Dubins and Reeds-Shepp paths

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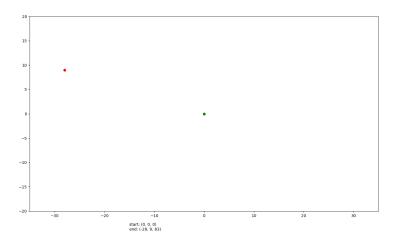


Figure: The shortest path in the case of a human being

Introduction

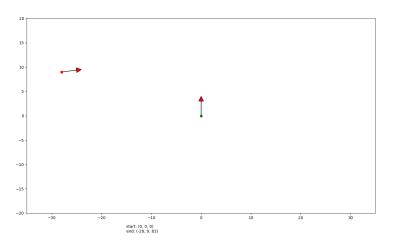


Figure: A shortest path in the case of an oriented human being

Introduction

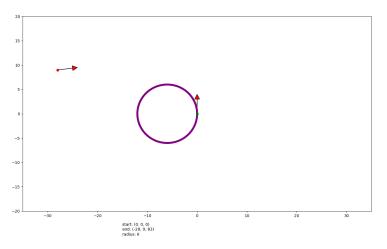


Figure: Robot's minimal turning circle

Dubins' paths

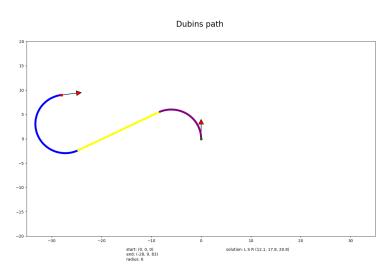


Figure: Dubins' path descriebing a shortest path

Dubins' paths





(b) All combinaisons of CCC paths

(a) All combinaisons of CSC paths

C: means "curve" (i.e. turning left or right)

S: means "straight"

R: means "right" (i.e. turning right)

L: means "left" (i.e. turning left)

Dubins' paths

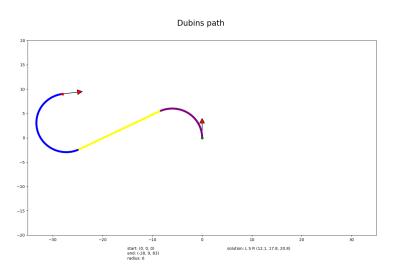


Figure: Dubins' path LSR descriebing the shortest path

1. Constant complexity

- 1. Constant complexity
- 2. Use of a table on angles

- 1. Constant complexity
- 2. Use of a table on angles
- 3. Outer and inner tangents

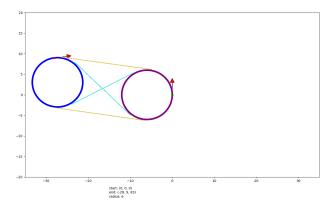


Figure: Same radius internal and external circles tangents

$$\dot{x} = \cos \theta$$

$$\dot{\mathbf{y}} = \sin \theta$$

$$\dot{\theta} = \frac{1}{r_{min turning}}$$

$$ightharpoonup x_{new} = x_{prev} + \delta * cos(\theta)$$

$$ightharpoonup y_{new} = y_{prev} + \delta * sin(\theta)$$

With δ a small value, typically below 0.05.

Reeds-Shepp paths

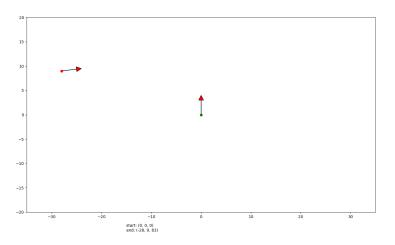


Figure: What is a shortest path for a robot able to go backward?

