Contents


```
%{

@author: Benjamin Bemis Ph.D Student,
Advisor: Dr Juliano


Description:
AME 60614: Numerical Methods
Homework: 7
Due: 12/5/2024


%}
```

Preperation of the workspace

```
clear all
clc
close all
fontsize = 16;


% set(0,'DefaultFigureWindowStyle','default')
set(0,'DefaultTextInterpreter','latex')
set(0,'DefaultAxesFontSize',fontsize)
set(0,'DefaultLegendFontSize',fontsize)
colors  = ["#000000","#1b9e77","#d95f02","#7570b3","#0099FF"]';
```

Setting data paths

Make sure to update this for the machine that you are working on. (Maybe, This should now run on any machine without change. 7/24/24) Change the current folder to the folder of this m-file.

```
if(~isdeployed)
  cd(fileparts(matlab.desktop.editor.getActiveFilename));
end

addpath(cd)
% cd ..; % Moving up a directory (from processing_code)
basepath = cd; % Pulling the current directory


imagepath = [basepath filesep 'images' filesep];
mkdir(imagepath);
```

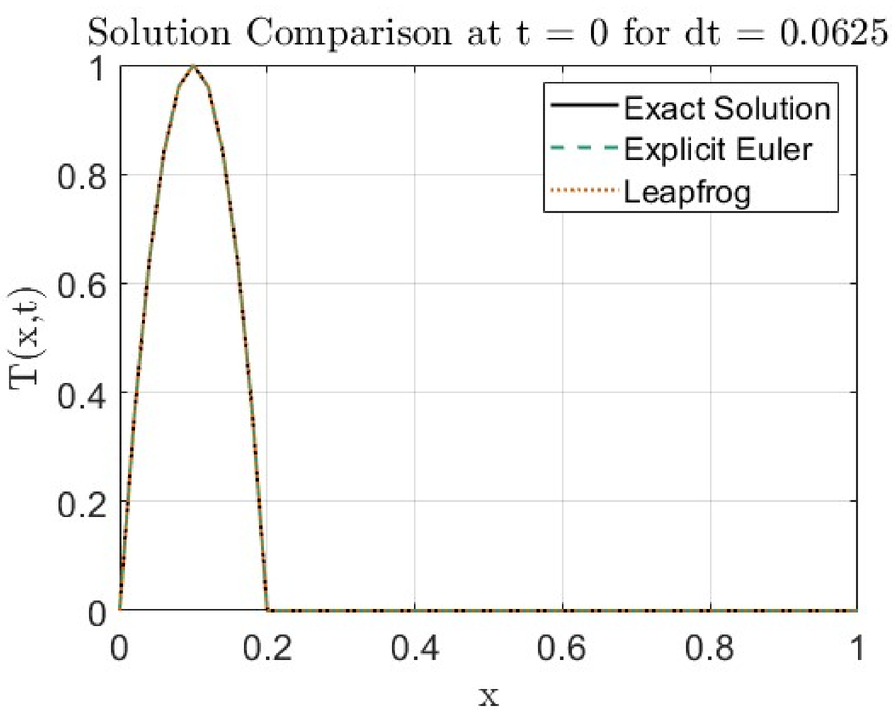
```
Warning: Directory already exists.
```

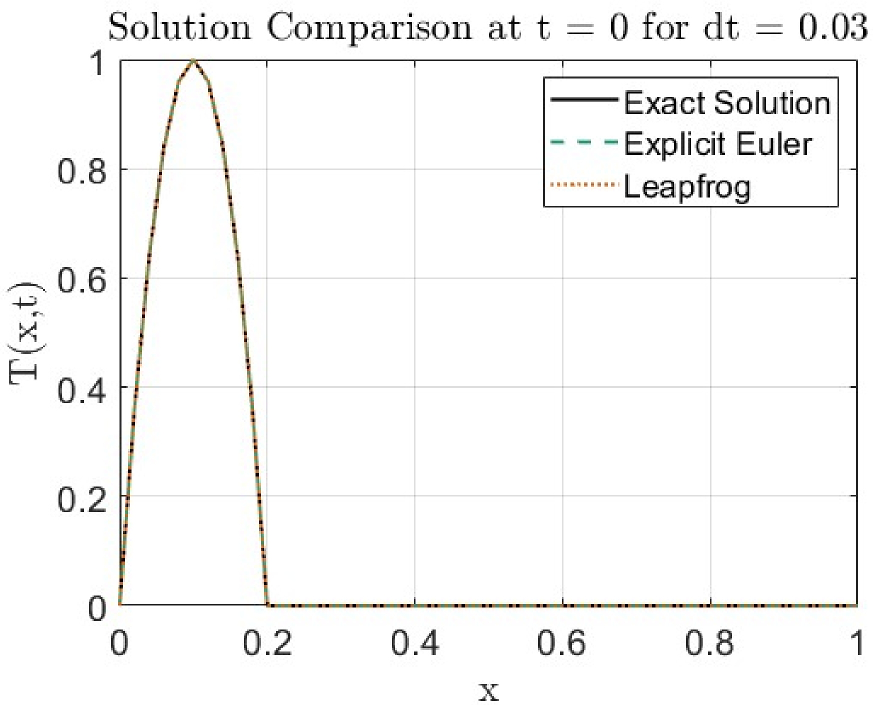
Problem 9 Chapter 5 Part a

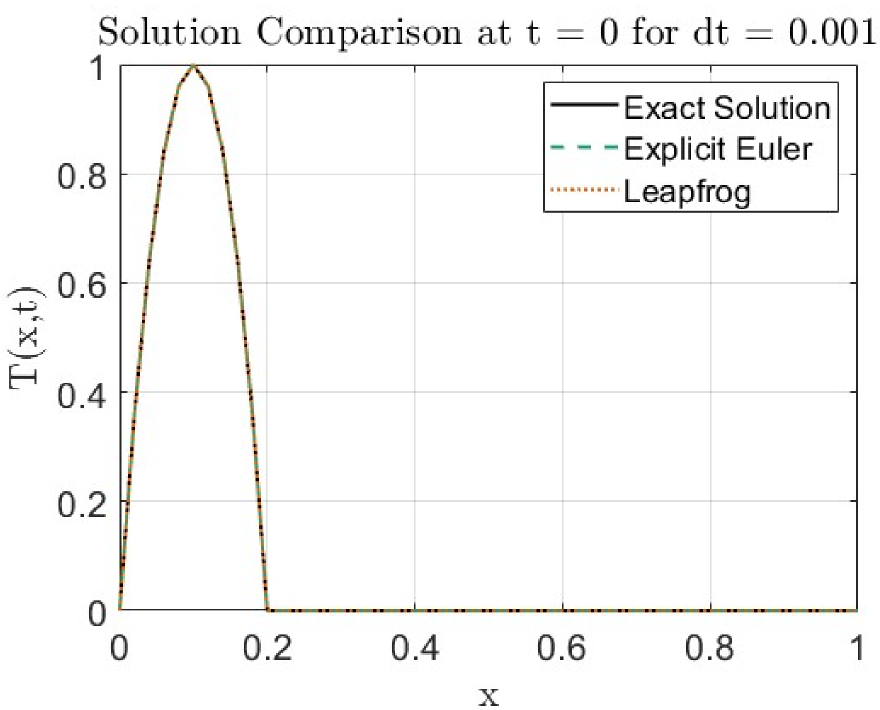
```
L = 1;                  % Length of the domain (0 ≤ x ≤ 1)
```

