Contents


```
%{

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Description:
AME 60614: Numerical Methods
Homework: 4
Due: 10/31/2024


%}
```

Preperation of the workspace

```
clear all
clc
close all
fontsize = 16;


% set(0,'DefaultFigureWindowStyle','docked')
set(0,'DefaultTextInterpreter','latex')
set(0,'DefaultAxesFontSize',fontsize)
set(0,'DefaultLegendFontSize',fontsize)
colors  = ["#000000","#1b9e77","#d95f02","#7570b3","#0099FF"]';
```

Setting data paths

Make sure to update this for the machine that you are working on. (Maybe, This should now run on any machine without change. 7/24/24) Change the current folder to the folder of this m-file.

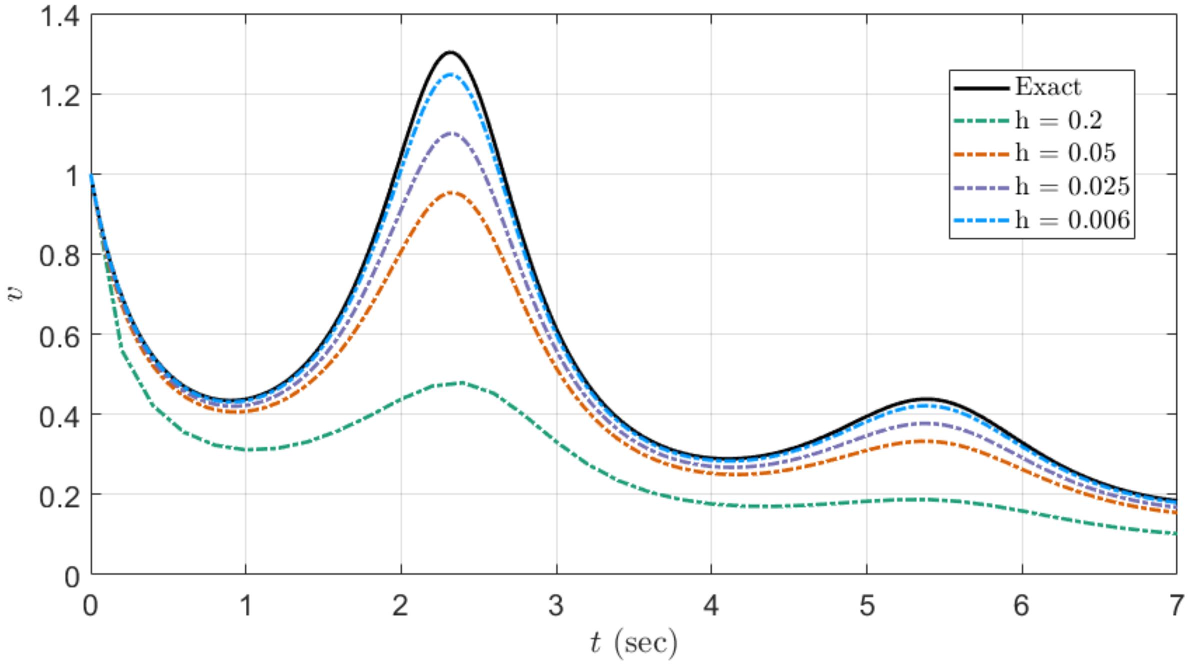
```
if(~isdeployed)
  cd(fileparts(matlab.desktop.editor.getActiveFilename));
end

addpath(cd)
% cd ..; % Moving up a directory (from processing_code)
basepath = cd; % Pulling the current directory

if isunix

    imagepath = [basepath '/images/']; % Unix
    mkdir(imagepath);

elseif ispc

    imagepath = [basepath '\images\']; % Windows
    mkdir(imagepath);

else
    disp('Platform not supported')
end
```

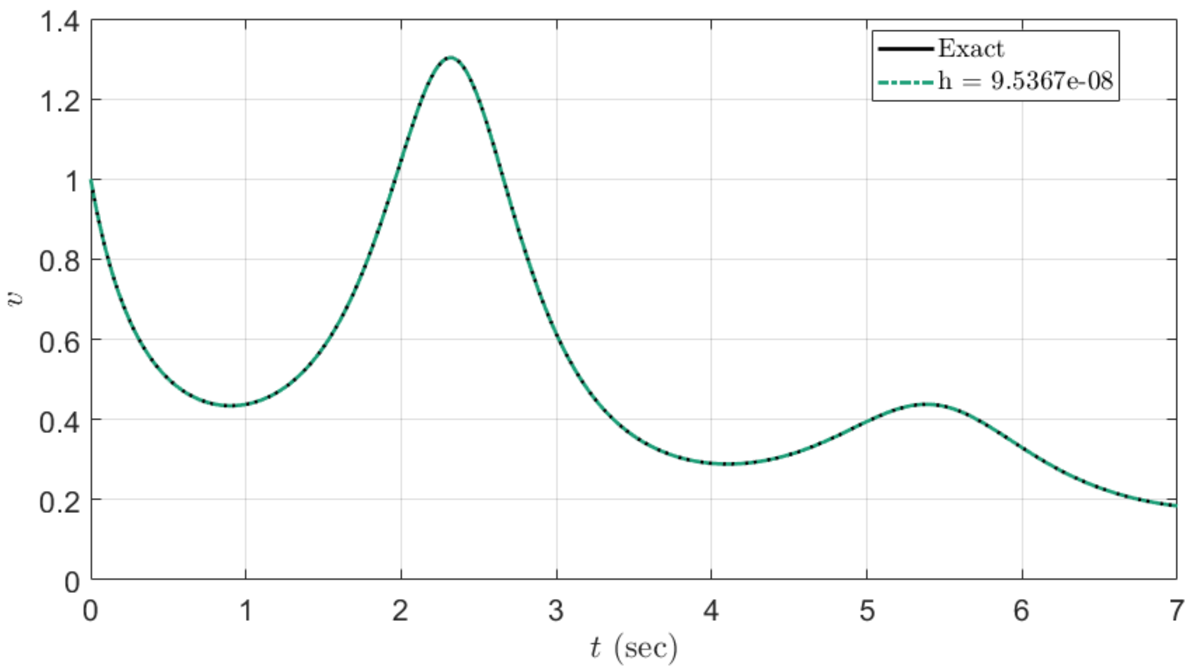
```
Warning: Directory already exists.
```

Problem 2 in Chapter 4

```
exact = @(t) 0.90991./(exp(0.2*t) + 0.900901*sin(2*t) - 0.0900901*cos(2*t));
f = @(t,y) (-0.2*y) - (2*cos(2*t)*y^2);
y0 = 1;
t0 = 0;
```

```
tf = 7;
h = [0.2 0.05 0.025 0.006];

figure
plot(linspace(t0,tf,1e3),exact(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
hold on
for n = 1:size(h,2)
       [t,y] = explicitEuler(f, y0, t0, tf, h(n));
       plot(t,y,"-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
       hold on
end

xlabel('$t$ (sec)')
ylabel('$v$')
grid on
xlim([t0 tf])
ylim([0 1.4])
legend(Location="best",Interpreter="latex")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q2.png'],'-dpng');


emax=1e-6;      % Convergence Tolerance
residual_error = 1;
h = 0.2
y_prev = [];
converged = false;
while ~converged

 [t,y] = explicitEuler(f, y0, t0, tf, h);

    % If there is a previous solution, compute the L2 norm difference
    if ~isempty(y_prev)
        % Interpolate previous solution onto the current time points for comparison
        y_prev_interp = interp1(t_prev, y_prev, t, 'linear', 'extrap');

        % Calculate the L2 norm of the difference between current and previous solution
        residual_error = sqrt(sum((y - y_prev_interp).^2) * h);

        % Check if residual error is below the tolerance and greater than
        % initial step
        if residual_error < emax && h < 0.1
            converged = true;
            disp(['Converged with step size h = ', num2str(h), ' and L2 norm error = ', num2str(residual_error)]);
        else
            % If not converged, halve the step size
            h = h / 2;
            disp(['Step size halved to h = ', num2str(h), ' with L2 norm error = ', num2str(residual_error)]);
        end
    end

    % Update previous solution and time points for the next iteration
    y_prev = y;
    t_prev = t;

end

figure
plot(linspace(t0,tf,1e3),exact(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
hold on

plot(t,y,"-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h)) , color=colors(2,:))

xlabel('$t$ (sec)')
ylabel('$v$')
grid on
xlim([t0 tf])
ylim([0 1.4])
legend(Location="best",Interpreter="latex")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q2_c.png'],'-dpng');
```

