Contents


```
%{

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Description:
AME 60614: Numerical Methods
Homework: 5
Due: 11/12/2024


%}
```

Preperation of the workspace

```
clear all
clc
close all
fontsize = 16;


% set(0,'DefaultFigureWindowStyle','docked')
set(0,'DefaultTextInterpreter','latex')
set(0,'DefaultAxesFontSize',fontsize)
set(0,'DefaultLegendFontSize',fontsize)
colors  = ["#000000","#1b9e77","#d95f02","#7570b3","#0099FF"]';
```

Setting data paths

Make sure to update this for the machine that you are working on. (Maybe, This should now run on any machine without change. 7/24/24) Change the current folder to the folder of this m-file.

```
if(~isdeployed)
  cd(fileparts(matlab.desktop.editor.getActiveFilename));
end

addpath(cd)
% cd ..; % Moving up a directory (from processing_code)
basepath = cd; % Pulling the current directory

if isunix

    imagepath = [basepath '/images/']; % Unix
    mkdir(imagepath);

elseif ispc

    imagepath = [basepath '\images\']; % Windows
    mkdir(imagepath);

else
    disp('Platform not supported')
end
```

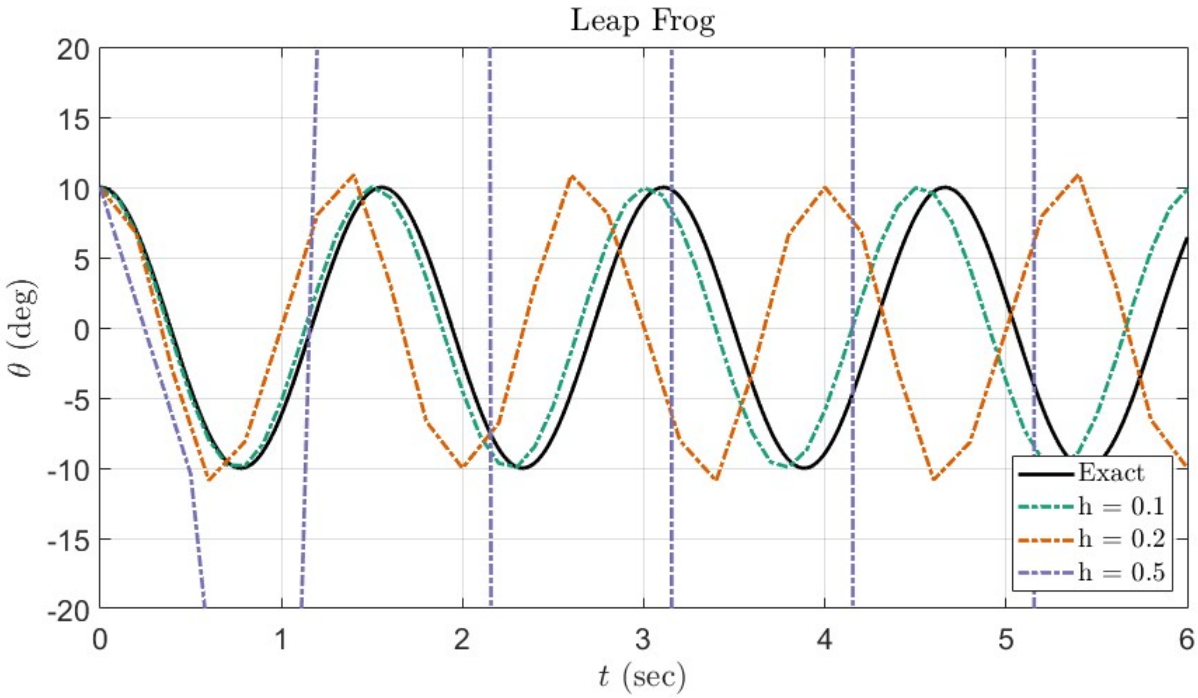
```
Warning: Directory already exists.
```