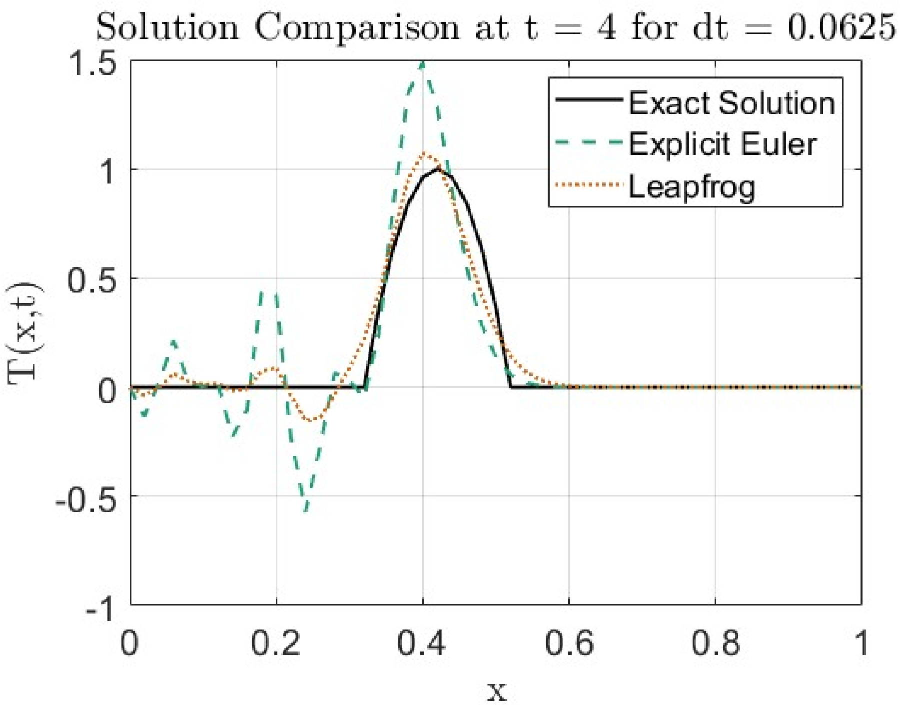
```
Nx = 51;                % Number of spatial points
dx = L / (Nx - 1);      % Spatial step size
x = linspace(0, L, Nx); % Spatial grid
u = 0.08;               % Convection velocity
dt_values = [0.0625, 0.03, 0.001];
t_end = 8;              % End time

% Exact solution function
exact_solution = @(x, t, u) (x - u * t >= 0 & x - u * t <= 0.2) .* (1 - (10 * (x - u * t) - 1).^2);

% Initial condition
T = zeros(Nx, 1);
for i = 1:Nx
    if x(i) <= 0.2
        T(i) = 1 - (10 * x(i) - 1)^2;
    end
end


% Loop through different time steps and plot solutions
for t = [0, 4, 8]
    for k = 1:length(dt_values)
        dt = dt_values(k);
        T_num_e = T;  % Solution for Explicit Euler
        T_num_l = T;  % Solution for Leapfrog

        Nt = round(t / dt); % Number of time steps

        % Time marching using Explicit Euler scheme
        for n = 1:Nt
            T_new_e = T_num_e;  % Temporary array to store updated values

            % Update T using the central difference for spatial derivative
            for i = 2:Nx-1
                T_new_e(i) = T_num_e(i) - (u * dt / (2 * dx)) * (T_num_e(i+1) - T_num_e(i-1));
            end

            % Boundary Conditions: Zero at the endpoints
            T_new_e(1) = 0;
            T_new_e(Nx) = 0;

            % Update the solution for the next time step
            T_num_e = T_new_e;
        end

        T_prev = T;
        for n = 1:Nt
            if n == 1
                T_new_l = T_num_l;
                for i = 2:Nx-1
                    T_new_l(i) = T_num_l(i) - (u * dt / (2 * dx)) * (T_num_l(i+1) - T_num_l(i-1));
                end
                T_prev = T_num_l;  % Save first step for Leapfrog
            else
                T_new_l = T_prev;
                for i = 2:Nx-1
                    T_new_l(i) = T_prev(i) - (u * dt / dx) * (T_num_l(i+1) - T_num_l(i-1));
                end
                T_prev = T_num_l;  % Update previous time step
            end

            % Boundary Conditions: Zero at the endpoints
            T_new_l(1) = 0;
            T_new_l(Nx) = 0;

            % Update the solution for the next time step
            T_num_l = T_new_l;
```

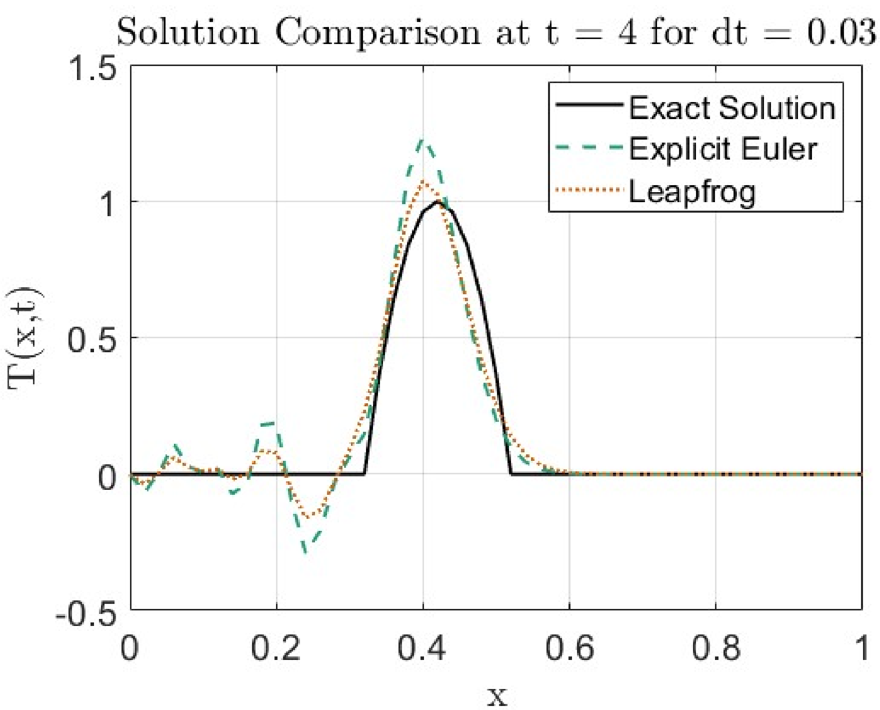
```
        end

        % Plot the results for both methods along with the exact solution
        T_exact = exact_solution(x, t, u);  % Exact solution
        figure
        plot(x, T_exact, '-', 'LineWidth', 1.5, 'Color',colors(1));
        hold on
        plot(x, T_num_e, '--', 'LineWidth', 1.5, 'Color',colors(2));
        plot(x, T_num_l, ':', 'LineWidth', 1.5, 'Color',colors(3));
        title(['Solution Comparison at t = ', num2str(t), ' for dt = ', num2str(dt)]);
        xlabel('x')
        ylabel('T(x,t)')
        legend('Exact Solution', 'Explicit Euler', 'Leapfrog');
        grid on;
        print(gcf,[imagepath,'Solution',num2str(t),'_',num2str(dt),'.png'],'-dpng');

    end

end
```

