LINEAR MODELS SESSION III

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Least Squares Problem

What is "Least"

RELATIONSHIPS USING

VARIANCES AND COVARIANCES!

MATH - ENGLISH - ART!

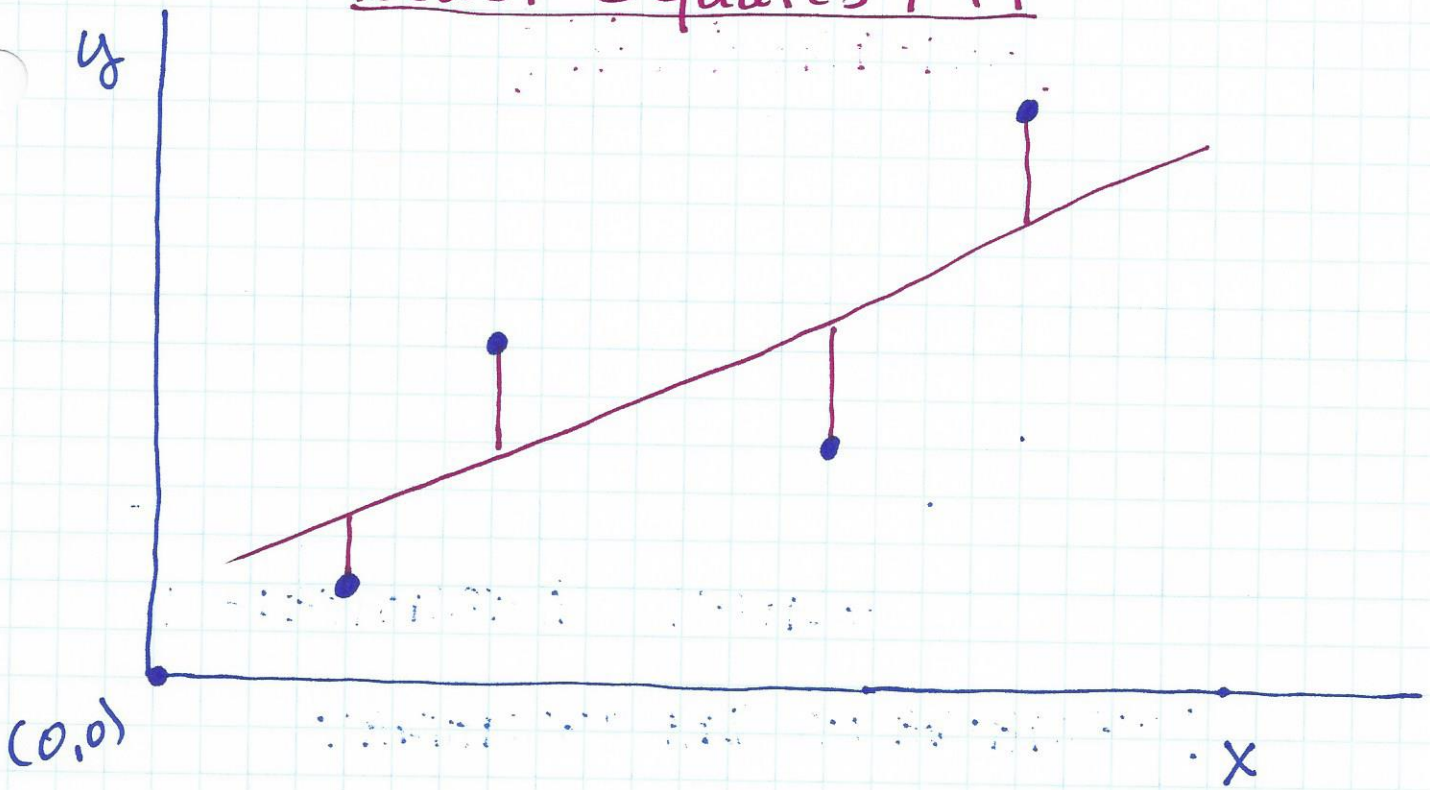
PERCEPTRONS

MACHINE LEARNING

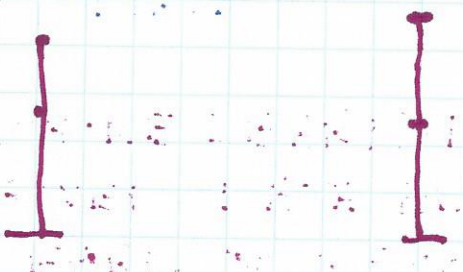
GIVEN DATA AND TRUTH

LEARN TO RECOGNIZE COMPLEX INPUT

Least Squares Fit

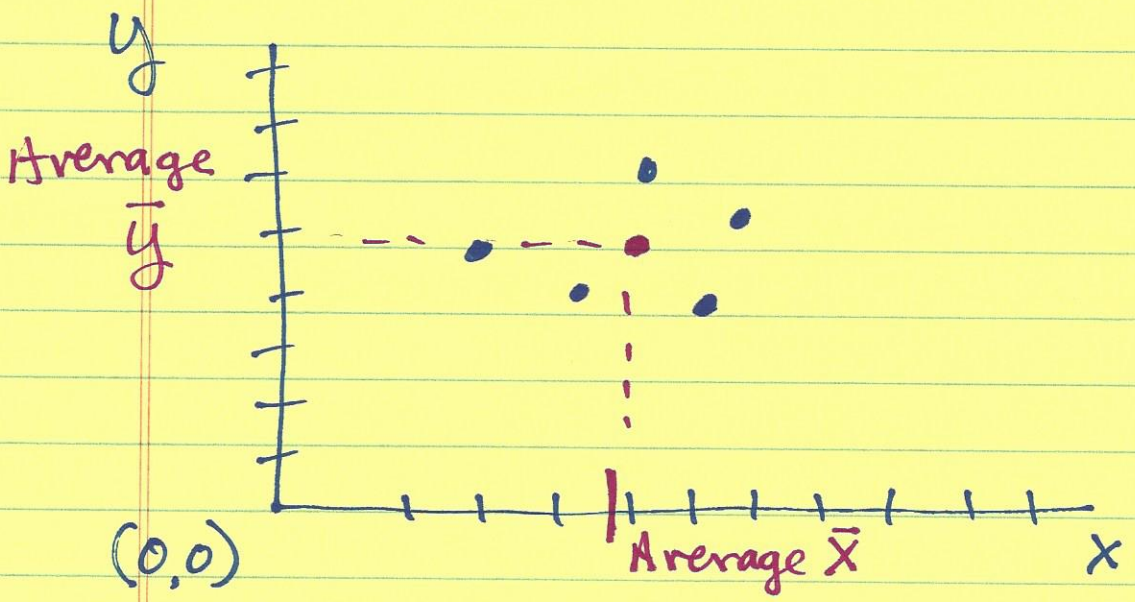


NEGATIVE POSITIVE



These are equal in "Least Square" SOLUTION

VARIANCE - COVARIANCE



Point Cluster

X values have average value.

Y values have average value.

VARIANCE - COVARIANCE

VARIANCE in X.

$$\frac{1}{N}\sum \left(x_i - \bar{x}_m\right)^2$$

$$\bar{x}_m = \frac{\sum x_i}{N} \Rightarrow \text{average X value}$$

VARIANCE in Y.

$$\frac{1}{N}\sum \left(y_i - \bar{y}_m\right)^2$$

$$\bar{y}_m = \frac{\sum y_i}{N} \Rightarrow \text{average Y value}$$

Variances

DISTANCE of X Points from Mean

$|X - \bar{X}|$ = DISTANCE

Y POINTS from Mean

$|Y - \bar{Y}|$ = DISTANCE