Reeds-Shepp paths

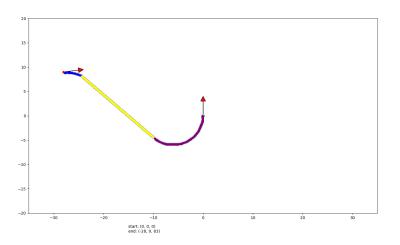


Figure: A shortest Reeds-Shepp path for a robot able to go backward

Reeds-Shepp paths

explicit	(1,1) form	(1.2) form	Section 8 formula
I*r-I*	C+C-C+	CICIC	
	C-C+C-		(8.3), two roots
r*l-r*	C+C-C+	CICIC	(8.3), two roots
	C-C+C-		(8.3), two roots
r-l+r-		CICIC	(8.3), two roots
I*r~I~	C+C-C-	CICC	(8.4), two roots
l-r+l+	C-C+C+	CICC	(8.4), two roots
r+l-r-	C+C-C-	CICC	(8.4), two roots
r-l+r+	C-C+C+	CICC	(8.4), two roots
1-r-1+	c-c-c+	ccic	(8.4), two roots
1+++1-	C*C*C-	CCIC	(8.4), two roots
r-1-r+	C-C-C+	ccic	
			(8.4), two roots
r+I+r-	C*C*C-	CCIC	(8.4), two roots
l+r+l-r-	C+C+CC-	CC _u IC _u C	(8.7), two roots
l-r=ul+r+	C-C-C+C+	CC, IC,C	(8.7), two roots
r+l+r-u1-	C+C+CC-	CC_IC_C	(8.7), two roots
r-l-urul+	C~C_uCuC+	CC _u IC _u C	(8.7), two roots
l+r-ul-ur+	C+C-,C-,C+	CIC_C_IC	(8.8)
l-r+l+r-	C-C+C+C-	CIC,C,IC	(8.8)
r+1-ar-al+	C+C=_C=_C+	CIC,C,IC	(8.8)
r-l*r*l-	C-C+C+C-	CIC,C,IC	(8.8)
I+r_m2s-I-	C+C=n/2S=C=	CIC _{n/2} SC	(8.9)
l"r#/25*l*	$C^{-}C_{\pi/2}^{+}S^{+}C^{+}$	CIC _{m/2} SC	(8.9)
r+L_12s-r-	C+C-m/2S-C-	CIC _{B/2} SC	(8.9)
r-l+128+r+	$C^-C^+_{\pi/2}S^+C^+$	CIC _{n/2} SC	(8.9)
l's'r_mol*	C-S-CC+	CSC #21C	(8,9)
	C+S+C+2C-		(8.9)
l+s+r+12l-		CSC #12 I C	
r-s-l- ₂₂ r+	C S C _ 1/2 C +	CSC w2 IC	(8.9)
r+s+l+12r-	C*S*C*2C-	CSC #12 I C	(8.9)
I+r-m/25-r-	C+C-R/2S-C-	CIC x/2SC	(8.10)
I"r#125+r+	$C^-C^*_{\pi/2}S^*C^*$	CIC _{m/2} SC	(8.10)
r*I=n/2s"1"	C*C-4/2S-C-	$C \mid C_{\pi/2}SC$	(8.10)
$r^{-}l_{\pi/2}^{+}s^{+}l^{+}$	C-C**2S+C+	$C \mid C_{\pi/2}SC$	(8.10)
r-s-r-4/2l+	C-S-C-22C+	CSC _{x/2} C	(8.10)
r+s+r+12l	C+S+C+2C-		(8.10)
	C"S"C=2C*	CSC #/2 IC	
1"s"1_=12r+		CSC m/2 IC	(8.10)
l+s+l+12r-	C+S+C+v2C-	CSC _{R/2} IC	(8.10)
l+s+r+	C+S+C+	CSC	(8.2)
1's'r'	C-S-C-	CSC	(8.2)
r*s*l*	C+S+C+	CSC	(8.2)
r-s-l-	C-S-C-	CSC	(8.2)
l*s*l*	C*S*C*	CSC	(8.1)
1-3-1-	C-S-C-	CSC	(8.1)
r+s+r+	C+S+C+	CSC	(8.1)
r+s+r+ r-s-r-	C+S+C+ C-S-C-	CSC	(8.1)
r+s+r+ r-s-r- l+r-z/2s-l-z/2r+	C+S+C+ C-S-C- C+C- _{8/2} S-C- _{8/2} C+	CSC C1C _{m2} SC _{m2} 1C	(8.1) (8.11), two roots
r*s*r* r*s*r* l*r=z2s*l=z2r* l*r*z2s*l*z7r*	C+S+C+ C-S-C- C+C- _{\pi/2} S-C- _{\pi/2} C+ C-C+ _{\pi/2} S+C+ _{\pi/2} C-	CSC C1C ₁₁₂ SC ₁₁₂ 1C C1C ₁₁₂ SC ₁₁₂ 1C	(8.1) (8.11), two roots (8.11), two roots
r+s+r+ r-s-r- l+r-z/2s-l-z/2r+	C+S+C+ C-S-C- C+C- _{8/2} S-C- _{8/2} C+	CSC C1C _{m2} SC _{m2} 1C	(8.1) (8.11), two roots

Reeds-Shepp paths: timeflip simplification

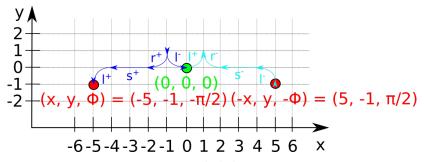


Figure: Timeflip simplification: $I^-r^+s^+I^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $I^+r^-s^-I^-$ goes from (0, 0, 0) to $(-x, y, -\phi)$