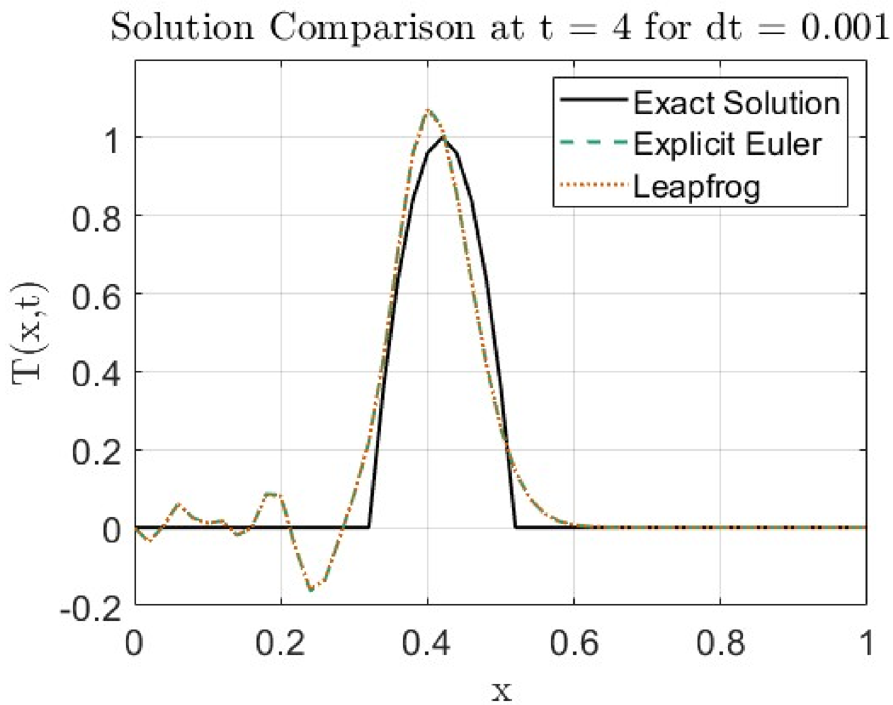


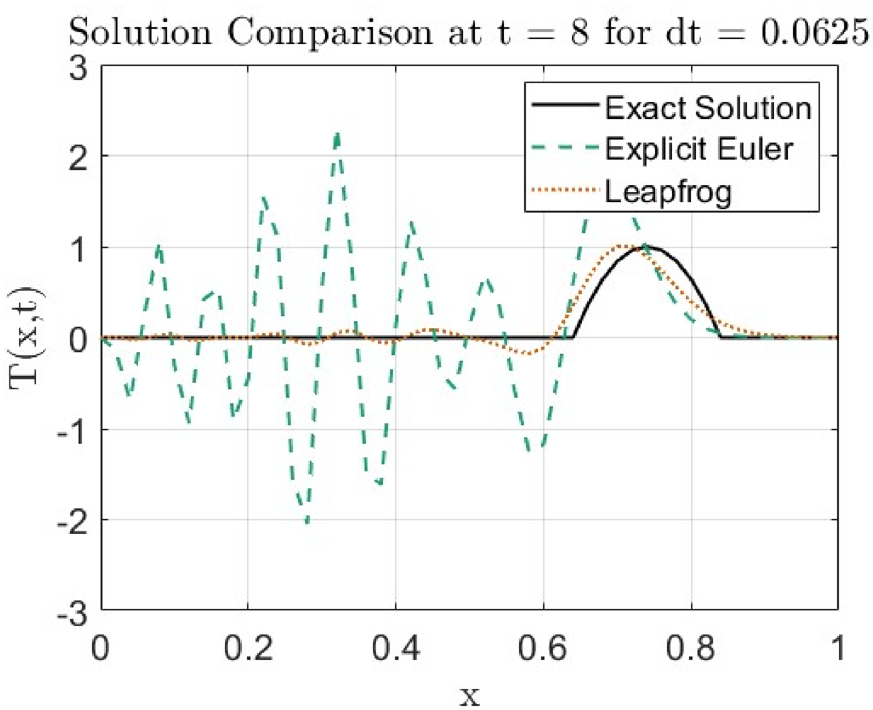

Solution Comparison at t = 0 for dt = 0.03
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
0
0.2
0.4
0.6
0.8
1

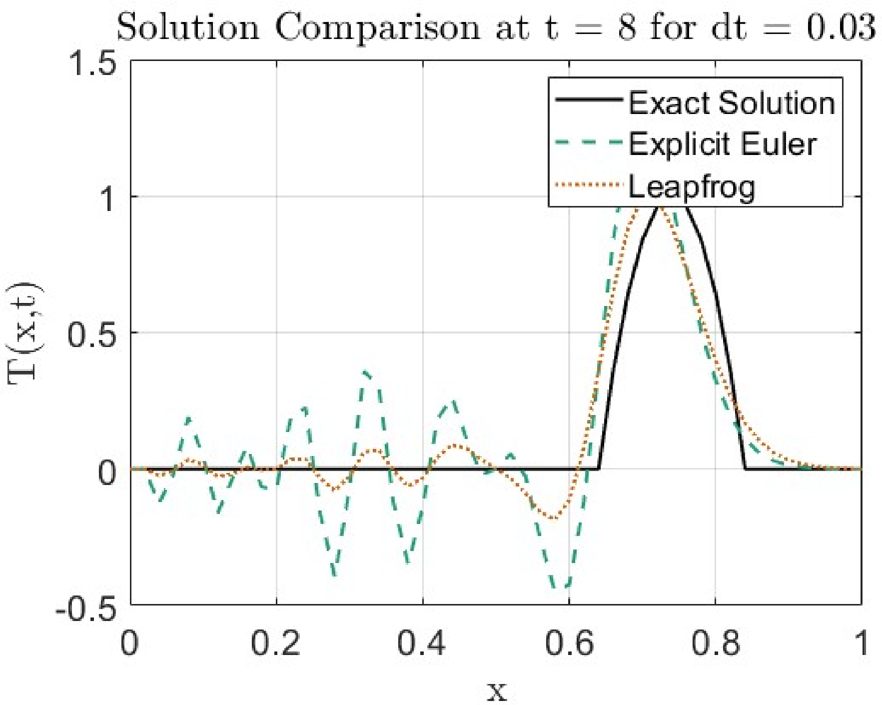

Solution Comparison at t = 0 for dt = 0.001
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
0
0.2
0.4
0.6
0.8
1

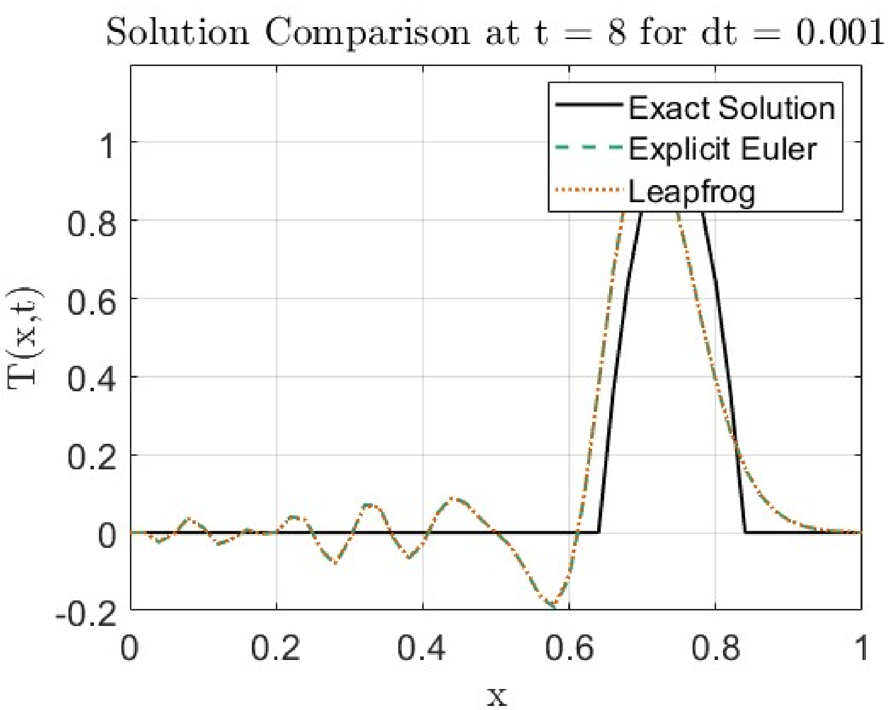

Solution Comparison at t = 4 for dt = 0.0625
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
1


Solution Comparison at t = 4 for dt = 0.03
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
1.5
1
0.5
0
-0.5
0
0.2
0.4
0.6
0.8
1


Solution Comparison at t = 4 for dt = 0.001
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x


Solution Comparison at t = 8 for dt = 0.0625
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x