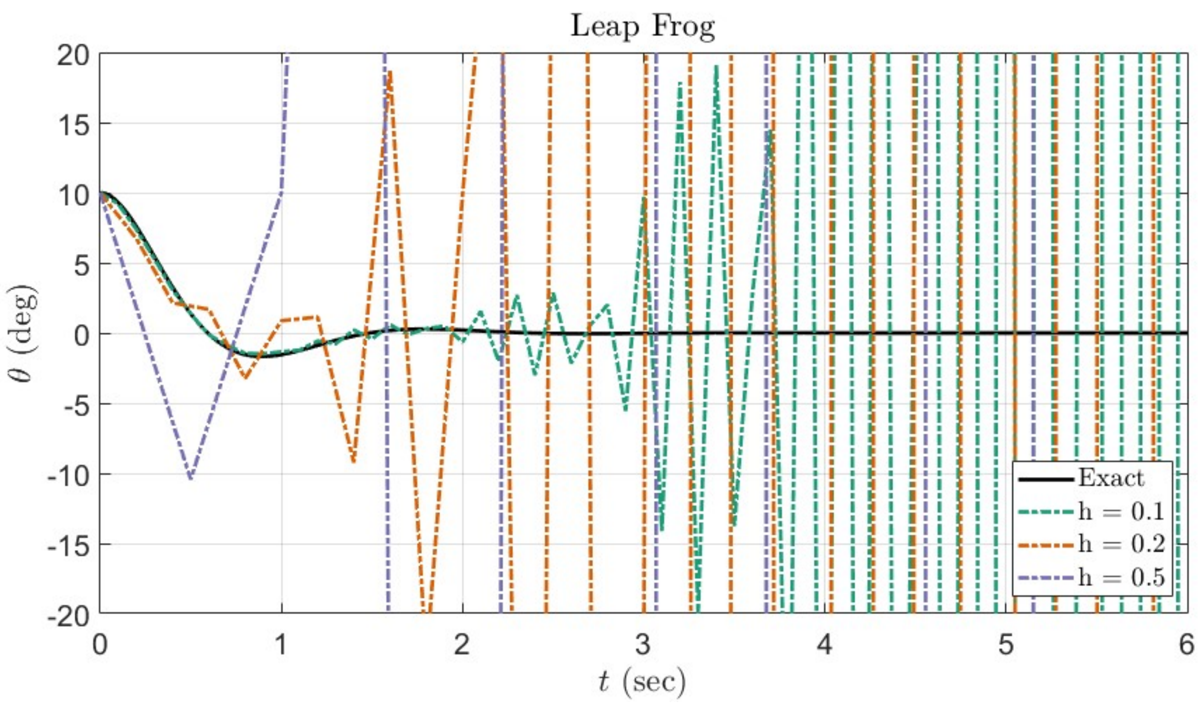
Problem 9 in Chapter 4

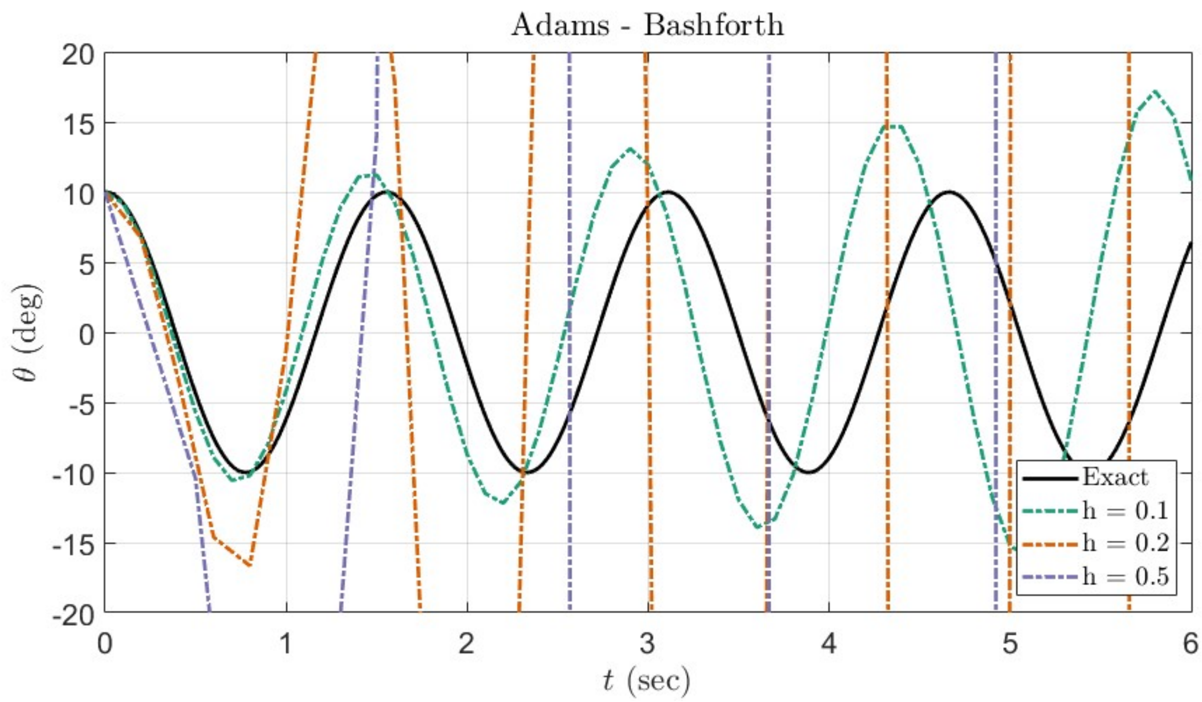
```
funlist = {@leapfrog, @AB2};
funlist_str = ["Leap Frog", "Adams - Bashforth"];
% funlist = {@leapfrog};
% funlist_str = ["Leap Frog"];
```