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Reeds-Shepp paths: timeflip simplification

	(1.1) 6	1 000	
explicit	(1.1) form	(1.2) form	Section 8 formula
I+r-I+	C+C-C+	CICIC	(8.3), two roots
1-+1-	C"C*C"	CICIC	(8.3), two roots
r*l-r*	C+C-C+	CICIC	(8.3), two roots
r-l+r-	C-C+C-	CICIC	(8.3), two roots
I*r~I~	C+C-C-	CICC	(8.4), two roots
1"r+1+	C-C+C+	CICC	(8.4), two roots
r+l-r-	C+C-C-	CICC	(8.4), two roots
+-1+++	C-C+C+	CICC	(8.4), two roots
1-r-t+	C-C-C+	CCIC	(8.4), two roots
l+r+l-	C*C*C-	CCIC	(8.4), two roots
-1-+	C-C-C+	CCIC	(8.4), two roots
r+l+r-	C+C+C-	ccic	(8.4), two roots
l+r+l-r-	C+C+CC-	CC, IC,C	(8.7), two roots
		CCMICKC	
l-r-ulur+	C-C-C+C+	CC _u IC _u C	(8.7), two roots
r+l+r-ul-	C+C+CC-	CC_u IC_uC	(8.7), two roots
r-l-urul+	C-C-uC+C+	CC_IC_C	(8.7), two roots
l+rl_r+	C+C-,C-,C+	CIC _H C _H IC	(8.8)
1-r+1+r-	C-C+C+C-	CICECIIC	(8.8)
$r^{+}l^{-}_{-u}r^{-}_{-u}l^{+}$	C+C=_C=_C+	CIC,C,IC	(8.8)
r-1+r+1-	C C C C C	CICACAIC	(8.8)
l+r_m2s-l-	C+C-n/2S"C"	CIC _{n/2} SC	(8.9)
I"r#125"1"	C C 228 C*	C I C R/2 SC	(8.9)
r+L _{x/2} s-r-	C*C-n/2S-C-	$C \mid C_{\pi/2}SC$	(8.9)
r-1*x/25+r+	C-C*25+C+	CIC _{R/2} SC	(8.9)
157-8/21*	C-S-C-5/2C+	ccc 1c	(8.9)
		CSC = 2 I C	
$l^{+}s^{+}r_{\pi r 2}^{+}l^{-}$	C+S+C+2C	CSC ₁₁₂ 1C	(8.9)
r s l-w2r+	C S C - n/2 C +	CSC _{E/2} I C	(8.9)
$r^+s^+l_{\pi/2}^+r^-$	C*S*C*v2C-	CSC w2 IC	(8.9)
I+r-m/25-r-	C+C-8/2S-C-	$C \mid C_{\pi/2}SC$	(8.10)
I"r#25*r*	C-C*25*C*	CIC E/2SC	(8.10)
r*1-225-1-	C*C-5/25-C-	CICEZSC	(8.10)
r-1+23+1+	C-C*/2S*C*	CIC _{E/2} SC	(8.10)
1 18020	0 0 _{8/2} 5 0	C 10 g/200	(0.10)
r-s-r-4/2(+	C-S-C-5/2C+	CSC w2+C	(8.10)
r+s+r+12l-	C+S+C+2C-	CSC w/2 IC	(8.10)
13 1-22	C S C - 1/2 C *	CSC 1/2 C	(8.10)
	Ctstct C-		
l+s+l+12r-	C+S+C+12C-	CSC _{n/2} I C	(8.10)
l+s+r+	C+S+C+	CSC	(8.2)
1-s-r-	C-S-C-	CSC	(8.2)
r+s+l+	C*S*C*	CSC	(8.2)
r"s"l"	C-S-C-	CSC	(8.2)
1*s*1*	C*S*C*	CSC	(8.1)
131	C-S-C-	CSC	(8.1)
r+s+r+	C+S+C+	CSC	(8.1)
r-s-r-	C-S-C-	CSC	(8.1)
r_z/25 - 1-z/2r	$C^+C^{-\pi/2}S^-C^{-\pi/2}C^+$	$C \mid C_{\pi i 2} S C_{\pi i 2} \mid C$	(8.11), two roots
1-r+128+1+12r-	$C^{-}C_{8/2}^{+}S^{+}C_{8/2}^{+}C^{-}$	CICWSCWIC	(8.11), two roots
+1=x/2s=r=x/2l+	C+C-=2S-C-=2C+	CIC #2SC #2IC	(8.11), two roots
r-1025+r021-	C-C+2S+C+2C-	CICWSCWIC	(8.11), two roots
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Reeds-Shepp paths: reflect simplification