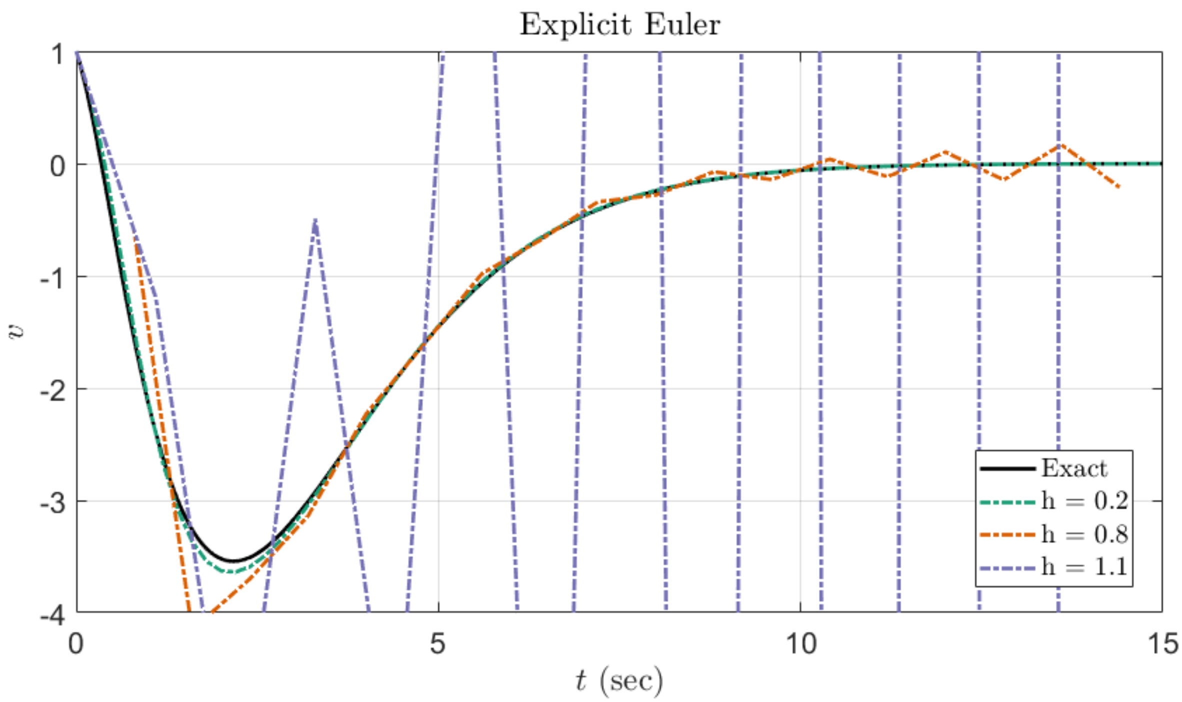


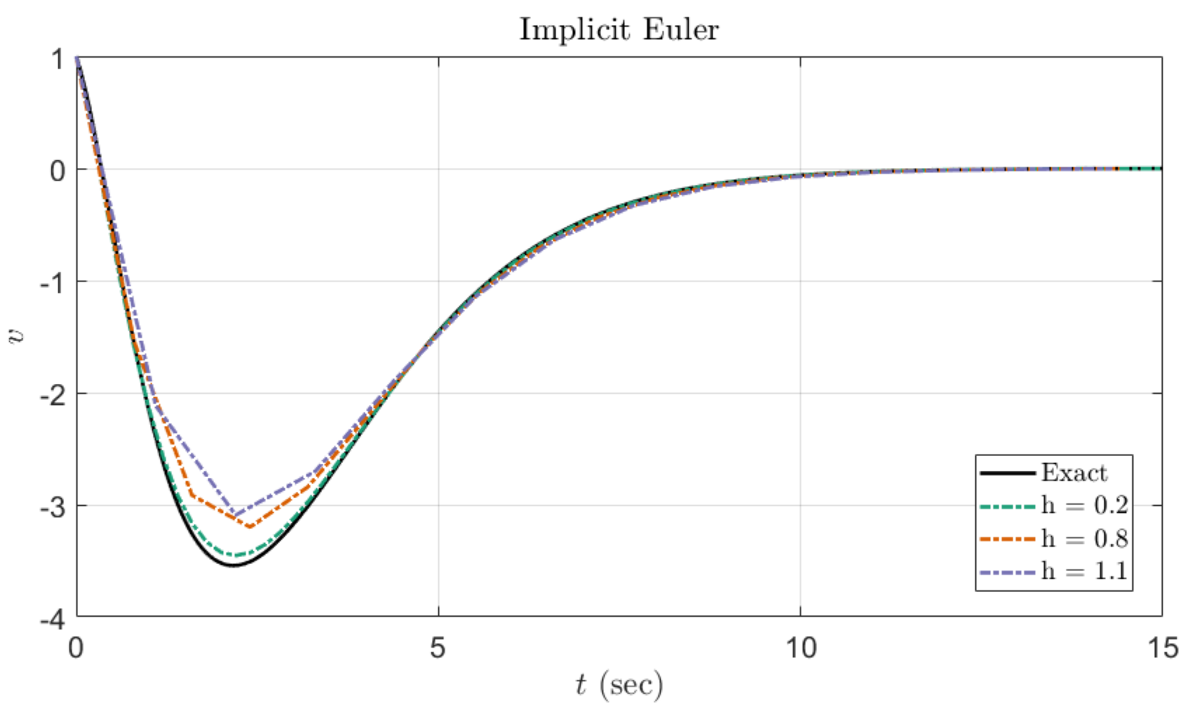


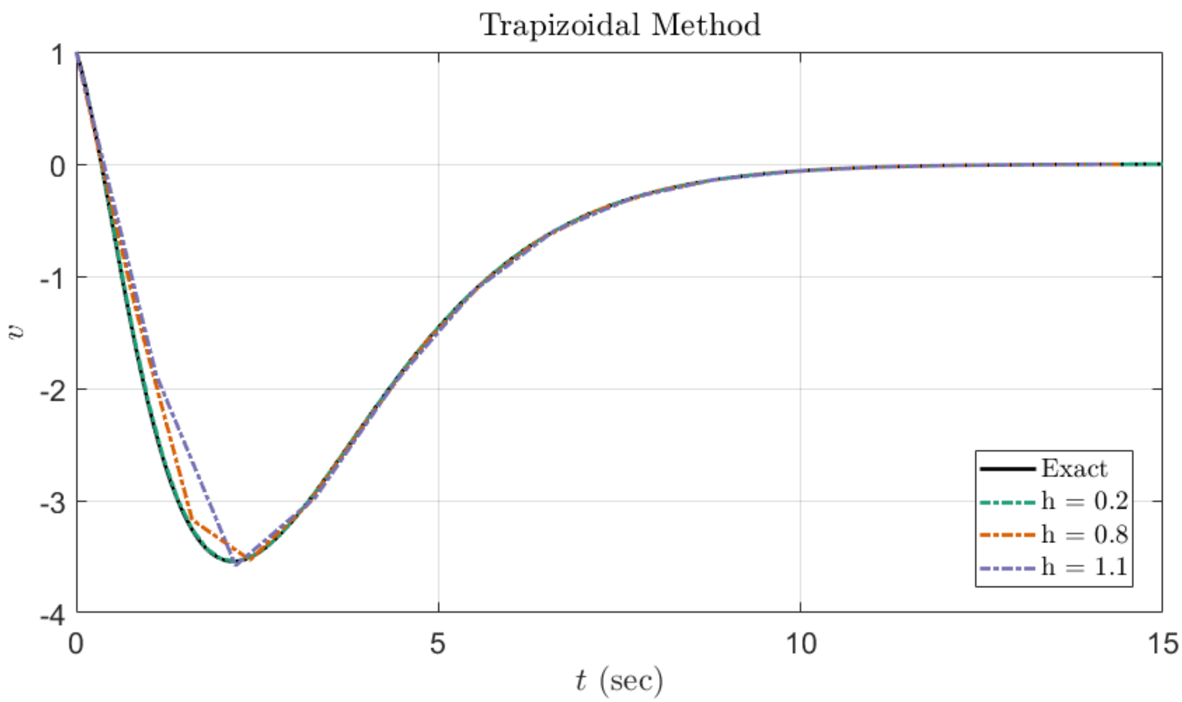
Problem 6 in Chapter 4

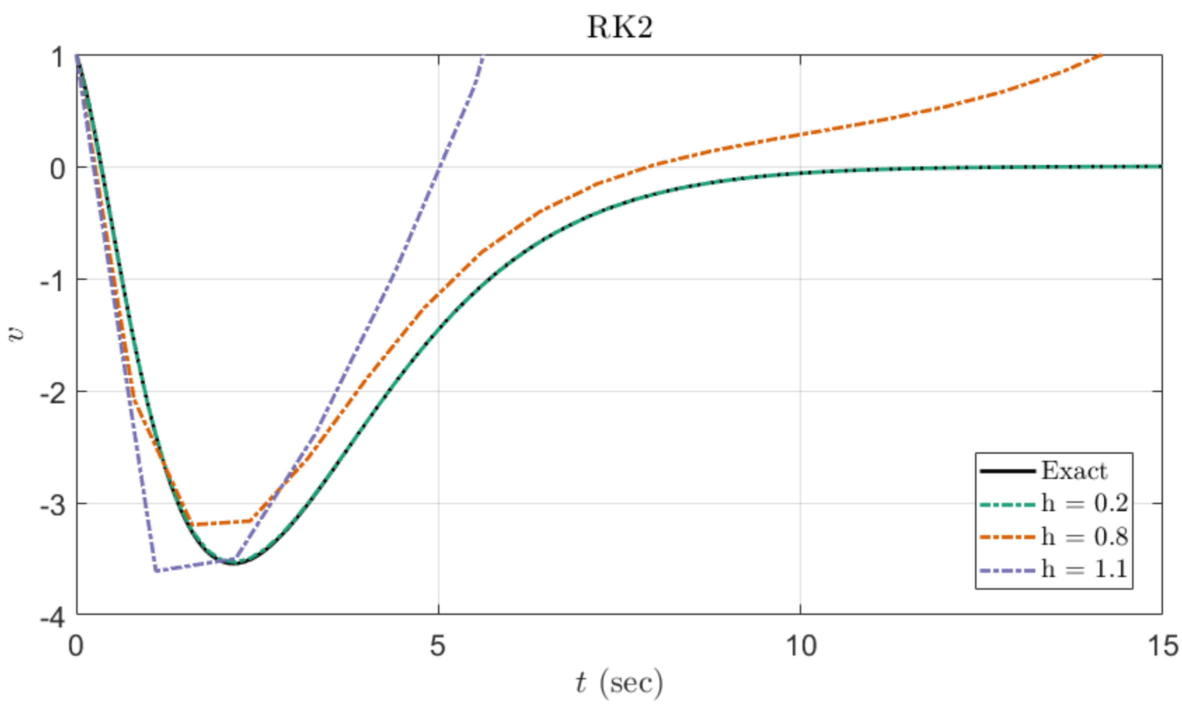
```
funlist = {@explicitEuler, @implicitEuler,@trapMethod,@RK2,@RK4};
funlist_str = ["Explicit Euler", "Implicit Euler","Trapizoidal Method","RK2","RK4"];
f = @(t,y) -((3*t)/(1+t))*y - (2*(1+t)^3*exp(-t));
y0 = 1;
t0 = 0;
tf = 15;
h = [0.2 0.8 1.1];
exact = @(t) -exp(-3*t).*(exp(2*t) - 2).*(t + 1).^3;
for i = 1:length(funlist)
    figure
    plot(linspace(t0,tf,1e3),exact(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(h)

    [t,y] = funlist{i}(f, y0, t0, tf, h(n));
    plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
    hold on
```