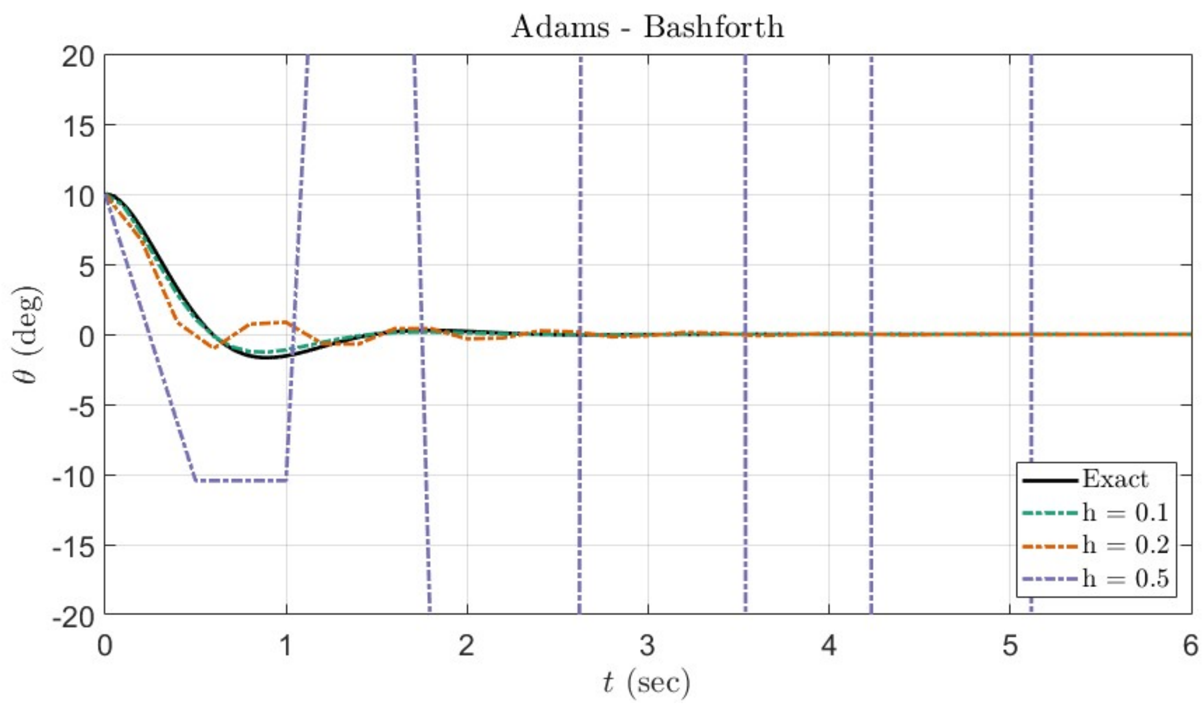
```
exact = @(t) 10*cos(4.04351*t);
exact2 = @(t) exp(-2* t).* (5.6911* sin(3.51426 *t) + 10 *cos(3.51426*t));
g = 9.81; %m/s^2
l = 0.6; %m
c = 4;
theta0= 10; %deg
thetap0 = 0; % assume at rest
f_theta = @(t, theta, thetap) -g/l *theta ;
f_theta_2 = @(t, theta, thetap) -g/l *theta -c*thetap ;
start = 0;


t0 = 0;
tf = 6;
h = [0.1 0.2 0.5];
for i = 1:length(funlist)
    figure
    plot(linspace(t0,tf,1e3),exact(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(h)

        [t,y,yp] = funlist{i}(f_theta,theta0,thetap0, t0, tf, h(n), start);
        plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
        hold on


    end
    xlabel('$t$ (sec)')
    ylabel('$\theta$ (deg)')
    legend(Location="southeast",Interpreter="latex")
    xlim([t0 tf])
    ylim([-20 20])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q9_a_RK',char(string(i)),'.png'],'-dpng');



    figure
    plot(linspace(t0,tf,1e3),exact2(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
    hold on
    for n = 1:length(h)

        [t,y,yp] = funlist{i}(f_theta_2,theta0,thetap0, t0, tf, h(n),start);
        plot(t,y, "-.", "LineWidth",2, 'DisplayName',strcat("h = ",string(h(n))) , color=colors(n+1,:))
        hold on


    end
    xlabel('$t$ (sec)')
    ylabel('$\theta$ (deg)')
    legend(Location="southeast",Interpreter="latex")
    xlim([t0 tf])
    ylim([-20 20])
    grid on
    title(funlist_str(i))
    set(gcf,'Position',[0,0,1000,500])
    print(gcf,[imagepath,'Q9_b_RK',char(string(i)),'.png'],'-dpng');



end


A = [0 1; -g/l -c];

lamda_9 = eig(A)
```

```
lamda_9 =

  -2.0000 + 3.5143i
  -2.0000 - 3.5143i
```


Leap Frog
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.1
h = 0.2
h = 0.5


Leap Frog
20
15
10
5
0
-5
-10
-15
-20
θ (deg)
0
1
2
3
4
5
6
t (sec)
Exact
h = 0.1
h = 0.2
h = 0.5