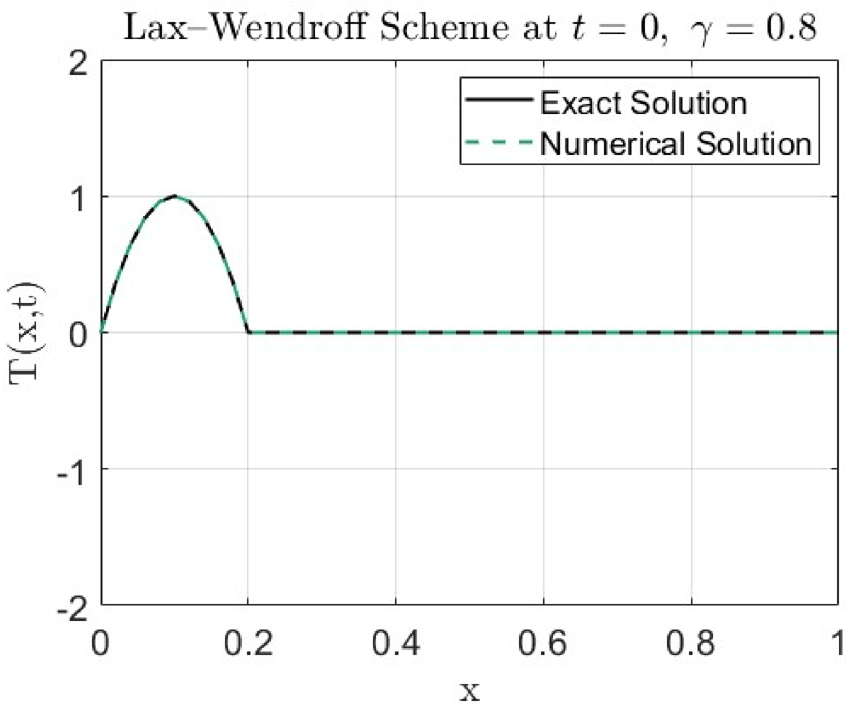


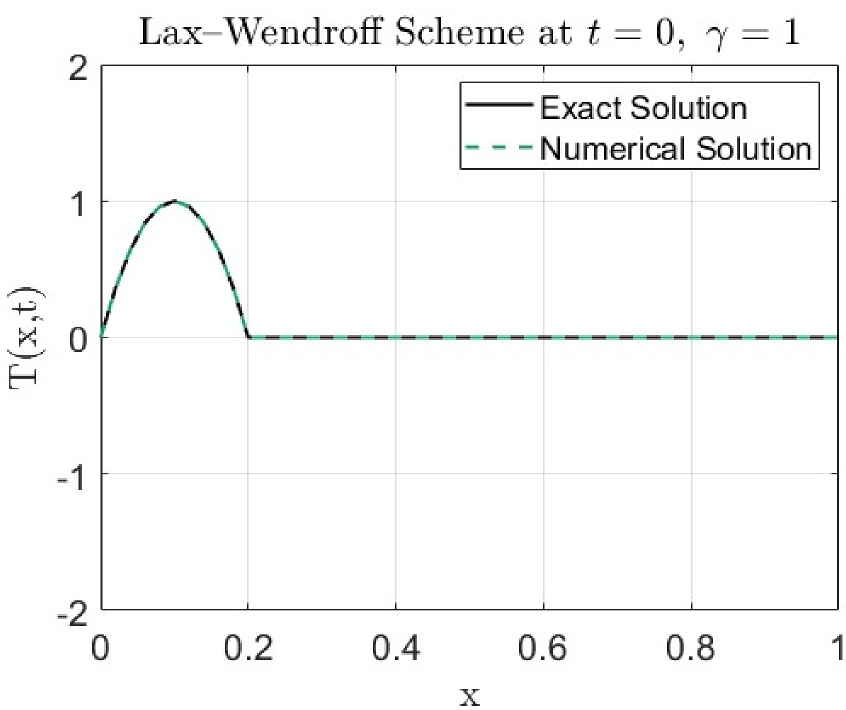


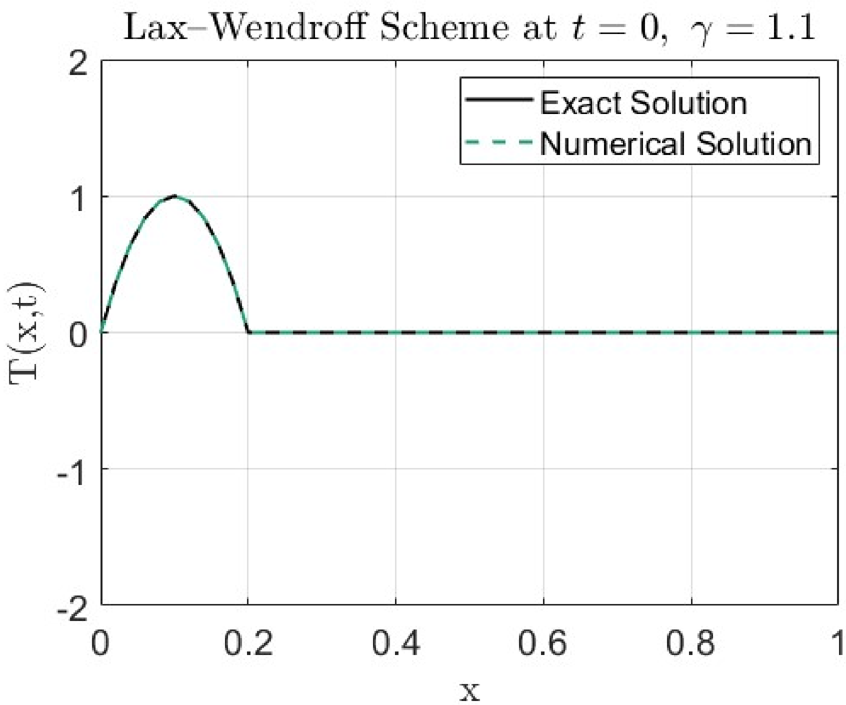
Problem 9 Chapter 5 Part c

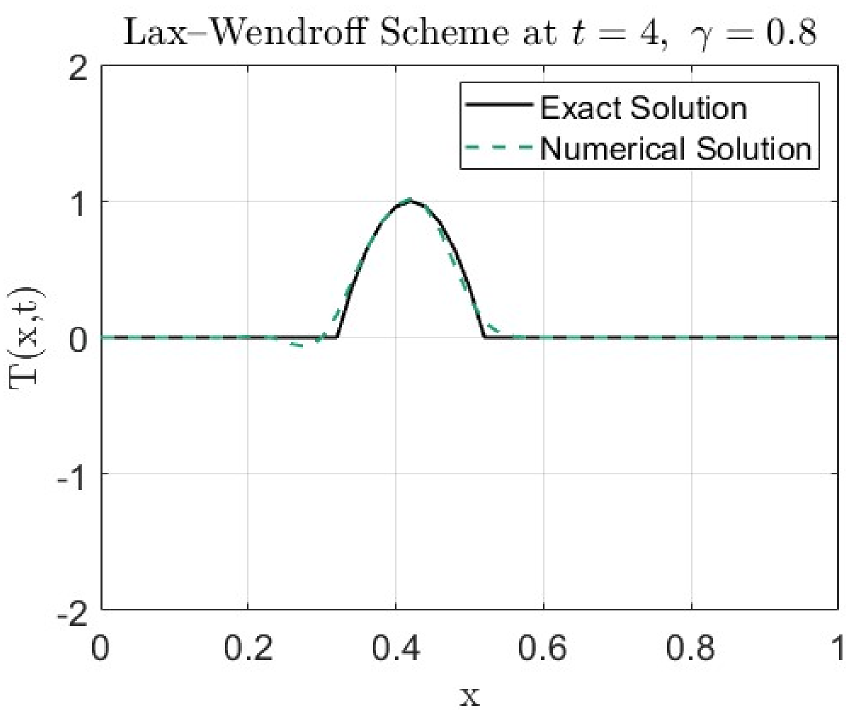
```
L = 1;                    % Length of the domain (0 ≤ x ≤ 1)
Nx = 51;                  % Number of spatial points
dx = L / (Nx - 1);        % Spatial step size
x = linspace(0, L, Nx);  % Spatial grid
u = 0.08;                 % Convection velocity
dt_values = [0.8, 1, 1.1] * dx / u;  % Time steps based on gamma
t_end = 8;                % End time

% Exact solution function
exact_solution = @(x, t, u) (x - u * t >= 0 & x - u * t <= 0.2) .* (1 - (10 * (x - u * t) - 1).^2);

% Initial condition
```