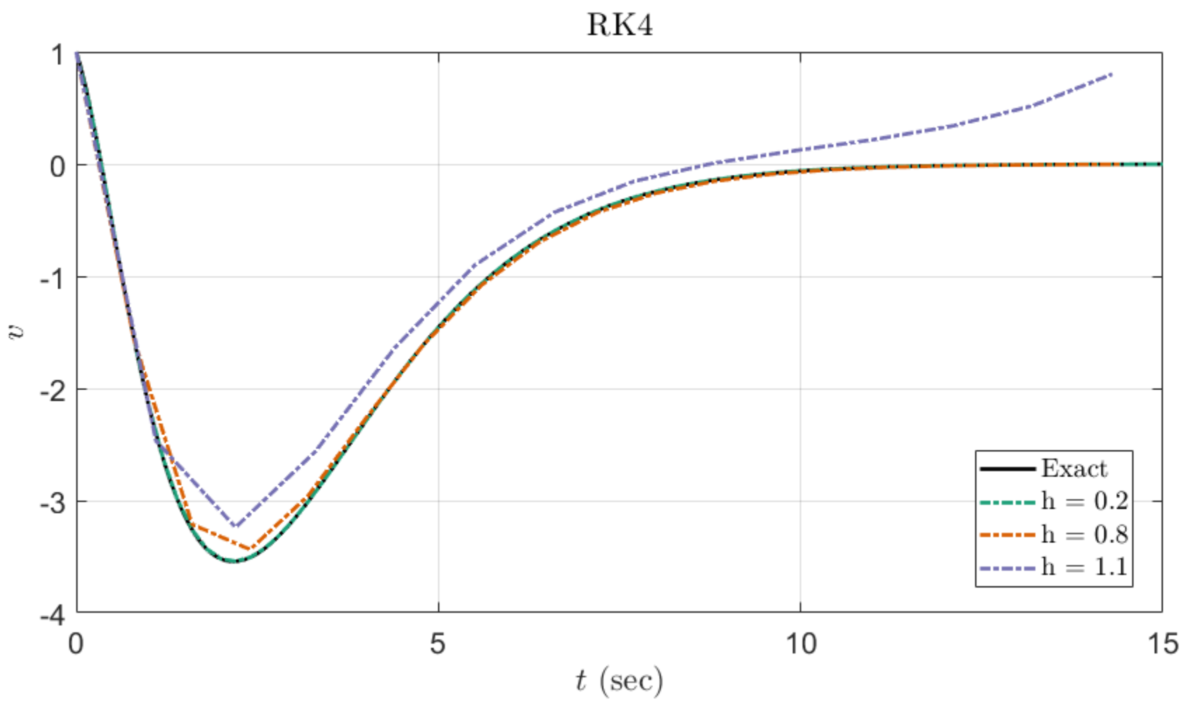
```
    end
    xlabel('$t$ (sec)')
    ylabel('$v$')
    legend(Location="best",Interpreter="latex")
    xlim([t0 tf])
    ylim([-4 1])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q6',char(string(i)),'.png'],'-dpng');
end
```






Trapizoidal Method
1
0
-1
-2
-3
-4
v
0
5
10
15
t (sec)
Exact
h = 0.2
h = 0.8
h = 1.1


RK2
1
0
-1
-2
-3
-4
v
0
5
10
15
t (sec)
Exact
h = 0.2
h = 0.8
h = 1.1

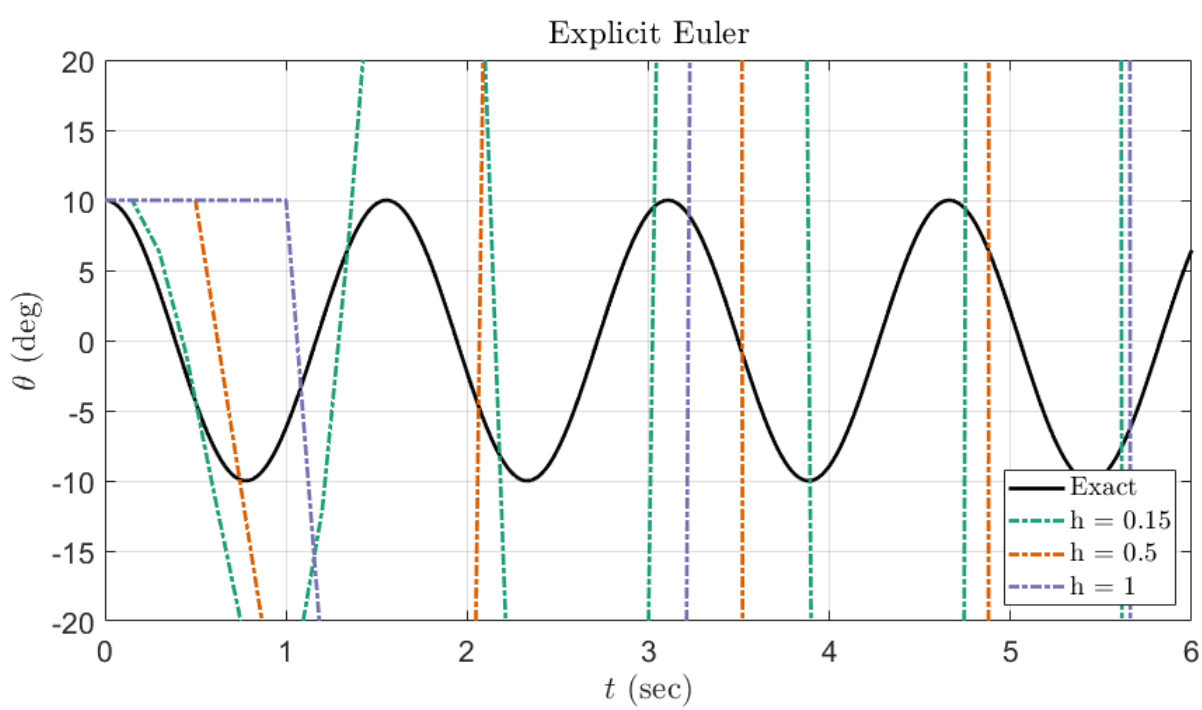


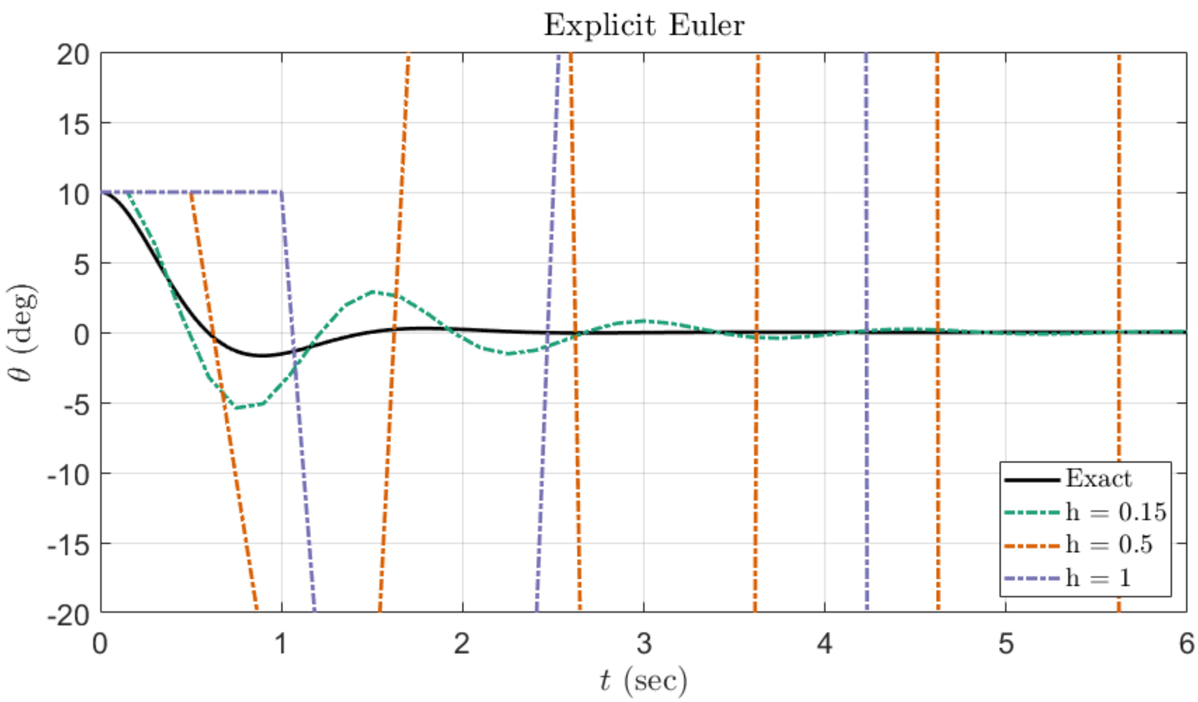
Problem 8 in Chapter 4

```
funlist2 = {@explicitEuler_2, @implicitEuler_2,@trapMethod_2,@RK2_2,@RK4_2};
funlist_str = ["Explicit Euler", "Implicit Euler","Trapizoidal Method","RK2","RK4"];
exact = @(t) 10*cos(4.04351*t);
exact2 = @(t) exp(-2* t).* (5.6911* sin(3.51426 *t) + 10 *cos(3.51426*t));
exact3 = @(t) pi/3 *cos(4.04351 *t);
g = 9.81; %m/s^2
l = 0.6; %m
c = 4;
theta0= 10; %deg
thetap0 = 0; % assume at rest
f_theta = @(t, theta, thetap) -g/l *theta ;
f_theta_2 = @(t, theta, thetap) -g/l *theta -c*thetap ;
f_theta_3 = @(t, theta, thetap) -g/l *sin(theta) ;



t0 = 0;
tf = 6;
h = [.15 .5 1];
hi = [.15 .05 .001]
for i = 1:length(funlist2)
    figure
    plot(linspace(t0,tf,1e3),exact(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(h)

        [t,y,yp] = funlist2{i}(f_theta,theta0,thetap0, t0, tf, h(n));
        plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
        hold on


    end
    xlabel('$t$ (sec)')
    ylabel('$\theta$ (deg)')
    legend(Location="southeast",Interpreter="latex")
    xlim([t0 tf])
    ylim([-20 20])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q8_a',char(string(i)),'.png'],'-dpng');



    figure
    plot(linspace(t0,tf,1e3),exact2(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(h)
```