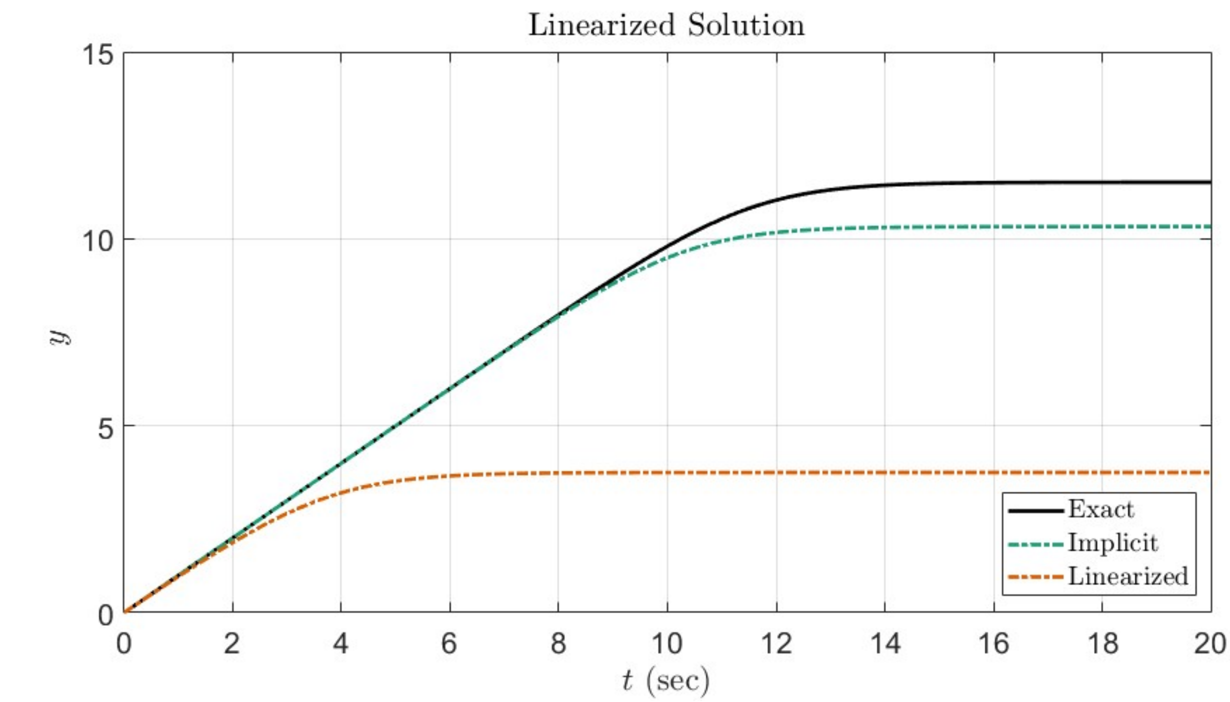


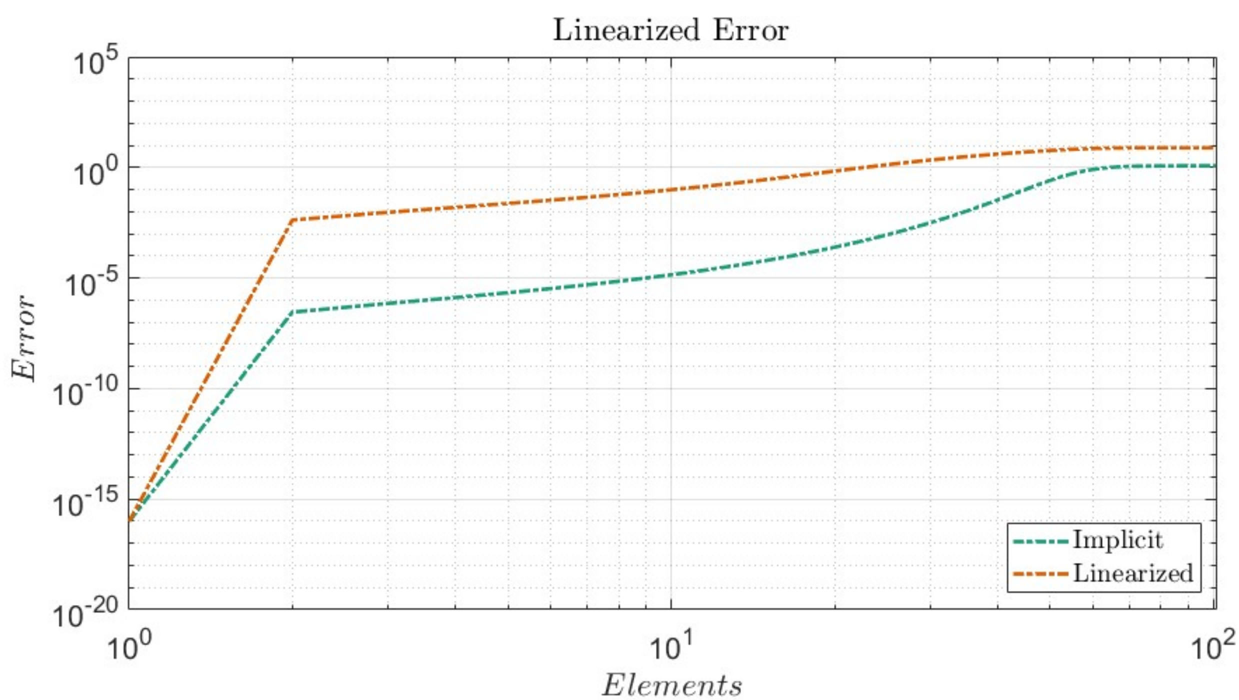


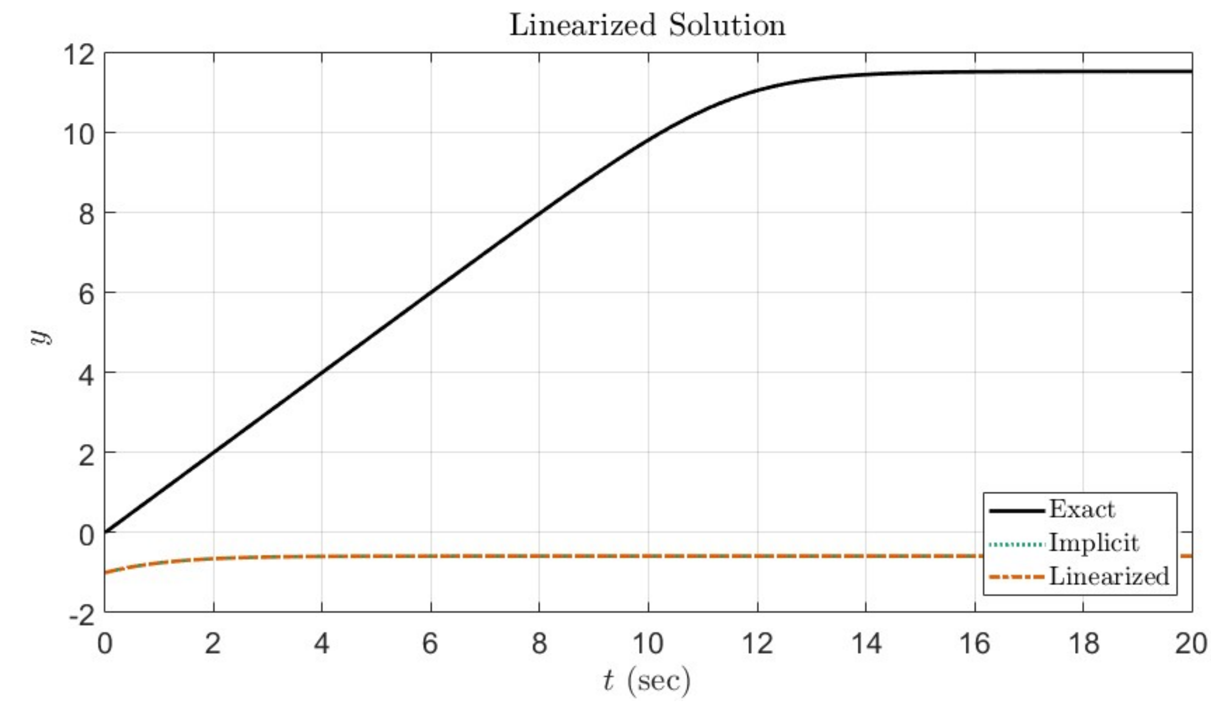
Problem 12

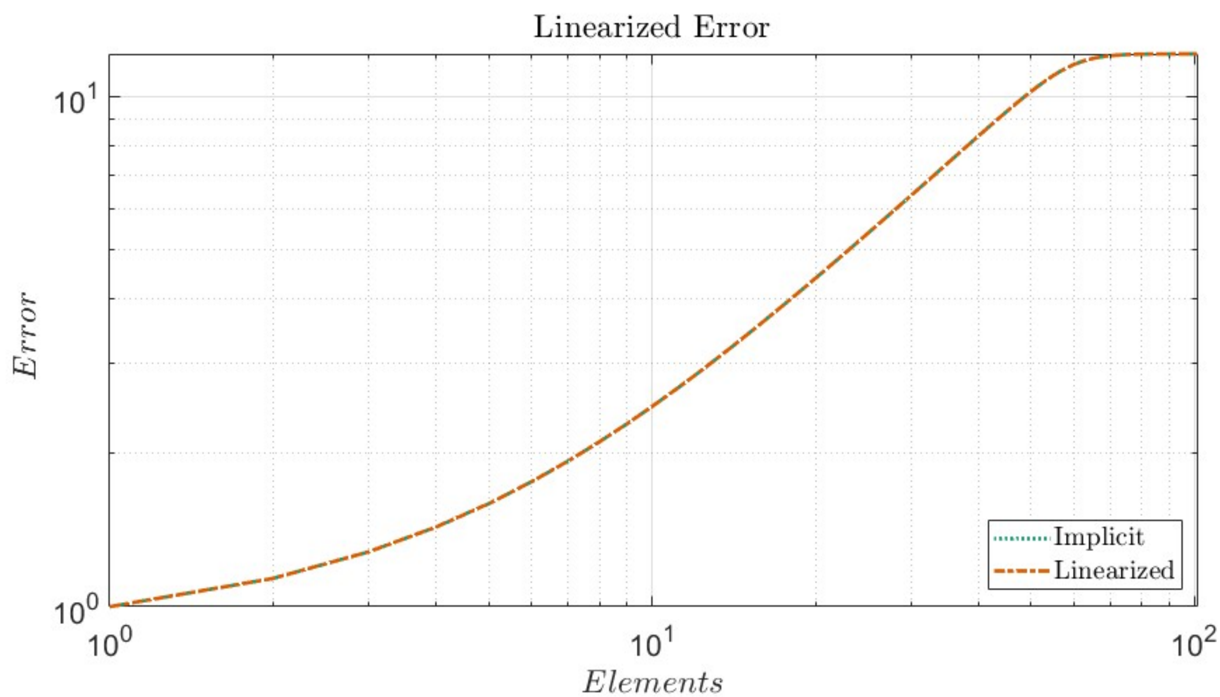
```
y0 = - 1e-5;
f_12 = @(t, y) exp(y-t);
dfdy = @(t, y) exp(y-t);
exact_12 = @(t) -log(exp(-y0) + exp(-t) -1);
h = 0.2;

t0 = 0;
tf = 20;

[t_imp,y_imp] = implicitEuler(f_12, y0, t0, tf, h);
[t_lin,y_lin] = limplicitEuler(f_12, dfdy, y0, t0, tf, h);

figure
plot(linspace(t0,tf,1e3),exact_12(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
hold on
plot(t_imp, y_imp,"-.","LineWidth",2, 'DisplayName', "Implicit", color=colors(2,:))
hold on
```

```
plot(t_lin, y_lin,"-.","LineWidth",2, 'DisplayName', "Linearized", color=colors(3,:))

xlabel('$t$ (sec)')
ylabel('$y$ ')
legend(Location="southeast",Interpreter="latex")
xlim([t0 tf])
ylim([0 15])
grid on
title("Linearized Solution")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q12_c',char(string(i)),'.png'],'-dpng');



figure