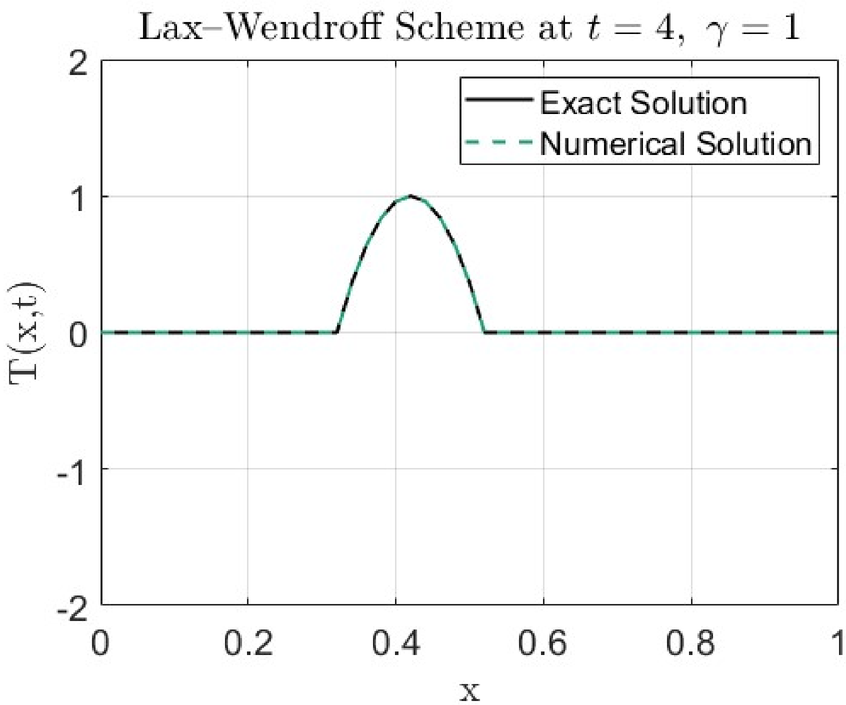
```
T = zeros(Nx, 1);
for i = 1:Nx
    if x(i) <= 0.2
        T(i) = 1 - (10 * x(i) - 1)^2;
    end
end

% Loop through different time steps (gamma values) and plot solutions
for t = [0, 4, 8]
    for k = 1:length(dt_values)
        dt = dt_values(k);
        gamma = u * dt / dx;  % Courant number
        T_num = T;       % Solution array
        Nt = round(t / dt);   % Number of time steps

        % Time marching using Lax-Wendroff scheme
        for n = 1:Nt
            T_new = T_num;  % Temporary array to store updated values

            for i = 2:Nx-1
                T_new(i) = T_num(i) ...
                         - 0.5 * gamma * (T_num(i+1) - T_num(i-1)) ...
                         + 0.5 * gamma^2 * (T_num(i+1) - 2*T_num(i) + T_num(i-1));
            end

            % Boundary Conditions: Zero at the endpoints
            T_new(1) = 0;
            T_new(Nx) = 0;

            % Update the solution for the next time step
            T_num = T_new;
        end

        % Plot the results for each gamma value
        T_exact = exact_solution(x, t, u);  % Exact solution
        figure
        plot(x, T_exact, '-', 'LineWidth', 1.5, 'Color', colors(1));
        hold on
        plot(x, T_num, '--', 'LineWidth', 1.5, 'Color', colors(2));
        title(['Lax--Wendroff Scheme at $t = ', num2str(t), ', \ \gamma = ', num2str(gamma), '$'], 'Interpreter', 'latex');
        xlabel('x');
        ylabel('T(x,t)');
        ylim([-2 2])
        legend('Exact Solution', 'Numerical Solution');
        grid on
        print(gcf,[imagepath,'C_',num2str(t),'_',num2str(gamma),'.png'],'-dpng');

    end
end
```


Lax–Wendroff Scheme at $t = 0,\ \gamma = 0.8$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1

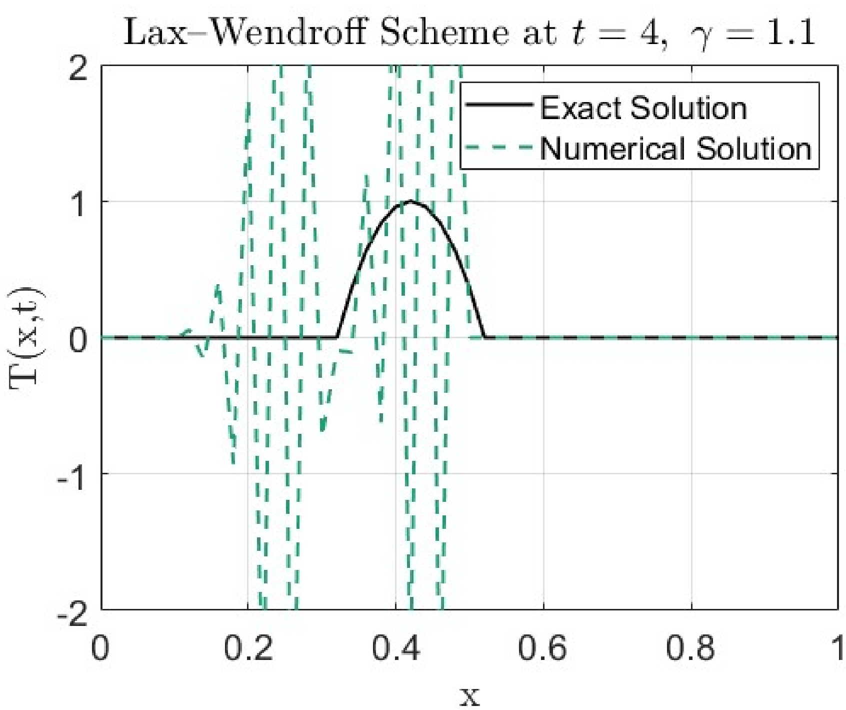

Lax–Wendroff Scheme at $t = 0,\ \gamma = 1$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1


Lax–Wendroff Scheme at $t = 0,\ \gamma = 1.1$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1

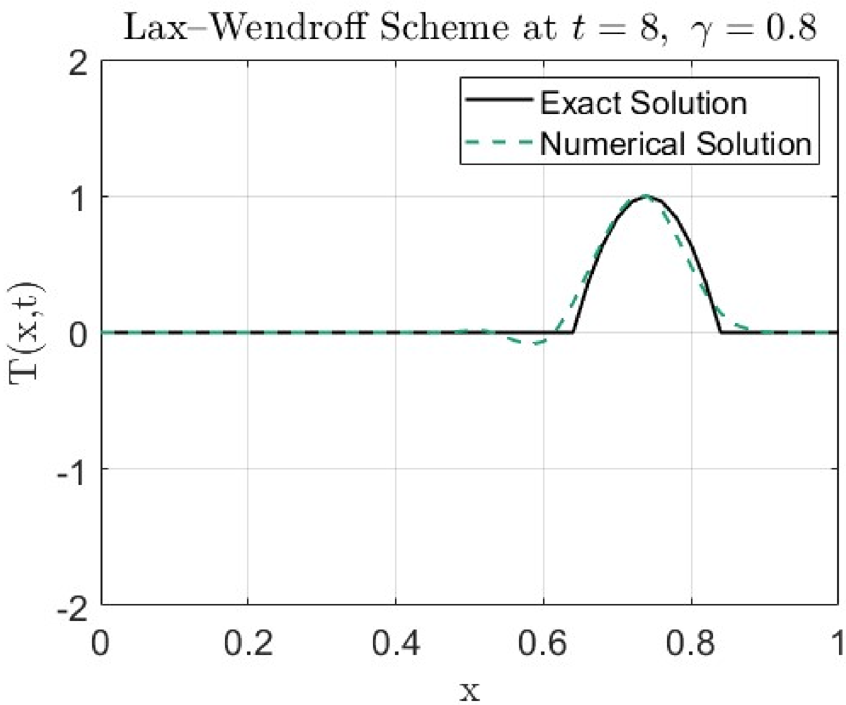

Lax–Wendroff Scheme at $t = 4,\ \gamma = 0.8$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1