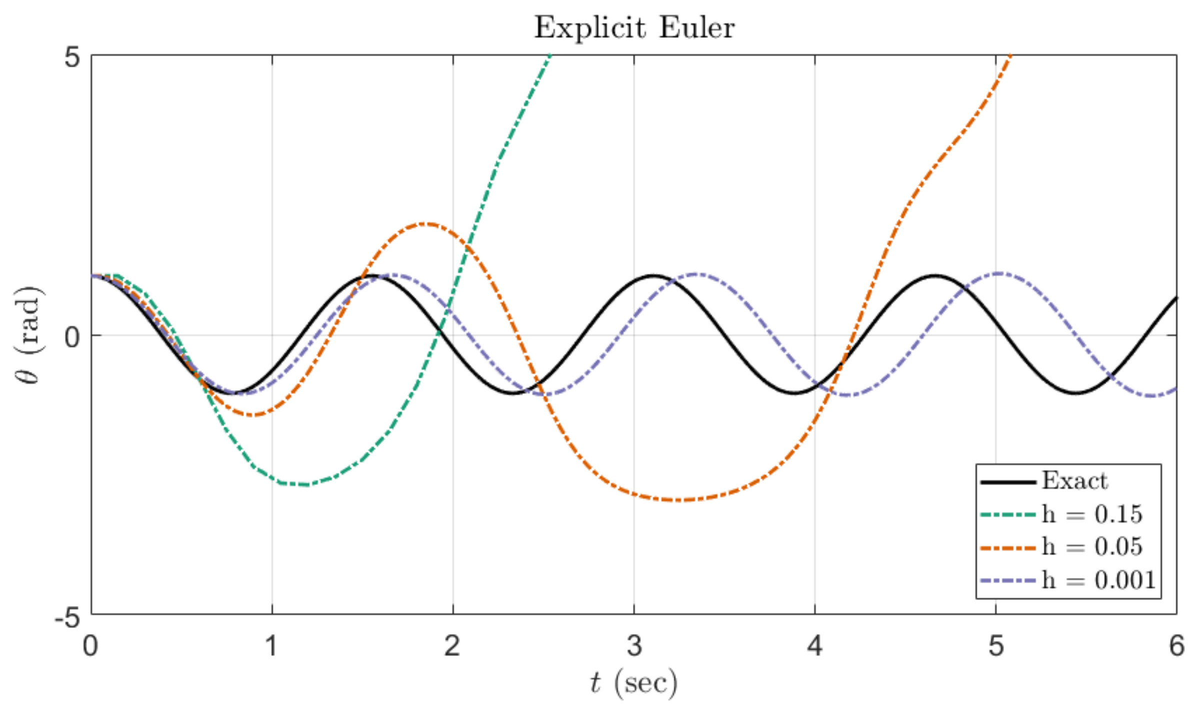
```
        [t,y,yp] = funlist2{i}(f_theta_2,theta0,thetap0, t0, tf, h(n));
        plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
        hold on


    end
    xlabel('$t$ (sec)')
    ylabel('$\theta$ (deg)')
    legend(Location="southeast",Interpreter="latex")
    xlim([t0 tf])
    ylim([-20 20])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q8_b',char(string(i)),'.png'],'-dpng');



    figure
    plot(linspace(t0,tf,1e3),exact3(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(hi)

        [t,y,yp] = funlist2{i}(f_theta_3,pi/3,thetap0, t0, tf, hi(n));
        plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(hi(n))) , color=colors(n+1,:))
        hold on


    end
    xlabel('$t$ (sec)')
    ylabel('$\theta$ (rad)')
    legend(Location="southeast",Interpreter="latex")
    xlim([t0 tf])
    ylim([-5 5])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q8_c',char(string(i)),'.png'],'-dpng');

end
```

```
hi =

    0.1500    0.0500    0.0010
```