% plot(linspace(t0,tf,1e3),exact_12(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
% hold on
loglog(exact_12(t0:h:tf) - y_imp,"-.","LineWidth",2, 'DisplayName', "Implicit", color=colors(2,:))
hold on
loglog(exact_12(t0:h:tf) - y_lin,"-.","LineWidth",2, 'DisplayName', "Linearized", color=colors(3,:))

xlabel('$Elements$')
ylabel('$Error$ ')
legend(Location="southeast",Interpreter="latex")
% xlim([t0 tf])
% ylim([0 15])
grid on
title("Linearized Error")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q12_c_err',char(string(i)),'.png'],'-dpng');


y0 = -1;
[t_imp,y_imp_2] = implicitEuler(f_12, y0, t0, tf, h);
[t_lin,y_lin_2] = limplicitEuler(f_12, dfdy, y0, t0, tf, h);


figure
plot(linspace(t0,tf,1e3),exact_12(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
hold on
plot(t_imp, y_imp_2,":","LineWidth",2, 'DisplayName', "Implicit", color=colors(2,:))
hold on
plot(t_lin, y_lin_2,"-.","LineWidth",2, 'DisplayName', "Linearized", color=colors(3,:))

xlabel('$t$ (sec)')
ylabel('$y$ ')
legend(Location="southeast",Interpreter="latex")