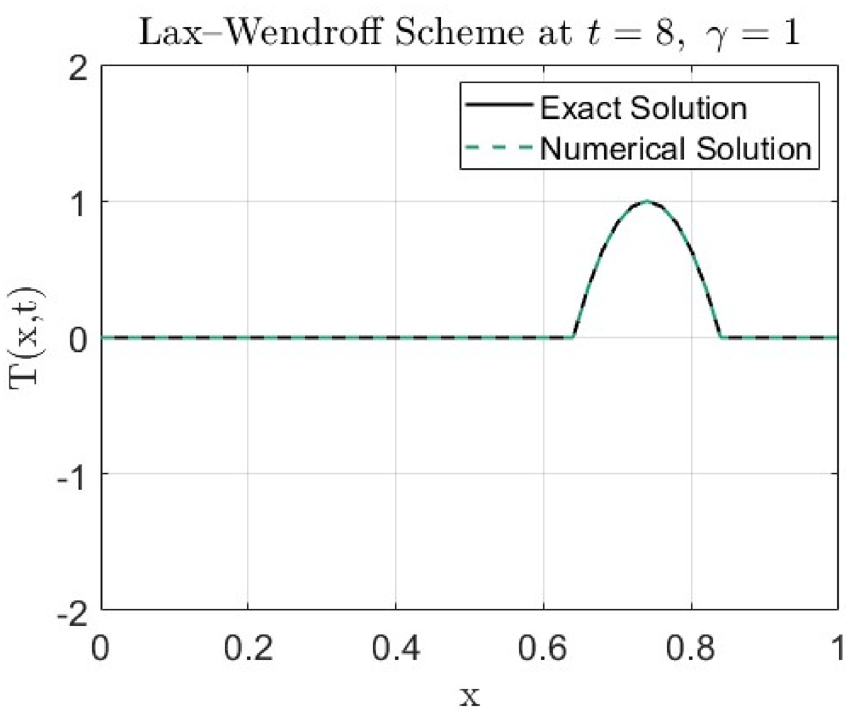


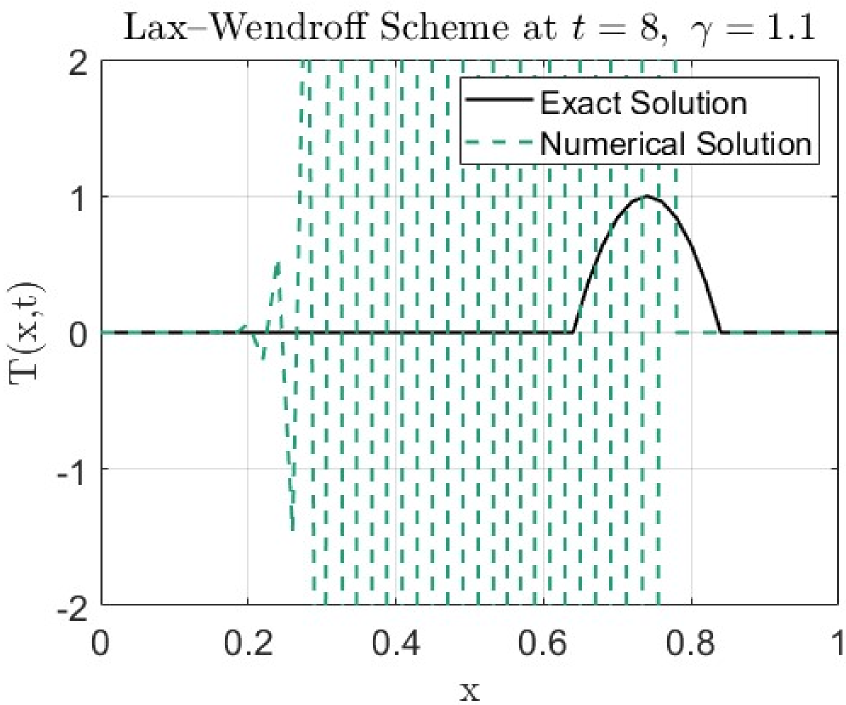
Lax–Wendroff Scheme at $t = 4,\ \gamma = 1$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1



Lax–Wendroff Scheme at $t = 4,\ \gamma = 1.1$
Exact Solution
Numerical Solution
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1


Lax–Wendroff Scheme at $t = 8,\ \gamma = 0.8$
Exact Solution
Numerical Solution
T(x,t)
x

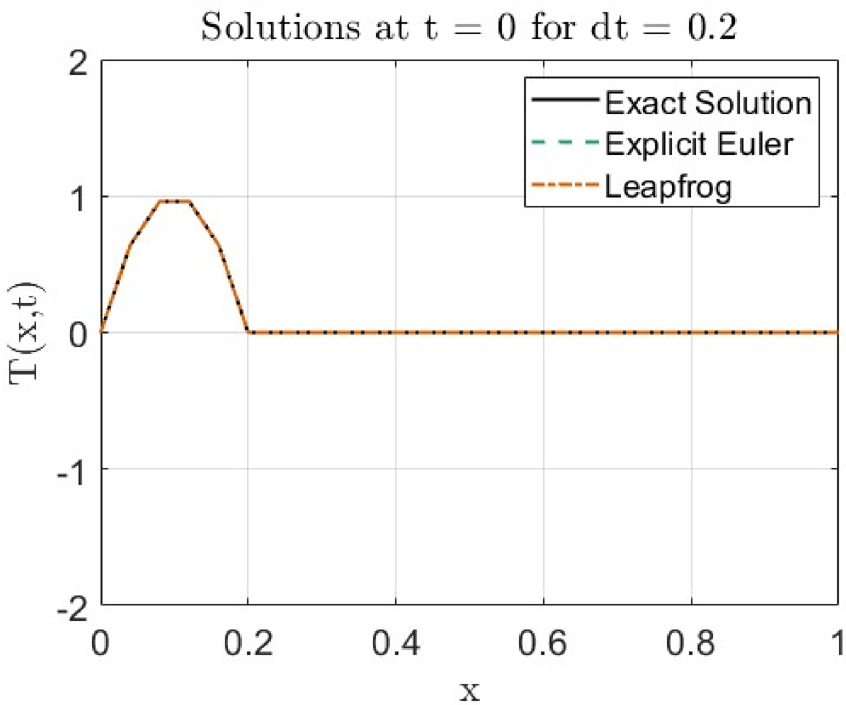

Lax–Wendroff Scheme at $t = 8,\ \gamma = 1$
Exact Solution
Numerical Solution
T(x,t)
x



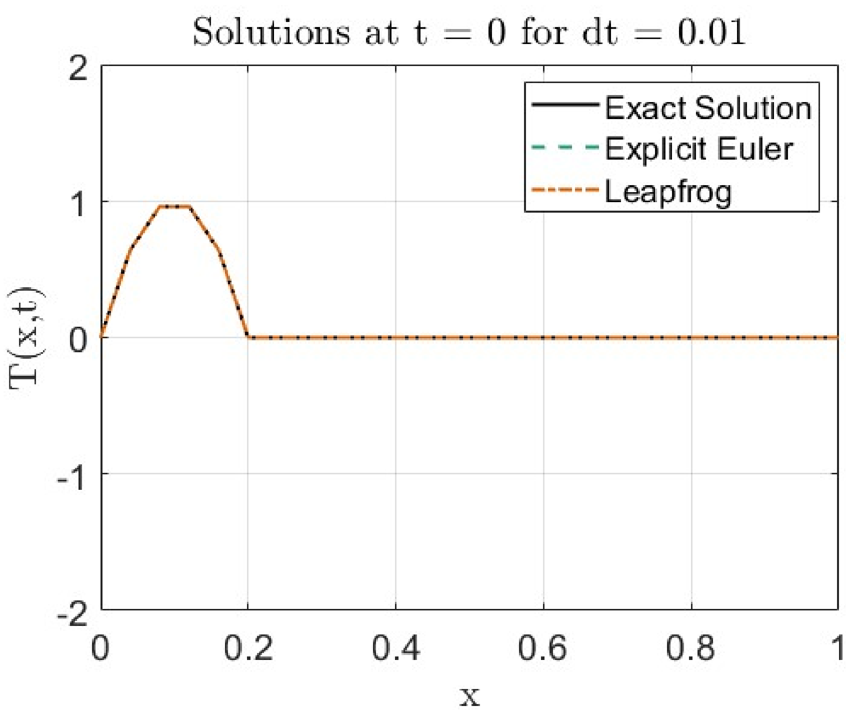
Problem 9 Chapter 5 Part d

```
% Parameters
L = 1;                    % Length of the domain (0 ≤ x ≤ 1)
Nx = 26;                  % Number of spatial points
dx = L / (Nx - 1);        % Spatial step size
x = linspace(0, L, Nx);   % Spatial grid
u = 0.08;                 % Convection velocity
alpha = 0.001;            % Diffusion coefficient
dt_values = [0.2, 0.01, 0.00001];  % Time step values
t_end = 8;                % End time

% Exact solution function
exact_solution = @(x, t, u) (x - u * t >= 0 & x - u * t <= 0.2) .* (1 - (10 * (x - u * t) - 1).^2);

% Initial condition
T = zeros(Nx, 1);
for i = 1:Nx
    if x(i) <= 0.2
        T(i) = 1 - (10 * x(i) - 1)^2;
    end
end

for t = [0 4 8]
for k = 1:length(dt_values)
    dt = dt_values(k);
    Nt = round(t / dt);  % Number of time steps
    gamma = u * dt / (2*dx);
    beta = alpha * dt / dx^2;

    % Initialize solutions
    T_e = T;
    T_l = T;
    T_prev = T;

    % Time-stepping loop
    for n = 1:Nt
        % Explicit Euler update
        T_new_e = T_e;
        for j = 2:Nx-1
```