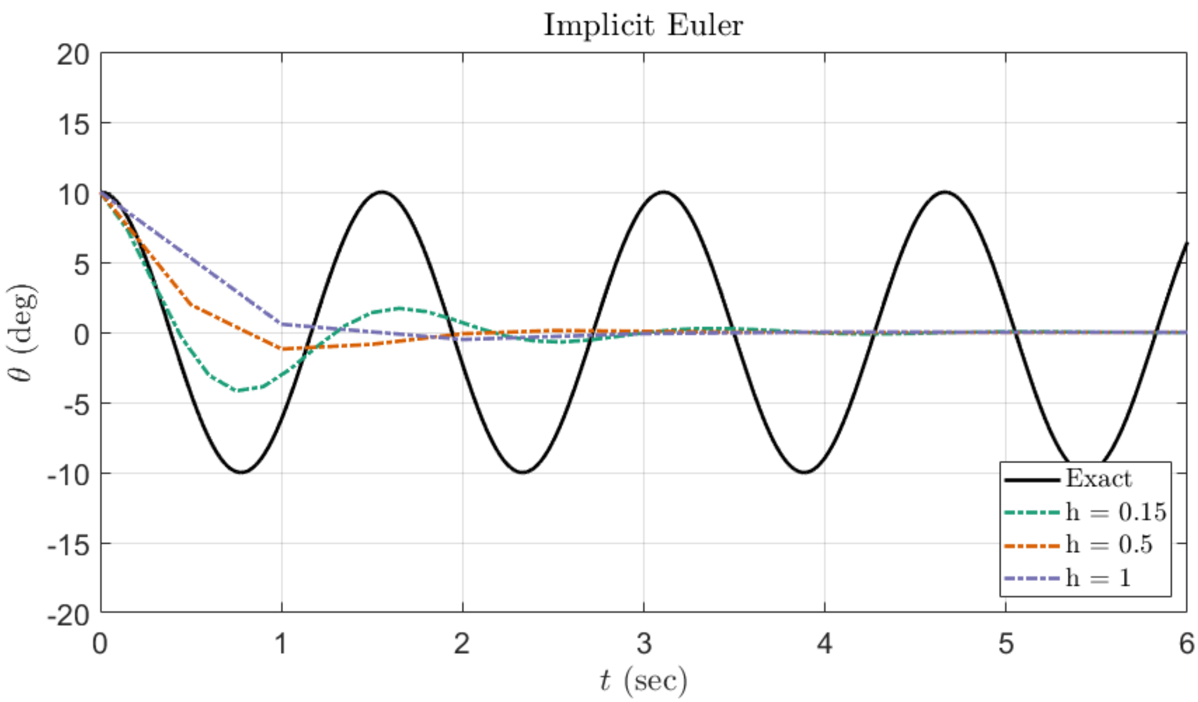


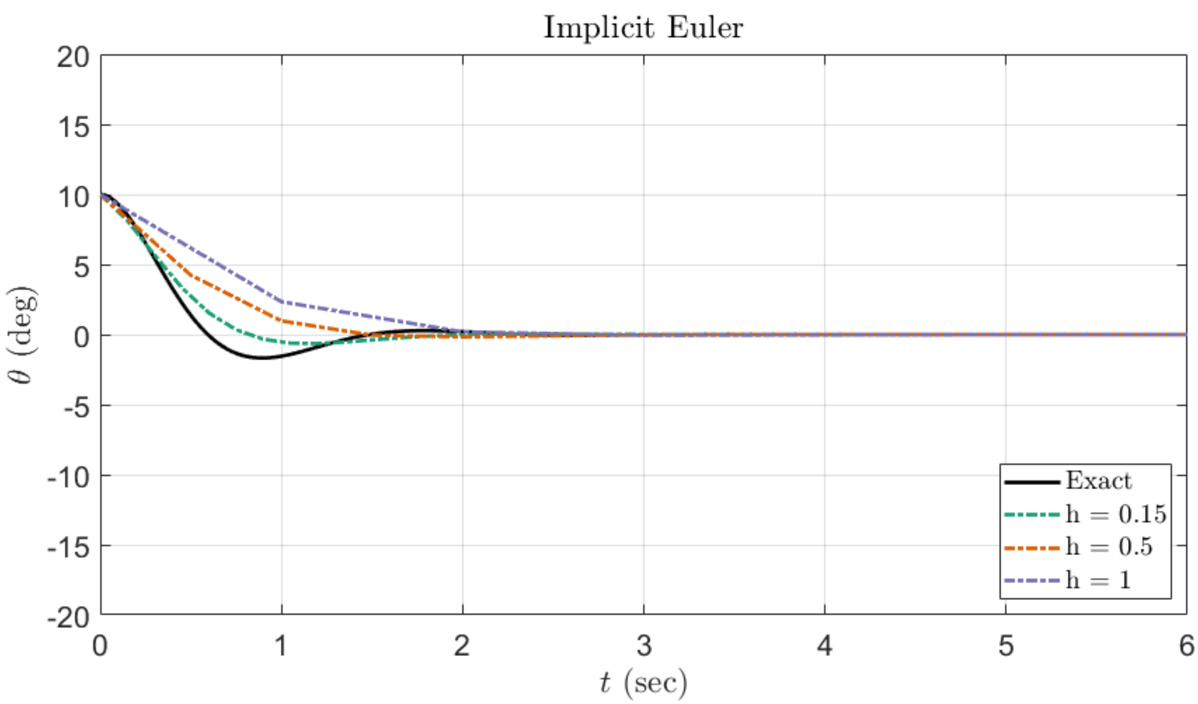


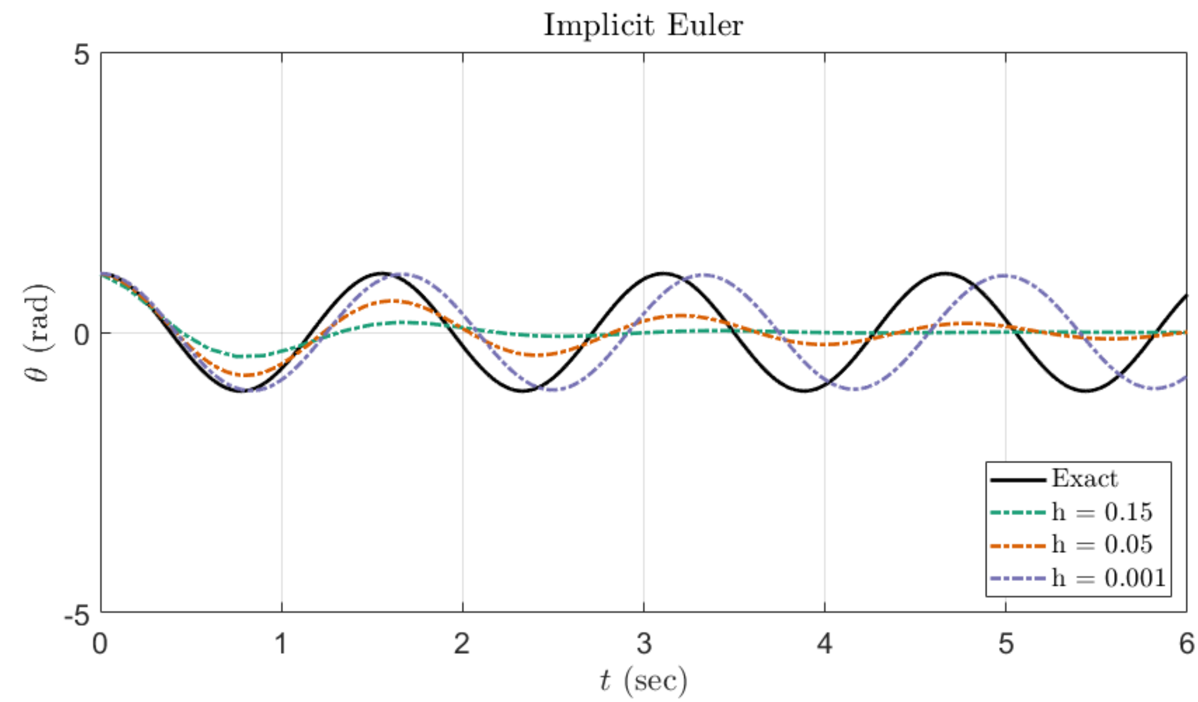
Explicit Euler
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.5
h = 1

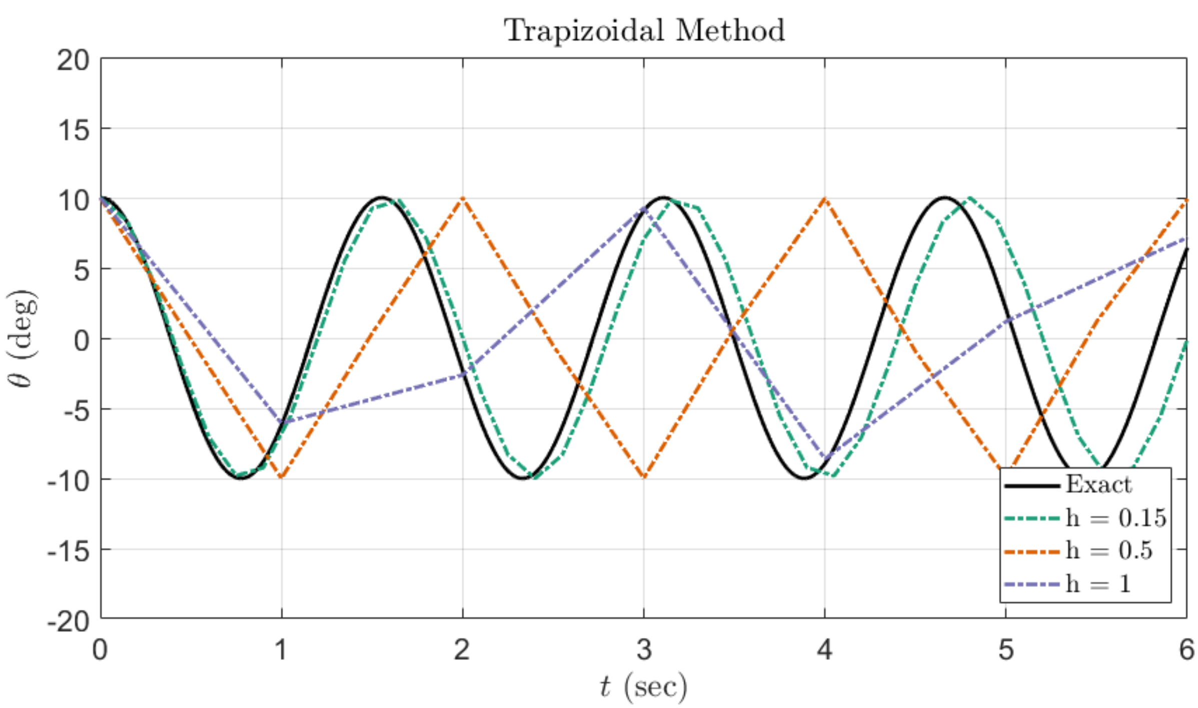


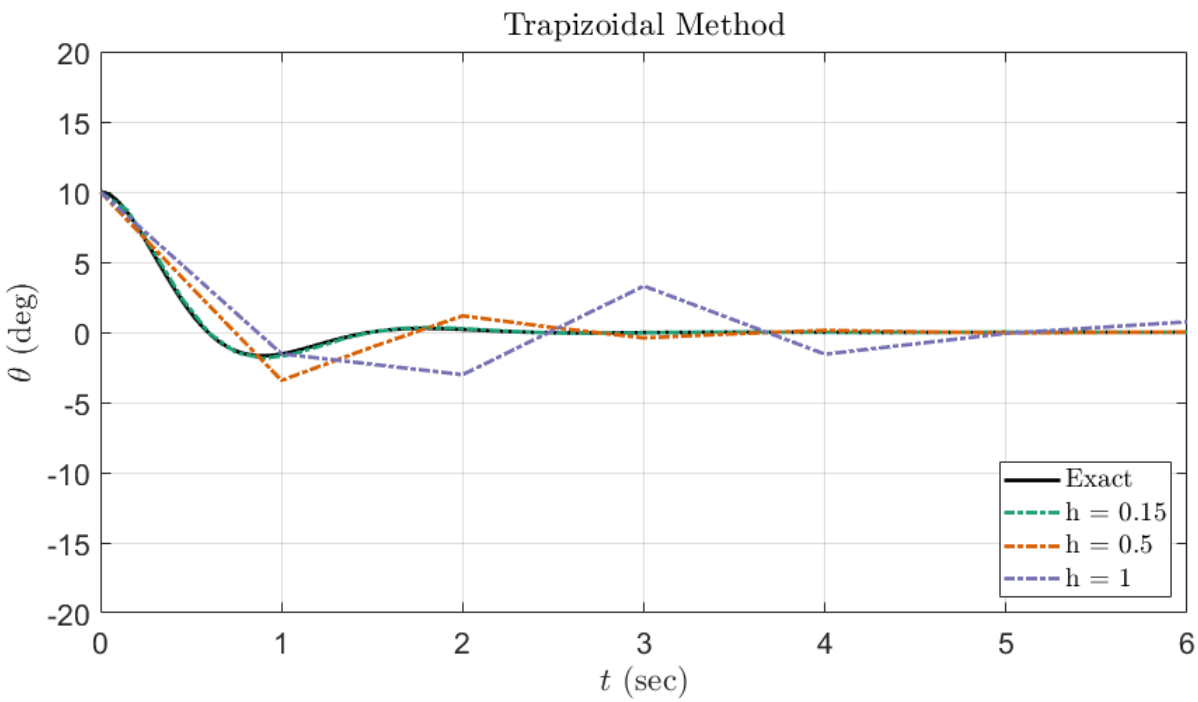
Explicit Euler
5
0
-5
θ (rad)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.05
h = 0.001