xlim([t0 tf])
% ylim([0 15])
grid on
title("Linearized Solution")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q12_d',char(string(i)),'.png'],'-dpng');


figure
% plot(linspace(t0,tf,1e3),exact_12(linspace(t0,tf,1e3)),"LineWidth",2, 'DisplayName', "Exact", color=colors(1,:))
% hold on
loglog(exact_12(t0:h:tf) - y_imp_2,":","LineWidth",2, 'DisplayName', "Implicit", color=colors(2,:))
hold on
loglog(exact_12(t0:h:tf) - y_lin_2,"-.","LineWidth",2, 'DisplayName', "Linearized", color=colors(3,:))

xlabel('$Elements$')
ylabel('$Error$ ')
legend(Location="southeast",Interpreter="latex")
% xlim([t0 tf])
% ylim([0 15])
grid on
title("Linearized Error")
set(gcf,'Position',[0,0,1000,500])
print(gcf,[imagepath,'Q12_d_err',char(string(i)),'.png'],'-dpng');
```


Linearized Solution
15
10
5
0
y
0
2
4
6
8
10
12
14
16
18
20
t (sec)
Exact
Implicit
Linearized


Linearized Error
10^{5}
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
Error
10^{0}
10^{1}
10^{2}
Elements
Implicit
Linearized