$$\text{Variance} = \frac{\sum \text{DISTANCES}^2}{N}$$

COVARIANCES

$$\frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{N} = \text{COVARIANCE}$$

USE X Versus Y Products

Example of Covariances

STUDENT	MATH	ENGLISH	ART
1	90	60	90
2	90	90	30
3	60	60	60
4	60	60	90
5	30	30	30

MATRIX DATA - AVERAGE VALUES

$$\begin{bmatrix} 90 & 60 & 90 \\ 90 & 90 & 30 \\ 60 & 60 & 60 \\ 60 & 60 & 90 \\ 30 & 30 & 30 \end{bmatrix} - \begin{bmatrix} 66 & 60 & 60 \\ 66 & 60 & 60 \\ 66 & 60 & 60 \\ 66 & 60 & 60 \\ 66 & 60 & 60 \end{bmatrix} = C$$

$$C = \begin{bmatrix} 24 & 0 & 30 \\ 24 & 30 & -30 \\ -6 & 0 & 0 \\ -6 & 0 & 30 \\ -36 & -30 & -30 \end{bmatrix}$$

$$C^T C = \begin{bmatrix} 24 & 24 & -6 & -6 & -36 \\ 0 & 30 & 0 & 0 & -30 \\ 30 & -30 & 0 & 30 & -30 \end{bmatrix} \times \begin{bmatrix} 24 & 0 & 30 \\ 24 & 30 & -30 \\ -6 & 0 & 0 \\ -6 & 0 & 30 \\ -36 & -30 & -30 \end{bmatrix} =$$

$$C^T C = \begin{bmatrix} 2520 & 1800 & 900 \\ 1800 & 1800 & 0 \\ 900 & 0 & 3600 \end{bmatrix}$$

SYMMETRIC!