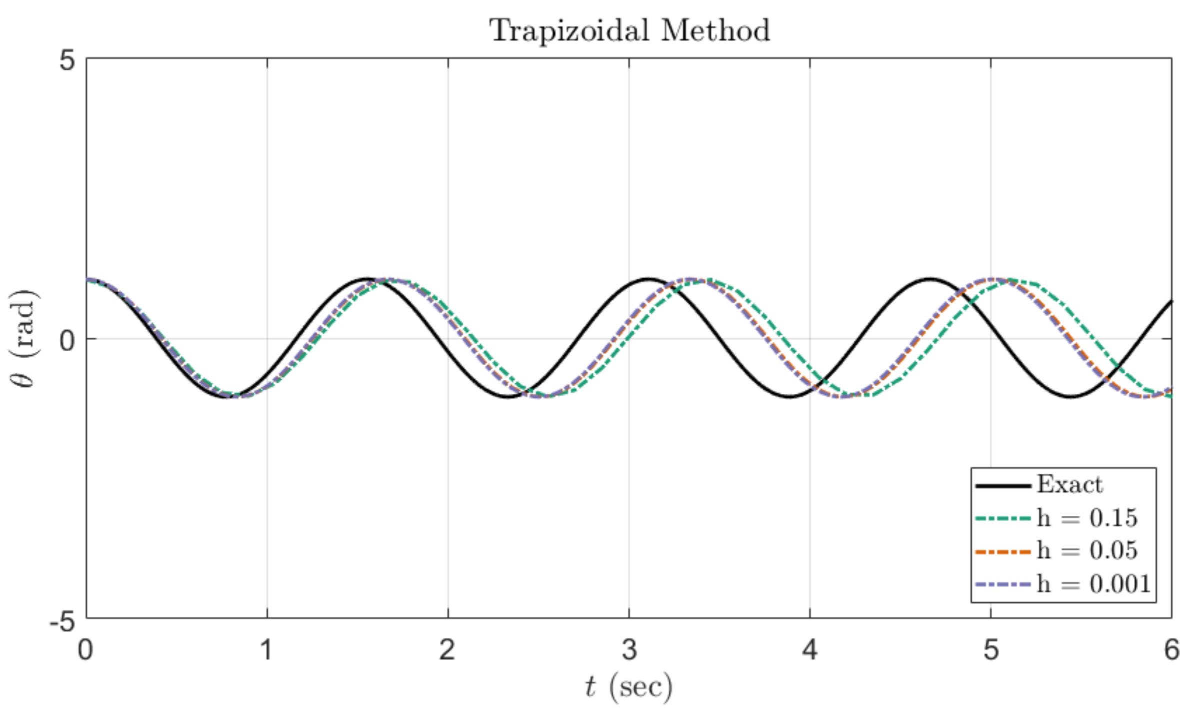

Implicit Euler
θ (deg)
t (sec)
Exact
h = 0.15
h = 0.5
h = 1

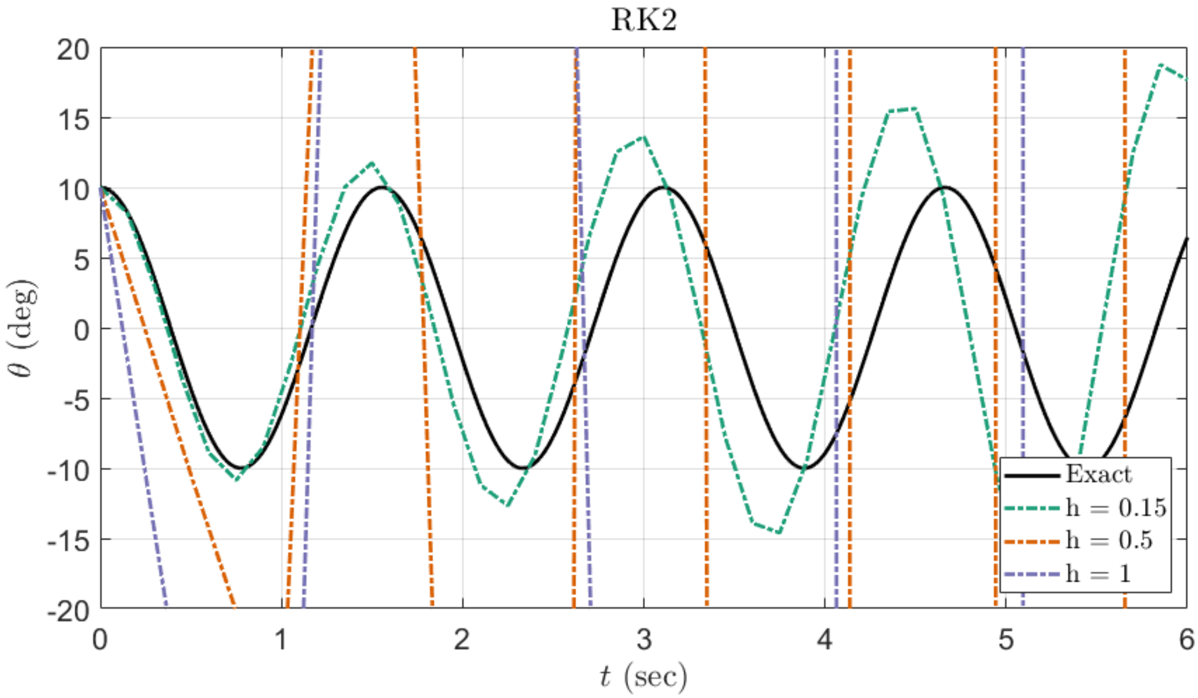

Implicit Euler
θ (deg)
t (sec)
Exact
h = 0.15
h = 0.5
h = 1

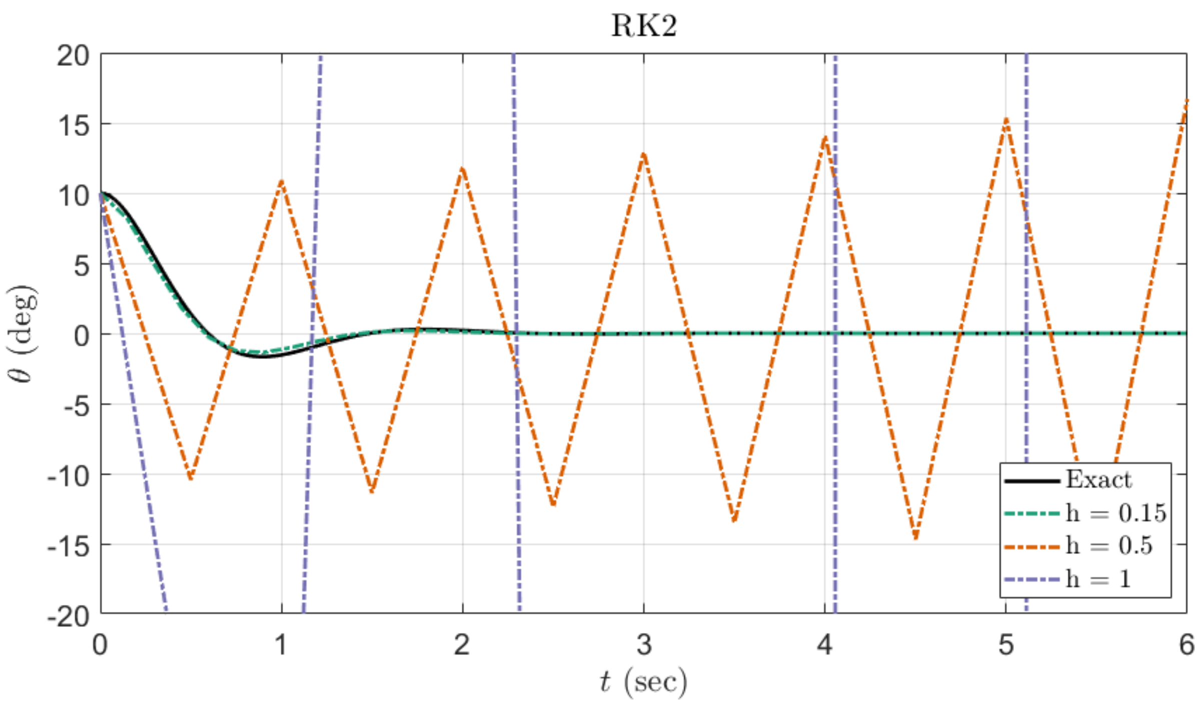

Implicit Euler
5
0
-5
θ (rad)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.05
h = 0.001

