}{7}} = \underline{0.0190}$$

$$\frac{\binom{27}{1}\binom{7}{6}}{\binom{34}{7}} \cdot \left(\frac{1}{27}\right) = 1.30 \cdot 10^{-6}$$

Oppgave 3

$$m = \binom{300}{5}$$

$$g = \binom{3}{1} \cdot \binom{297}{4}$$

$$\frac{g}{m} = 0.0487$$

$$P(\text{minst 1 Type A}) = 1 - P(\text{ingen Type A})$$

$$m = \binom{300}{5}$$

$$g = \binom{3}{0} \cdot \binom{297}{5}$$

$$P(\text{minst 1 Type A}) = 0.0493$$

$$P(\text{minst 1 gevinst}) = 1 - P(\text{ingen gevinst})$$

$$P(\text{ingen gevinst}) = \frac{(294 \cdot 293 \cdot 292 \cdot 291 \cdot 290)}{(300 \cdot 299 \cdot 298 \cdot 297 \cdot 296)}$$

$$P(\text{minst 1 gevinst}) = 0.0967$$

Oppgave 4

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.4 + 0.3 - 0.6$$

$$= 0.1$$

$$P(A \cap B \cap C) = P(A \cup B \cup C)$$
$$- (P(A) + P(B) + P(C))$$
$$+ (P(A \cap B) + P(A \cap C) + P(B \cap C))$$

$$= P(A \cup B \cup C) - (P(A) + P(B) + P(C))$$
$$+ (P(A) + P(B) - P(A \cup B))$$
$$+ (P(A) + P(C) - P(A \cup C))$$
$$+ (P(B) + P(C) - P(B \cup C))$$

$$= 0$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.3} = \frac{1}{3}$$

de er avhengige siden

$P(A|B) \neq P(A)$

B

Siden $P(A \cap B) \neq 0$ er de

ikke disjunkte.

Oppgave 5

F → farge blind
K → kvinne

$P(F|K) = 0,003$
$P(F|\bar{K}) = 0,008$

$P(K) = 2/3 \quad P(\bar{K}) = 1/3$

$$P(K|F) = \frac{P(F|K) \cdot P(K)}{P(K)\,P(F|K) + P(\bar{K})\,P(F|\bar{K})}$$

$$= \frac{0,003 \cdot \frac{2}{3}}{0,003 \cdot \frac{2}{3} + 0,008 \cdot \frac{1}{3}} \approx 0,0698$$

Oppgave 6

$$P(X \leq 2) = \sum_{x=0}^{2} f(k) = f(0) + f(1) + f(2)$$
$$= 0,05 + 0,1 + 0,25$$
$$= 0,4$$

$$P(X \leq 2 \mid X < 4) = \frac{P(x \leq 2,\ x < 4)}{P(x < 4)}$$

$$= \frac{0,4}{0,4+0,4} = 0.5$$

$$P(X \leq 2 \mid X \geq 1) = \frac{P(x \leq 2,\ X \geq 1)}{P(x \geq 1)}$$

$$= \frac{0.1 + 0,25}{0,1 + 0,25 + 0,4 + 0,15 + 0,05} = \frac{0,35}{0,95} \approx 0.368$$

Oppgave 7

a) $F_X(x) = 1 - \exp\left(-\frac{x^2}{\lambda}\right), \quad x \geq 0$

$$f_X(x) = F_X'(x) = -\exp\left(-\frac{x^2}{\lambda}\right) \cdot -\frac{2x}{\lambda}$$

$$= \frac{2x}{\lambda} \exp\left(-\frac{x^2}{\lambda}\right)$$

b)

La X og Y være to stokastiske variabler som betegner hvor lenge de to komponentene i instrumentet lever før de svikter.

Sannsynligheten for at en komponent der i kopet av z år beskrevet av

$$F_X(z)$$

For at instrumentet skal dø må begge komponentene dø, altså vil

$$F_X(z)\, F_Y(z) = F_Z(z)$$

beskrive sannsynligheten for at instrumentet har dødd innen z år. Da er

$$P(Z > z) = 1 - F_Z(z)$$
$$= 1 - F_X(z)\, F_Y(z)$$

$$= 1 - \left(1 - \exp\left(-\frac{z^2}{\lambda}\right)\right)^2$$

$$= 2\exp\left(-\frac{z^2}{\lambda}\right) - \exp\left(-\frac{2z^2}{\lambda}\right)$$

$$f_Z(z) = \frac{4x}{\lambda}\left(\exp\left(-\frac{z^2}{\lambda}\right) - \exp\left(-\frac{2z^2}{\lambda}\right)\right)$$

Ser at den får lengre forventet levetid som er forventet siden den vi har to komponenter.