COVARIANCE MATRIX $\div 5$

$$\begin{bmatrix} 504 & 360 & 180 \\ 360 & 360 & 0 \\ 180 & 0 & 720 \end{bmatrix}$$

$$\begin{bmatrix} VAR & COV & COV \\ COV & VAR & COV \\ COV & COV & VAR \end{bmatrix}$$

VARIANCE - COVARIANCE

	MATH	ENG.	ART
MATH	504	360	180
ENG	360	360	0
ART	180	0	720

DOES ENGLISH

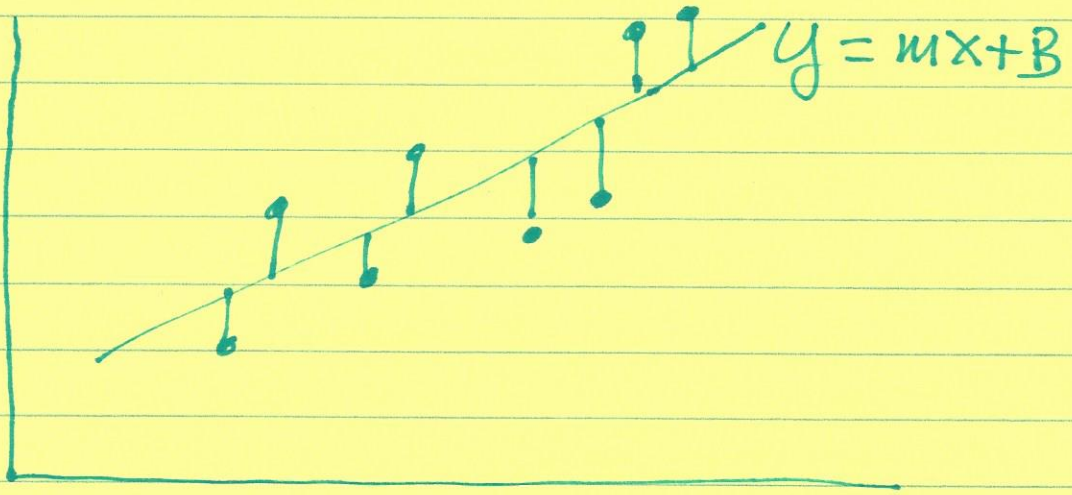
AND

ART

RELATE ?

NO PREDICTABLE RELATIONSHIP!

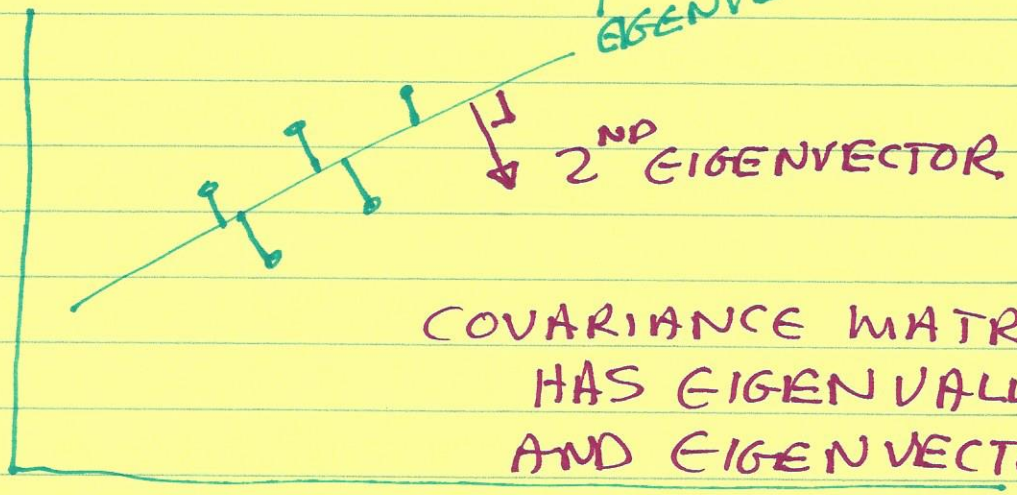
LEAST SQUARES



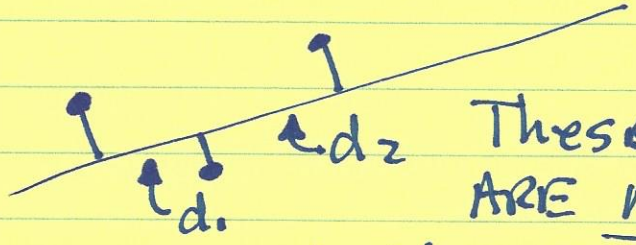
$$\sum (+\text{Error}) + (-\text{Error}) = 0$$

"LeasT"

EIGENVECTOR



COVARIANCE MATRIX
HAS EIGENVALUES
AND EIGENVECTORS



These Distances ARE MAXIMUM!

MAX!