Functions

```
function [t, y] = explicitEuler(f, y0, t0, tf, h)
    % explicitEuler solves an ODE using the explicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
```

```
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the explicit Euler method
    for n = 1:N-1
        y(n+1) = y(n) + h * f(t(n), y(n));
    end
end

function [t, y] = implicitEuler(f, y0, t0, tf, h)
    % implicitEuler solves an ODE using the implicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Options for fsolve to increase accuracy and ensure convergence
    options = optimoptions('fsolve', 'Display', 'off');
    % options = optimoptions('fmincon', 'Display', 'off');


    % Apply the implicit Euler method
    for n = 1:N-1
        % Define the function for the nonlinear equation at each step
        g = @(ynext) ynext - y(n) - h * f(t(n+1), ynext);

        % Use fsolve to solve for y(n+1)
        y(n+1) = fsolve(g, y(n), options);
    end
end

function [t, y] = limplicitEuler(f, dfdy, y0, t0, tf, h)
    % limplicitEuler solves a first-order ODE y' = f(t, y)
    % using a linearized implicit Euler method.
    %
    % Inputs:
    %   f    - Function handle for y' = f(t, y)
    %   dfdy - Function handle for the partial derivative of f with respect to y
    %   y0   - Initial condition for y
    %   t0   - Initial time
    %   tf   - Final time
    %   h    - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values for y at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for the solution

    % Set the initial condition
    y(1) = y0;

    % Apply the linearized implicit Euler method for each step
    for n = 1:N-1
        % Evaluate f and its derivative with respect to y at the current step
        fn = f(t(n+1), y(n));
        dfdyn = dfdy(t(n+1), y(n));

        % Calculate the next value of y using the linearized formula
        y(n+1) = y(n) + ((h * fn) / (1 - h * dfdyn));
    end
```

```
end

function [t, y] = trapMethod(f, y0, t0, tf, h)
    % trapMethod solves an ODE using the trapezoidal method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Options for fsolve to increase accuracy and ensure convergence
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the trapezoidal method
    for n = 1:N-1
        % Define the function for the nonlinear equation at each step
        g = @(ynext) ynext - y(n) - (h/2) * (f(t(n), y(n)) + f(t(n+1), ynext));

        % Use fsolve to solve for y(n+1)
        y(n+1) = fsolve(g, y(n), options);
    end
end

function [t, y] = RK2(f, y0, t0, tf, h)
    % RK2 solves an ODE using the 2nd-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the 2nd-order Runge-Kutta method
    for n = 1:N-1
        k1 = f(t(n), y(n));
        k2 = f(t(n) + h/2, y(n) + h/2 * k1);
        y(n+1) = y(n) + h * k2;
    end
end

function [t, y] = RK4(f, y0, t0, tf, h)
    % RK4 solves an ODE using the 4th-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for dy/dt = f(t, y)
    %   y0 - Initial condition (value of y at t = t0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t - Array of time steps
    %   y - Array of solution values at each time step
```

```
    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y = zeros(1, N); % Preallocate y for speed

    % Set the initial condition
    y(1) = y0;

    % Apply the 4th-order Runge-Kutta method
    for n = 1:N-1
        k1 = f(t(n), y(n));
        k2 = f(t(n) + h/2, y(n) + h/2 * k1);
        k3 = f(t(n) + h/2, y(n) + h/2 * k2);
        k4 = f(t(n) + h, y(n) + h * k3);
        y(n+1) = y(n) + (h/6) * (k1 + 2*k2 + 2*k3 + k4);
    end
end

function [t, y1, y2] = explicitEuler_2(f, y0, v0, t0, tf, h)
    % A general second-order ODE y'' = f(t, y, y')
    % using the explicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the explicit Euler method
    for n = 1:N-1
        % Update y1 and y2
        y1(n+1) = y1(n) + h * y2(n);
        y2(n+1) = y2(n) + h * f(t(n), y1(n), y2(n));
    end
end

function [t, y1, y2] = implicitEuler_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the implicit Euler method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Options for fsolve
```

```
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the implicit Euler method
    for n = 1:N-1
        % Define the system of equations to solve at each step
        func = @(Y_next) [
            Y_next(1) - y1(n) - h * Y_next(2);          % y1^{n+1} = y1^n + h * y2^{n+1}
            Y_next(2) - y2(n) - h * f(t(n+1), Y_next(1), Y_next(2))  % y2^{n+1} = y2^n + h * f(t^{n+1}, y1^{n+1}, y2^{n+1})
        ];

        % Initial guess for fsolve
        Y_guess = [y1(n), y2(n)];

        % Solve for Y_next = [y1^{n+1}; y2^{n+1}] using fsolve
        Y_next = fsolve(func, Y_guess, options);

        % Update y1 and y2 with the solved values
        y1(n+1) = Y_next(1);
        y2(n+1) = Y_next(2);
    end
end

function [t, y1, y2] = trapMethod_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the trapezoidal (implicit) method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Options for fsolve
    options = optimoptions('fsolve', 'Display', 'off');

    % Apply the trapezoidal method
    for n = 1:N-1
        % Define the system of equations to solve at each step