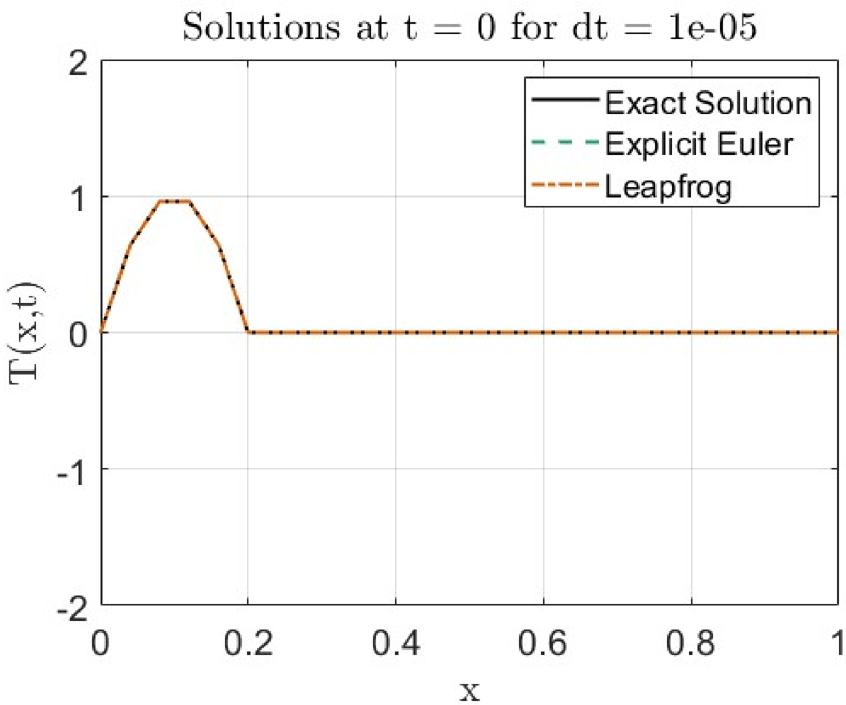
```
            T_new_e(j) = T_e(j) - gamma * (T_e(j+1) - T_e(j-1)) + beta * (T_e(j+1) - 2 * T_e(j) + T_e(j-1));
        end
        % Apply boundary conditions
        T_new_e(1) = 0;
        T_new_e(Nx) = 0;
        T_e = T_new_e;
    end

    for n = 1:Nt
        if n == 1
            T_new_l = T_l;
            for j = 2:Nx-1
                T_new_l(j) = T_l(j) -  gamma * (T_l(j+1) - T_l(j-1)) + beta * (T_l(j+1) - 2 * T_l(j) + T_l(j-1));
            end
            T_prev = T_l;  % Save first step for Leapfrog
        else
            T_new_l = T_prev;
            for j = 2:Nx-1
                T_new_l(j) = T_prev(j)  - 2*gamma * (T_l(j+1) - T_l(j-1)) + 2* beta * (T_l(j+1) - 2 * T_l(j) + T_l(j-1));
            end
            T_prev = T_l;  % Update previous time step
        end

        % Boundary Conditions: Zero at the endpoints
        T_new_l(1) = 0;
        T_new_l(Nx) = 0;

        % Update the solution for the next time step
        T_l = T_new_l;
    end
        % Plot solution at selected times

        T_exact = exact_solution(x,t,u);
        figure;
        plot(x, T_exact, '-', 'LineWidth', 1.5, 'Color', colors(1))
        hold on;
        plot(x, T_e, '--', 'LineWidth', 1.5, 'Color', colors(2));
        plot(x, T_l, '-.', 'LineWidth', 1.5, 'Color', colors(3));
        title(['Solutions at t = ', num2str(t), ' for dt = ', num2str(dt)])
        xlabel('x')
        ylabel('T(x,t)')
        legend('Exact Solution', 'Explicit Euler', 'Leapfrog');
        grid on
        ylim([-2 2])
        xlim([0,1])
        print(gcf,[imagepath,'D_',num2str(t),'_',num2str(gamma),'.png'],'-dpng');

end

end
```

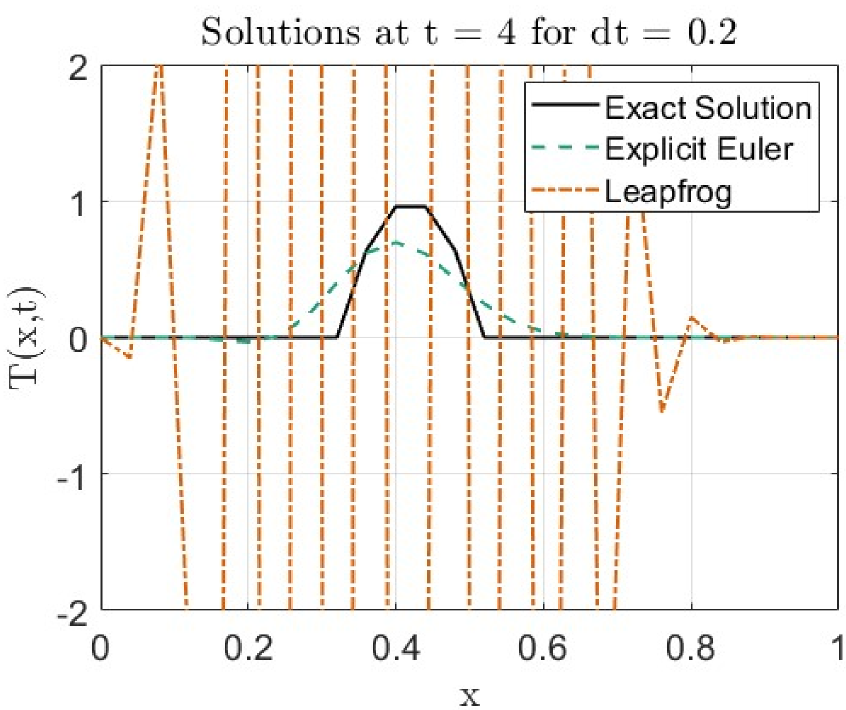


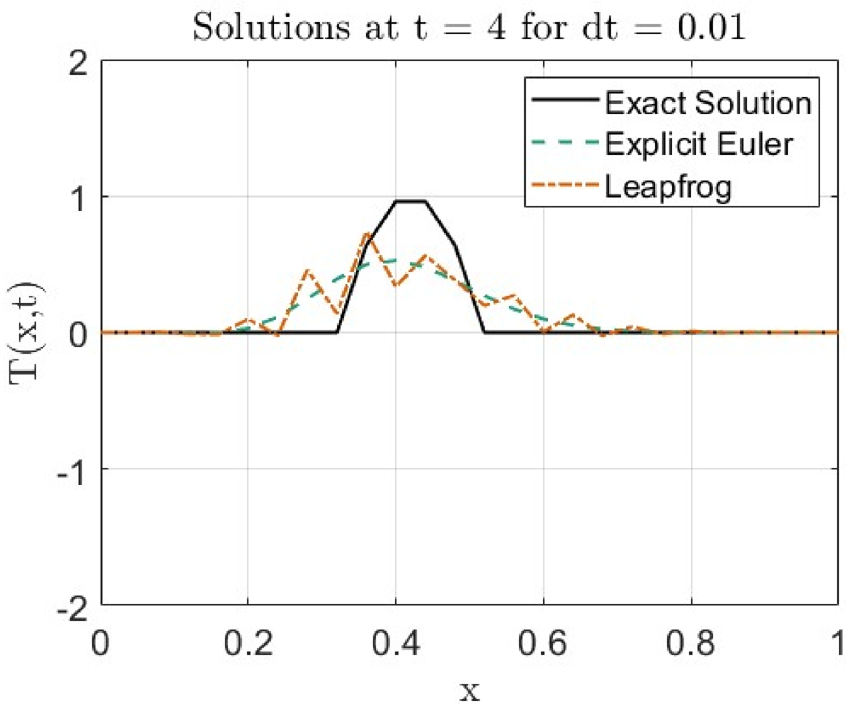
Solutions at t = 0 for dt = 0.2
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x