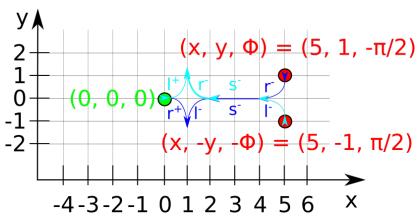


Figure: Reflect simplification: $r^+I^-s^-r^-$ goes from the (0, 0, 0) to (x, y, ϕ) and $I^+r^-s^-I^-$ goes from (0, 0, 0) to $(x, -y, -\phi)$

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Reeds-Shepp paths: reflect simplification

	(1.1) (1 000	Section 8 formula
explicit	(1.1) form	(1.2) form	
l+r-l+	C*C-C*	CICIC	(8.3), two roots
1-+1-	C"C*C"	CICIC	(8.3), two roots
r*!-r*	C+C-C+	CICIC	(8.3), two roots
r-1+r-	C-C+C-	CICIC	(8.3), two roots
I*r~I~	C+C-C-	CICC	(8.4), two roots
1"r+1+	C-C+C+	CICC	(8.4), two roots
r+1-r-	C+C-C-	CICC	(8.4), two roots
+-1+++	C-C+C+	CICC	(8.4), two roots
			(,,,
11+	C-C-C+	CCIC	(8.4), two roots
l+r+l-	C*C*C-	CCIC	(8.4), two roots
+-1-+	C-C-C+	CCIC	(8.4), two roots
r+I+r-	C+C+C-	CCIC	(8.4), two roots
l+r+lr-	C+C+CC-	CC _u 1C _u C	(8.7), two roots
l-r-ulur+	C-C-C+C+	CC, IC, C	(8.7), two roots
r+1+r-1-	C+C+CC-	CC _u C _u C	(8.7), two roots
r-1-urul+	C-C-uC+C+	CC, IC, C	(8.7), two roots
l+r-ul-ur+	C+C-,C-,C+	CIC _H C _H IC	(8.8)
I-r#I#r-	C-C+C+C-	CICECIIC	(8.8)
r+1r1+	C+C=,C=,C+	CIC,C,IC	(8.8)
r-1*r*1-	C=C C C	CIC,C,IC	(8.8)
	C+C=n/2S=C=		
I+r_=025 I	C'C-#/25 C	CIC _{R/2} SC	(8.9)
1" r # 25 * 1 *	C C 22 S + C +	C I C N/2 SC	(8.9)
r+1-1025-r	C*C=n/2S-C-	C I C RIZSC	(8.9)
$r^{-}l_{\pi/2}^{+}s^{+}r^{+}$	C-C*v2S+C+	C I C R/2 SC	(8.9)
15 1-4/21	C-S-C-4/2C+	CSC v2 IC	(8,9)
1+s+r+n1-	C+S+C+2C-	CSC w21C	(8.9)
r s l = v2r+	C S C C C+	CSC W2 IC	(8.9)
r+s+l+2r-	C*S*C*v2C-	CSC wz IC	(8.9)
	C+C-425-C-		
I+r_m/25 r		$C \mid C_{\pi/2}SC$	(8.10)
1"r" 25"r"	C-C*28*C*	CIC _{π/2} SC	(8.10)
r*1-w25-1-	C*C=_mr2S=C=	CIC _{R/2} SC	(8.10)
r-1+ 1+1+	C-C**/2S*C*	CIC _{R/2} SC	(8.10)
r-s-r-s/2!*	C-S-C-5/2C+	CSC v2 IC	(8.10)
r*s*r*121	C*S*C*2C-	CSC x/2 IC	(8.10)
131-227	C S C - R/2 C	CSC m/2 I C	(8.10)
l+s+l+12r-	C+S+C+v2C-	CSC _{n/2} C	(8.10)
l+s+r+	C+S+C+	CSC	(8.2)
1-5-1	C-S-C-	CSC	(8.2)
r+s+1+	C*S*C*	CSC	(8.2)
r"s"l"	C-S-C-	CSC	(8.2)
I*s*I*	C*S*C*	CSC	(8.1)
137	C-5-C-	CSC	(8.1)
r*s*r*	C+S+C+	CSC	(8.1)
	C-S-C-		
+ s +		CSC	(8.1)
+r-z/25 - 1-z/2r+	C+C-mr2S-C-mr2C+	$C \mid C_{\varpi 2} S C_{\varpi 2} \mid C$	(8.11), two roots
1-r+12s+1+12r-	$C^{-}C^{+}_{\pi/2}S^{+}C^{+}_{\pi/2}C^{-}$	$C \mid C_{w_2} S C_{w_2} \mid C$	(8.11), two roots
+1-m25-r-m21+	$C^{+}C_{-\pi/2}^{-}S^{-}C_{-\pi/2}^{-}C^{+}$	C C 1/2 S C 1/2 C	(8.11), two roots
r=1*025*r*021-	$C^{-}C^{+}_{\pi/2}S^{+}C^{+}_{\pi/2}C^{-}$	CIC NO SC NO IC	(8.11), two roots