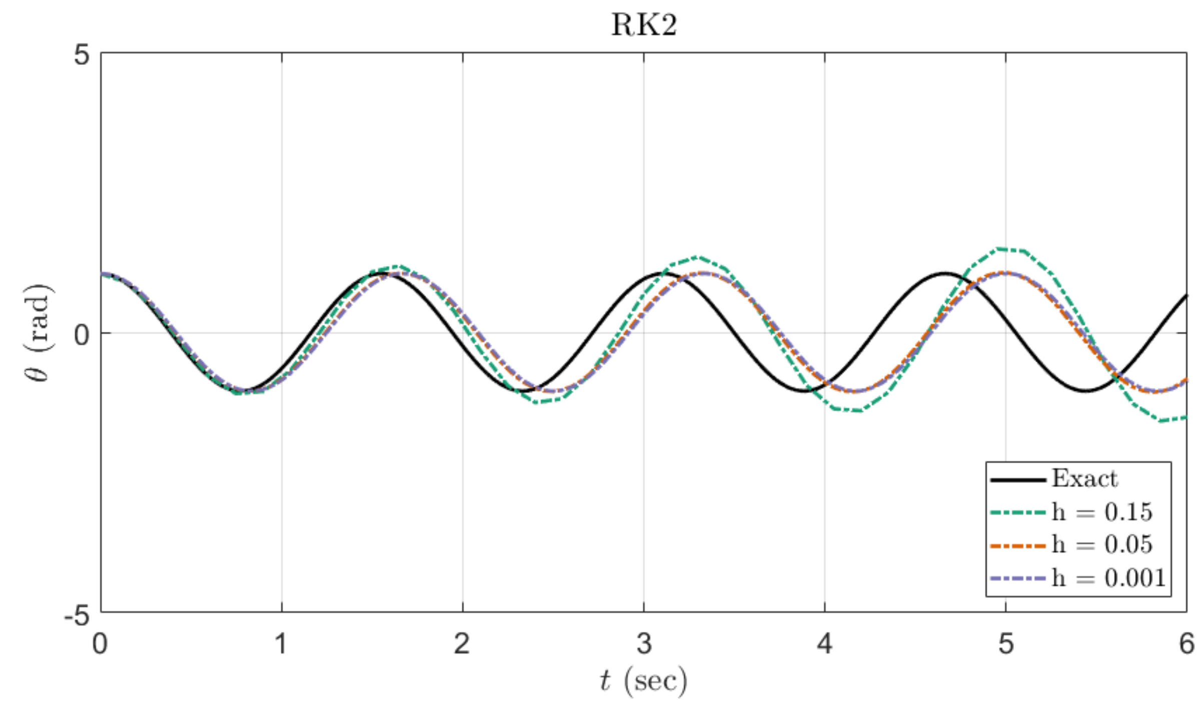

Trapizoidal Method
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.5
h = 1

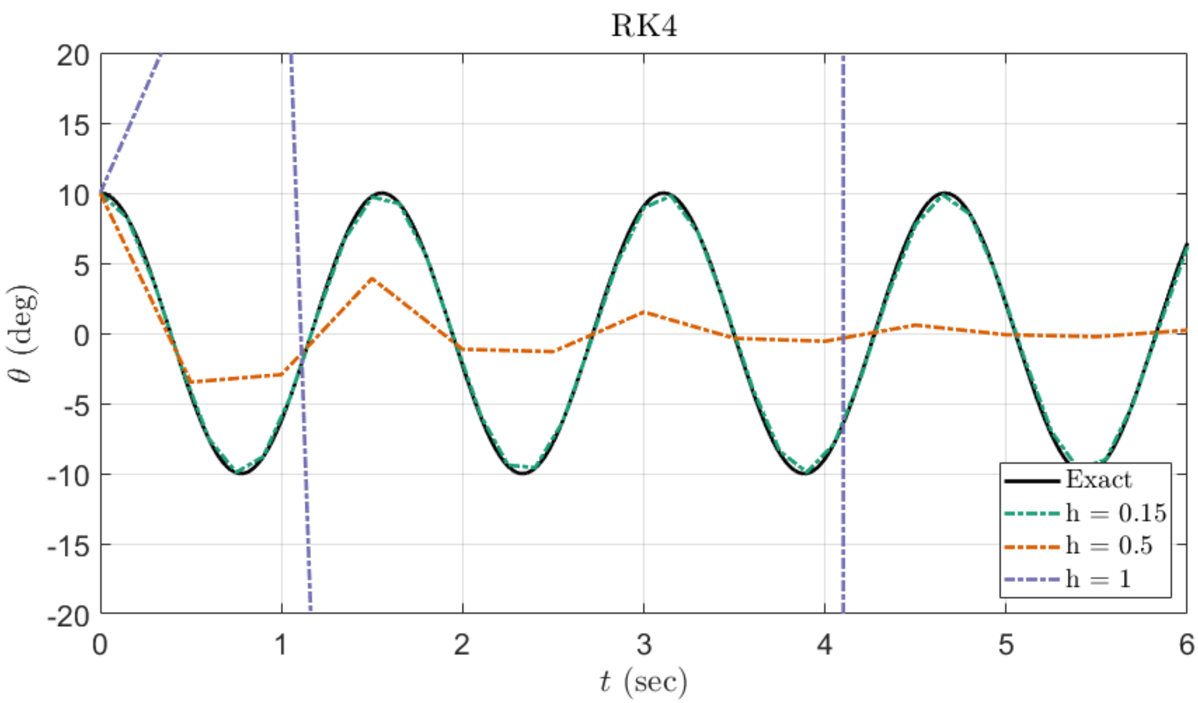

Trapizoidal Method
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.5
h = 1

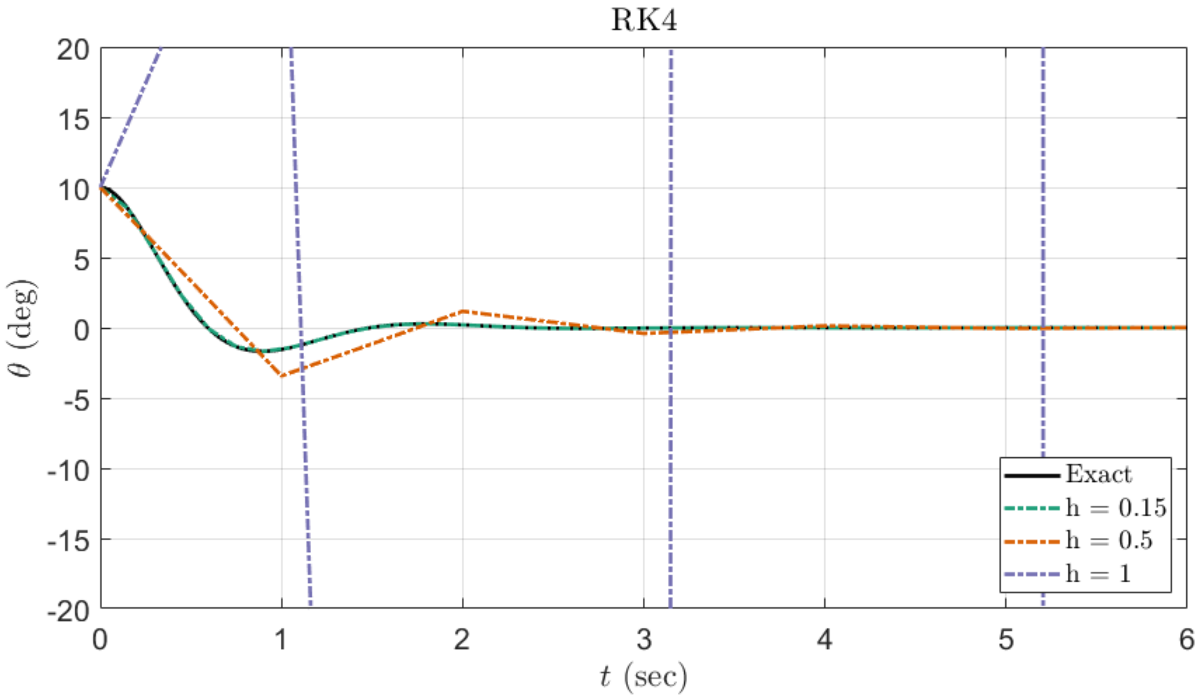

Trapizoidal Method
5
0
-5
θ (rad)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.05
h = 0.001

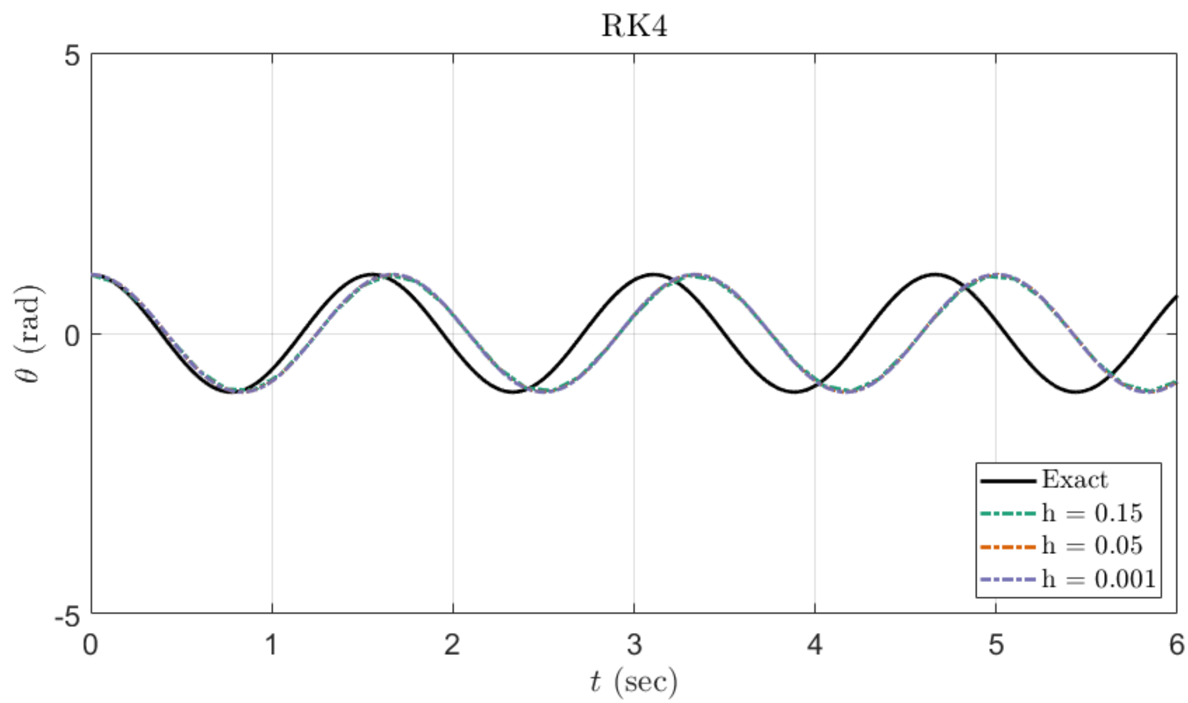

RK2
θ (deg)
t (sec)
Exact
h = 0.15
h = 0.5
h = 1


RK2
θ (deg)
t (sec)
Exact
h = 0.15
h = 0.5
h = 1


RK2
5
0
-5
θ (rad)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.05
h = 0.001