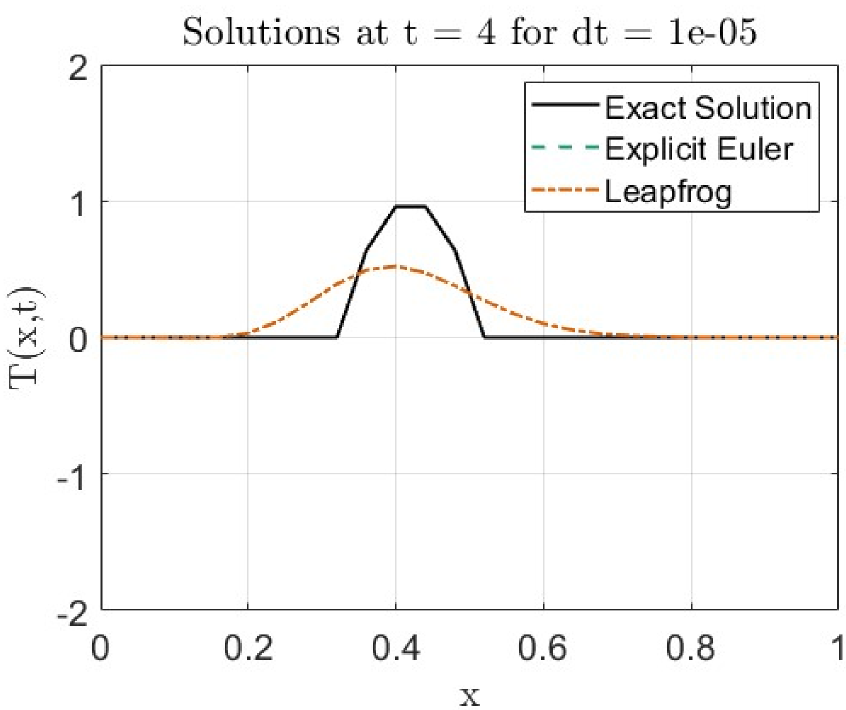
Solutions at t = 0 for dt = 0.01
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x


Solutions at t = 0 for dt = 1e-05
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1

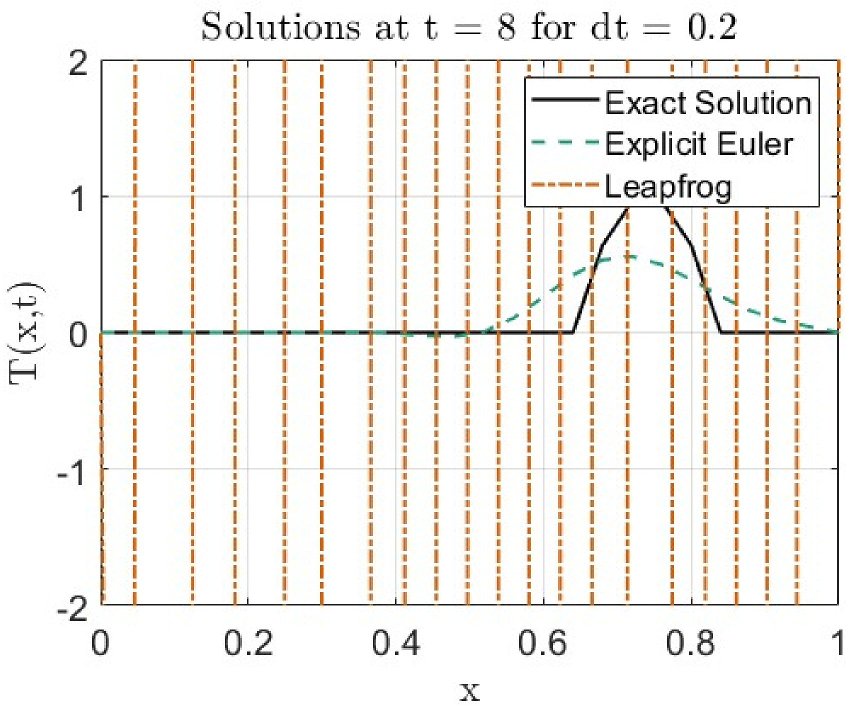

Solutions at t = 4 for dt = 0.2
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1

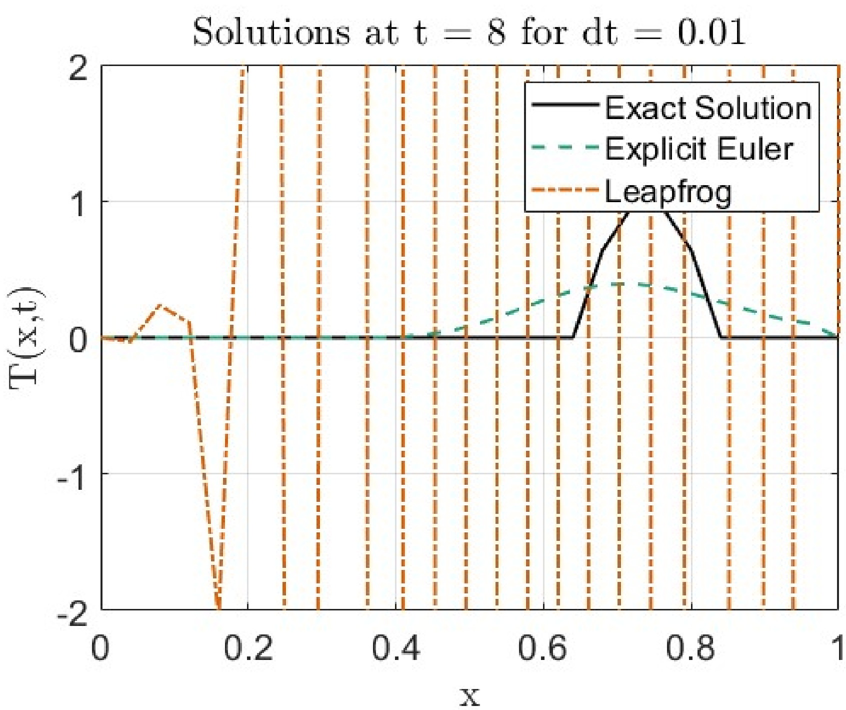


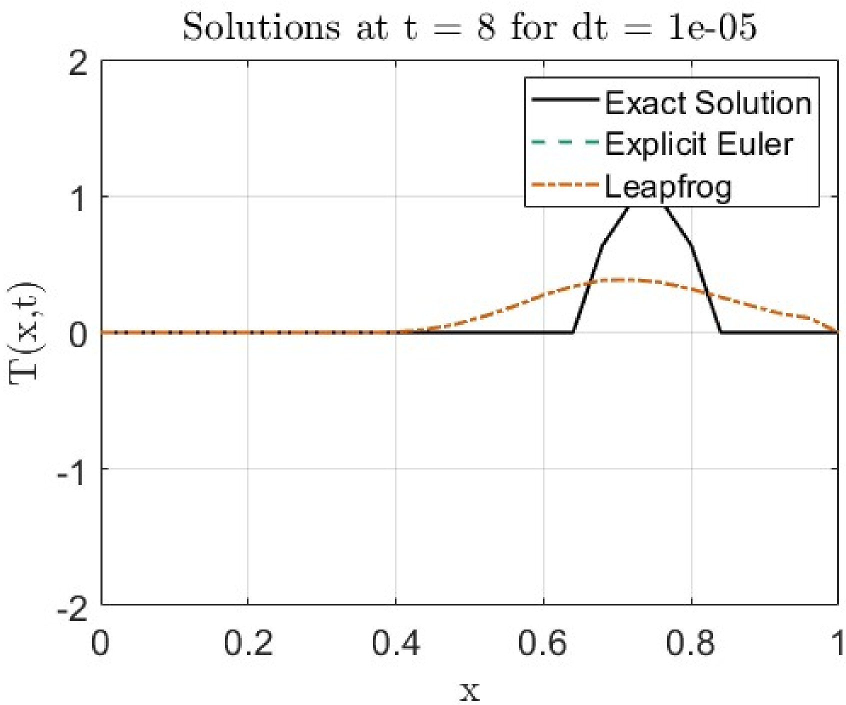
Solutions at t = 4 for dt = 0.01
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x



Solutions at t = 4 for dt = 1e-05
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x


Solutions at t = 8 for dt = 0.2
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1


Solutions at t = 8 for dt = 0.01
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1


Solutions at t = 8 for dt = 1e-05
Exact Solution
Explicit Euler
Leapfrog
T(x,t)
x
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1