        func = @(Y_next) [
            Y_next(1) - y1(n) - h/2 * (y2(n) + Y_next(2));  % y1^{n+1} = y1^n + h/2 * (y2^n + y2^{n+1})
            Y_next(2) - y2(n) - h/2 * (f(t(n), y1(n), y2(n)) + f(t(n+1), Y_next(1), Y_next(2))) % y2^{n+1} = y2^n + h/2 * (f_n + f_{n+1})
        ];

        % Initial guess for fsolve
        Y_guess = [y1(n), y2(n)];

        % Solve for Y_next = [y1^{n+1}; y2^{n+1}] using fsolve
        Y_next = fsolve(func, Y_guess, options);

        % Update y1 and y2 with the solved values
        y1(n+1) = Y_next(1);
        y2(n+1) = Y_next(2);
    end
end

function [t, y1, y2] = RK2_2(f, y0, v0, t0, tf, h)
    % General second-order ODE y'' = f(t, y, y')
    % using the second-order Runge-Kutta method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
```

```matlab
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the second-order Runge-Kutta method
    for n = 1:N-1
        % Calculate k1 values
        k1y1 = h * y2(n);
        k1y2 = h * f(t(n), y1(n), y2(n));

        % Calculate k2 values
        k2y1 = h * (y2(n) + k1y2 / 2);
        k2y2 = h * f(t(n) + h / 2, y1(n) + k1y1 / 2, y2(n) + k1y2 / 2);

        % Update y1 and y2
        y1(n+1) = y1(n) + k2y1;
        y2(n+1) = y2(n) + k2y2;
    end
end

function [t, y1, y2] = RK4_2(f, y0, v0, t0, tf, h)

    % RK4_2

    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (y(t0) = y0)
    %   v0 - Initial condition for y' (y'(t0) = v0)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y
    y2 = zeros(1, N); % Preallocate y2 for y'

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

    % Apply the 4th-order Runge-Kutta method
    for n = 1:N-1
        % Calculate k1 values
        k1_y1 = y2(n);
        k1_y2 = f(t(n), y1(n), y2(n));

        % Calculate k2 values
        k2_y1 = y2(n) + h/2 * k1_y2;
        k2_y2 = f(t(n) + h/2, y1(n) + h/2 * k1_y1, y2(n) + h/2 * k1_y2);

        % Calculate k3 values
        k3_y1 = y2(n) + h/2 * k2_y2;
        k3_y2 = f(t(n) + h/2, y1(n) + h/2 * k2_y1, y2(n) + h/2 * k2_y2);

        % Calculate k4 values
        k4_y1 = y2(n) + h * k3_y2;
        k4_y2 = f(t(n) + h, y1(n) + h * k3_y1, y2(n) + h * k3_y2);

        % Update y1 and y2 using weighted average of slopes
        y1(n+1) = y1(n) + (h/6) * (k1_y1 + 2*k2_y1 + 2*k3_y1 + k4_y1);
```

```
        y2(n+1) = y2(n) + (h/6) * (k1_y2 + 2*k2_y2 + 2*k3_y2 + k4_y2);
    end
end

function [t, y1, y2] = AB2(f, y0, v0, t0, tf, h, start)
    % AB2 solves a general second-order ODE y'' = f(t, y, y')
    % using the second-order Adams-Bashforth method.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %   start - Start method
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

if start == 1

    % Use explicit Euler for the first step to get y1(2) and y2(2)
    y1(2) = y1(1) + h * y2(1);
    y2(2) = y2(1) + h * f(t(1), y1(1), y2(1));

else
    % Use RK2 for the first step
    k1y1 = h * y2(1);
    k1y2 = h * f(t(1), y1(1), y2(1));

    % Calculate k2 values
    k2y1 = h * (y2(1) + k1y2 / 2);
    k2y2 = h * f(t(1) + h / 2, y1(1) + k1y1 / 2, y2(1) + k1y2 / 2);

    % Update y1 and y2
    y1(2) = y1(1) + k2y1;
    y2(2) = y2(1) + k2y2;
end

    % Apply the Adams-Bashforth method for the rest of the steps
    for n = 2:N-1
        % Update y1 and y2 using the Adams-Bashforth formula
        y1(n+1) = y1(n) + h * (3/2 * y2(n) - 1/2 * y2(n-1));
        y2(n+1) = y2(n) + h * (3/2 * f(t(n), y1(n), y2(n)) - 1/2 * f(t(n-1), y1(n-1), y2(n-1)));
    end
end

function [t, y1, y2] = leapfrog(f, y0, v0, t0, tf, h, start)
    % leapfrog solves a general second-order ODE y'' = f(t, y, y')
    % using the leapfrog method with an explicit Euler or RK2 start.
    %
    % Inputs:
    %   f  - Function handle for y'' = f(t, y, y')
    %   y0 - Initial condition for y (position)
    %   v0 - Initial condition for y' (velocity)
    %   t0 - Initial time
    %   tf - Final time
    %   h  - Step size
    %   start - Start method
    %
    % Outputs:
    %   t  - Array of time steps
    %   y1 - Array of solution values for y at each time step
    %   y2 - Array of solution values for y' at each time step

    % Define the time vector from t0 to tf with step size h
    t = t0:h:tf;
    N = length(t); % Number of time steps
```

```
    y1 = zeros(1, N); % Preallocate y1 for y (position)
    y2 = zeros(1, N); % Preallocate y2 for y' (velocity)

    % Set the initial conditions
    y1(1) = y0;
    y2(1) = v0;

if start == 1

    % Use explicit Euler for the first step to get y1(2) and y2(2)
    y1(2) = y1(1) + h * y2(1);
    y2(2) = y2(1) + h * f(t(1), y1(1), y2(1));

else
    % Use RK2 for the first step
    k1y1 = h * y2(1);
    k1y2 = h * f(t(1), y1(1), y2(1));

    % Calculate k2 values
    k2y1 = h * (y2(1) + k1y2 / 2);
    k2y2 = h * f(t(1) + h / 2, y1(1) + k1y1 / 2, y2(1) + k1y2 / 2);

    % Update y1 and y2
    y1(2) = y1(1) + k2y1;
    y2(2) = y2(1) + k2y2;
end

    % Apply the leapfrog method for the rest of the steps
    for n = 2:N-1
        % Full-step for position
        y1(n+1) = y1(n-1) + 2*h * y2(n);

        % Full-step for velocity
        y2(n+1) = y2(n-1) + 2*h * f(t(n), y1(n), y2(n));

    end
end
```

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