RK4
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.15
h = 0.5
h = 1





Functions

```
function [t, y] = explicitEuler(f, y0, t0, tf, h)
    % explicitEuler solves an ODE using the explicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
```

```
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the explicit Euler method
    for n = 1:N-1
        y(n+1) = y(n) + h * f(t(n), y(n));
    end
end

function [t, y] = implicitEuler(f, y0, t0, tf, h)
    % implicitEuler solves an ODE using the implicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Options for fsolve to increase accuracy and ensure convergence
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the implicit Euler method
    for n = 1:N-1
        % Define the function for the nonlinear equation at each step
        g = @(ynext) ynext - y(n) - h * f(t(n+1), ynext);

        % Use fsolve to solve for y(n+1)
        y(n+1) = fsolve(g, y(n), options);
    end
end

function [t, y] = trapMethod(f, y0, t0, tf, h)
    % trapMethod solves an ODE using the trapezoidal method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Options for fsolve to increase accuracy and ensure convergence
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the trapezoidal method
    for n = 1:N-1
        % Define the function for the nonlinear equation at each step
        g = @(ynext) ynext - y(n) - (h/2) * (f(t(n), y(n)) + f(t(n+1), ynext));

        % Use fsolve to solve for y(n+1)
        y(n+1) = fsolve(g, y(n), options);
    end
end
```

```
function [t, y] = RK2(f, y0, t0, tf, h)
    % RK2 solves an ODE using the 2nd-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the 2nd-order Runge-Kutta method
    for n = 1:N-1
        k1 = f(t(n), y(n));
        k2 = f(t(n) + h/2, y(n) + h/2 * k1);
        y(n+1) = y(n) + h * k2;
    end
end

function [t, y] = RK4(f, y0, t0, tf, h)
    % RK4 solves an ODE using the 4th-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the 4th-order Runge-Kutta method
    for n = 1:N-1
        k1 = f(t(n), y(n));
        k2 = f(t(n) + h/2, y(n) + h/2 * k1);
        k3 = f(t(n) + h/2, y(n) + h/2 * k2);
        k4 = f(t(n) + h, y(n) + h * k3);
        y(n+1) = y(n) + (h/6) * (k1 + 2*k2 + 2*k3 + k4);
    end
end

function [t, y1, y2] = explicitEuler_2(f, y0, v0, t0, tf, h)
    % A general second-order ODE y'' = f(t, y, y')
    % using the explicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
```

```
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the explicit Euler method
    for n = 1:N-1
        % Update y1 and y2
        y1(n+1) = y1(n) + h * y2(n);
        y2(n+1) = y2(n) + h * f(t(n), y1(n), y2(n));
    end
end

function [t, y1, y2] = implicitEuler_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the implicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Options for fsolve
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the implicit Euler method
    for n = 1:N-1
        % Define the system of equations to solve at each step
        func = @(Y_next) [
            Y_next(1) - y1(n) - h * Y_next(2);          % y1^{n+1} = y1^n + h * y2^{n+1}
            Y_next(2) - y2(n) - h * f(t(n+1), Y_next(1), Y_next(2))  % y2^{n+1} = y2^n + h * f(t^{n+1}, y1^{n+1}, y2^{n+1})
        ];

        % Initial guess for fsolve
        Y_guess = [y1(n), y2(n)];

        % Solve for Y_next = [y1^{n+1}; y2^{n+1}] using fsolve
        Y_next = fsolve(func, Y_guess, options);

        % Update y1 and y2 with the solved values
        y1(n+1) = Y_next(1);
        y2(n+1) = Y_next(2);
    end
end

function [t, y1, y2] = trapMethod_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the trapezoidal (implicit) method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
```

```
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Options for fsolve
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the trapezoidal method
    for n = 1:N-1
        % Define the system of equations to solve at each step
        func = @(Y_next) [
            Y_next(1) - y1(n) - h/2 * (y2(n) + Y_next(2));  % y1^{n+1} = y1^n + h/2 * (y2^n + y2^{n+1})
            Y_next(2) - y2(n) - h/2 * (f(t(n), y1(n), y2(n)) + f(t(n+1), Y_next(1), Y_next(2))) % y2^{n+1} = y2^n + h/2 * (f_n + f_{n+1})
        ];

        % Initial guess for fsolve
        Y_guess = [y1(n), y2(n)];

        % Solve for Y_next = [y1^{n+1}; y2^{n+1}] using fsolve
        Y_next = fsolve(func, Y_guess, options);

        % Update y1 and y2 with the solved values
        y1(n+1) = Y_next(1);
        y2(n+1) = Y_next(2);
    end
end

function [t, y1, y2] = RK2_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the second-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the second-order Runge-Kutta method
    for n = 1:N-1
        % Calculate k1 values
        k1y1 = h * y2(n);
        k1y2 = h * f(t(n), y1(n), y2(n));

        % Calculate k2 values
        k2y1 = h * (y2(n) + k1y2 / 2);
        k2y2 = h * f(t(n) + h / 2, y1(n) + k1y1 / 2, y2(n) + k1y2 / 2);

        % Update y1 and y2
        y1(n+1) = y1(n) + k2y1;
        y2(n+1) = y2(n) + k2y2;
    end
end

function [t, y1, y2] = RK4_2(f, y0, v0, t0, tf, h)
    % RK4_2
```

```
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (y(t0) = y0)
    %   v0 - Initial condition for y' (y'(t0) = v0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y
    y2 = zeros(1, N); % Preallocate y2 for y'

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the 4th-order Runge-Kutta method
    for n = 1:N-1
        % Calculate k1 values
        k1_y1 = y2(n);
        k1_y2 = f(t(n), y1(n), y2(n));

        % Calculate k2 values
        k2_y1 = y2(n) + h/2 * k1_y2;
        k2_y2 = f(t(n) + h/2, y1(n) + h/2 * k1_y1, y2(n) + h/2 * k1_y2);

        % Calculate k3 values
        k3_y1 = y2(n) + h/2 * k2_y2;
        k3_y2 = f(t(n) + h/2, y1(n) + h/2 * k2_y1, y2(n) + h/2 * k2_y2);

        % Calculate k4 values
        k4_y1 = y2(n) + h * k3_y2;
        k4_y2 = f(t(n) + h, y1(n) + h * k3_y1, y2(n) + h * k3_y2);

        % Update y1 and y2 using weighted average of slopes
        y1(n+1) = y1(n) + (h/6) * (k1_y1 + 2*k2_y1 + 2*k3_y1 + k4_y1);
        y2(n+1) = y2(n) + (h/6) * (k1_y2 + 2*k2_y2 + 2*k3_y2 + k4_y2);
    end
end
```

```
h =

    0.2000

Step size halved to h = 0.1 with L2 norm error = 0
Step size halved to h = 0.05 with L2 norm error = 0.28934
Step size halved to h = 0.025 with L2 norm error = 0.20358
Step size halved to h = 0.0125 with L2 norm error = 0.13601
Step size halved to h = 0.00625 with L2 norm error = 0.082385
Step size halved to h = 0.003125 with L2 norm error = 0.046103
Step size halved to h = 0.0015625 with L2 norm error = 0.024514
Step size halved to h = 0.00078125 with L2 norm error = 0.012658
Step size halved to h = 0.00039063 with L2 norm error = 0.0064345
Step size halved to h = 0.00019531 with L2 norm error = 0.0032442
Step size halved to h = 9.7656e-05 with L2 norm error = 0.001629
Step size halved to h = 4.8828e-05 with L2 norm error = 0.0008162
Step size halved to h = 2.4414e-05 with L2 norm error = 0.00040853
Step size halved to h = 1.2207e-05 with L2 norm error = 0.00020437
Step size halved to h = 6.1035e-06 with L2 norm error = 0.00010221
Step size halved to h = 3.0518e-06 with L2 norm error = 5.1113e-05
Step size halved to h = 1.5259e-06 with L2 norm error = 2.5558e-05
Step size halved to h = 7.6294e-07 with L2 norm error = 1.278e-05
Step size halved to h = 3.8147e-07 with L2 norm error = 6.3899e-06
Step size halved to h = 1.9073e-07 with L2 norm error = 3.195e-06
Step size halved to h = 9.5367e-08 with L2 norm error = 1.5975e-06
Converged with step size h = 9.5367e-08 and L2 norm error = 7.9875e-07
```