AME 70634 Flow Control

Fall, 2024

Homework No. 3

Problem 4.1 Experiments have indicated that a reduced frequency of $F^+ = fL/U_\infty$ is optimum to reattach a separation bubble whose streamwise length is L. Figure 4.14 indicates the maximum amplification of a flow separation with reverse flow ($d/\theta = 4.5$) occurs at $\omega\theta/U_\infty = 0.075$.

1. Based on this, determine the ratio of θ/L such that $F^+ = 1$ occurs where the spatial growth rate is a maximum. Note that $\omega = 2\pi f$.

2. Assuming that the ratio of the boundary layer thickness to momentum thickness is $\delta/\theta \simeq 10$, what is the ratio δ/L such that $F^+ = 1$ occurs where the spatial growth rate is a maximum.

Problem 4.2 Both wall-normal suction and blowing can reduce the tendency for flow separation. Can this be explained from the point of view of the separated shear layer instability? Refer to Figures 4.12 to 4.14.

1. Describe how is d/θ of the profile is affected by wall-normal suction or wall-normal blowing?

2. How does the profile d/θ affect the shear layer instability properties? Are the effects the same for suction and blowing?

3. Based on these observations, list the effects that reduce the tendency for flow separation in the cases.

Problem 4.3 Spectra of velocity fluctuations at different streamwise locations in a flow separation bubble shown in Figure 4.18 identifies dominant spectral peaks at f_0 and $f_0/2$. The growth in amplitude of fluctuations at the two frequencies is shown in the right part of the figure. Of particular interest is the enhanced growth at $f_0/2$ for $x/L > 0.63$ that is evidence of a "subharmonic resonance".

1. How is this relevant to the reducing the x-extent of the separation bubble?

2. Based on the temporal model for the effect of the growth of f_0 on $f_0/2$ given in Equations 4.8-4.10, how can this interaction be enhanced by unsteady excitation?

3. If fluctuations at both frequencies are excited, would the phase difference between them be important? If so, wha would be the optimum phase difference based on the temporal model.

4. In the figure, the disturbance growth occures in space (x). How could the temporal model be converted to a spatial model?

Problem 4.4 Figure 4.15 displays the normalized growth rates and phase velocities as a function of wave angle in a Falkner-Skan boundary layer just prior to separation. Considering this,

1. What is the most important instability characteristic for a triad resonance of the type described by Craik (1985) to occur between 2-D and 3-D waves?
2. Based on Figure 4.15, describe conditions that could lead to a triad resonance between a 2-D wave and a pair of 3-D waves of equal but opposite wave angles.
3. What effect do you expect this would have on the amplification of the 3-D waves?
4. Could this have some benefit in controlling flow separation? Describe how.

Problem 4.5 Table 4.3 indicates that the optimum spanwise spacing of vortex generators to suppress a flow separation is 0.8δ, where δ is the boundary layer thickness.

1. Considering a Falkner-Skan boundary layer just prior to separation, $d/\theta = 4.0$ in Figure 4.15, the most amplified dimensionless frequency is $\omega\theta/U_\infty = 0.12$. The streamwise wavelength of the instability waves is $\lambda_x = 2\pi/\omega$, write λ_x in terms of θ/U_{infty}.
2. Assuming that $\theta/\delta \simeq 0.1$, determine the effective wave angle, $\phi = tan^{-1}(\lambda_z/\lambda_x)$ produced by the vortex generators separated by $\lambda_x = 0.8\delta$ for a range of free-stream velocities of 5, 10, 15, and 20m/s.
3. Can you draw a conclusion on a connection between the optimum spacing of vortex generators and the boundary layer instability to 3-D disturbances?

Problem 4.6 Based on plasma vortex generators, Kelley et al. (2016) found that the streamwise length of the actuator, L, was based on the time scale of the vorticity, $T_c \geq (\partial\bar{U}/\partial z)^{-1}$, such that $L \geq T_c U_\infty$. The plasma vortex generators produced the equivalent of a spanwise-oriented tangential wall jet, and therefore the same scaling criterion should equally apply to a pressure-driven tangential wall jet that extends in the streamwise direction.

1. A typical spanwise velocity of a plasma actuator is about 3-5m/s. Based on Figure 4.26 for freestream Mach numbers of 0.15-0.20 (51-68m/s), a $T_c \simeq 1.5$ms met the criteria that $T_c \geq (\partial\bar{U}/\partial z)^{-1}$. Based on this, what is the required actuator length, L?
2. Using wall-jet vortex generators, Johnston (1990) found that $V_j/U_\infty > 0.8$ was effective. Based on this and assuming that that T_c scales linearly with the actuator velocity, what is the required actuator length?
3. Does this justify the typical size (diameter) of wall jet vortex generators?