Reeds-Shepp paths: reverse simplification

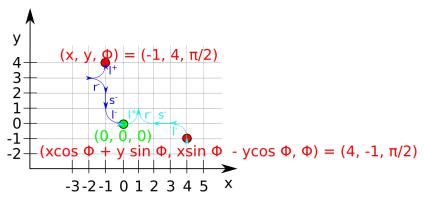


Figure: Reverse simplification: $l^-s^-r^-l^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+r^-s^-l^-$ goes from (0, 0, 0) to $(x\cos\phi + y\sin\phi, x\sin\phi - y\cos\phi, \phi)$

Reeds-Shepp paths: reverse simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
I*r=I*	C'C-C'	CICIC	(8.3), two roots
1-+1-	C"C*C"	CICIC	(8.3), two roots
r*1-r*	C+C-C+	CICIC	(8.3), two roots
r-1+r-	C-C+C-	CICIC	(8.3), two roots
I*r*I*	C+C-C-	CICC	(8.4), two roots
1-r+1+	C-C+C+	CICC	(8.4), two roots
y+1-y-	C+C-C-	CICC	(8.4), two roots
-1++	C-C+C+	CICC	(8.4), two roots
		0.00	(0.4), 140 10013
11+	C-C-C+	CCIC	(8,4), two roots
1+++-			
	C*C*C	CCIC	(8.4), two roots
+-1-++	C-C-C+	CCIC	(8.4), two roots
r+I+r-	C*C*C-	CCIC	(8.4), two roots
l*r"l="r"	C*C*C=_C-	CC, IC,C	(8.7), two roots
l-r-ulur+	C-C-C+C+C	CC_IC_C	(8.7), two roots
r+l+r-,l-	C+C+C-C-	CC_IC_C	(8.7), two roots
r-1-ur+1+	C-C-C+C+	CC_IC_C	(8.7), two roots
I+r-,I-,r+	C+C-,C-,C+	CIC_C_IC	(8.8)
1-r+1+r-	C-C'_C'_C	CIC,C,IC	(8.8)
r+1-ar-al+	C+C=_C=_C+	CIC,C,IC	(8.8)
r-1+r+1-	C"C"C"C"	CICHCHIC	(8.8)
I*r_m/25" ["	C+C=-nS=C=	CIC-nSC	(8.9)
1"7" 25" 1"	C "C + 2 S + C +	C I C WZ SC	(8.9)
r+1-025-r-	C*C-472S-C-	CICEZSC	(8.9)
r-l+25+r+	C-C+25+C+	C I C N/2 SC	(8.9)
/ 1g/20 /	C C#/25 C	C 10 8/200	(0.5)
15 7-8/21	C-S-CC*	CSC +12 C	(8.9)
1*5*/*/21	C*S*C*2C	CSC w2 IC	(8.9)
r-s-(-w)2r+			
	C*S*C*********************************	CSC w2 C	(8.9)
r+s+l+2r-	$C^*S^*C^*_{w_2}C^-$	CSC _{W2} IC	(8.9)
I+r_m/25 r	C+C-R/2S-C-	$C \mid C_{\pi/2}SC$	(8.10)
1"r" 25"r"	C-C*22S*C*	CIC _{E/2} SC	(8.10)
r*1-125-1-	C*C=e/2S=C=	CIC _{E/2} SC	(8.10)
r-1+123+1+	C-C+2S+C+	CICESC	(8.10)
F-5-F-1+	C-S-CC+	CSC ALC	(8-10)
r s r-x/2 (+	C*S*C*_C*	CSC JC	(8.10)
r*s*r*21	C*S*C**2C-	CSC x/2 I C	(8.10)
T*S*T*U2!	C*S*C**2C* C*S*C**2C*	CSC n/2 C	(8.10) (8.10)
r*s*r* _{x2} l* f*s*l _{x2} r* f*s*l _{x2} r*	C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₁₂ C*	CSC 112 C CSC 112 C CSC 112 C	(8.10) (8.10) (8.10)
T*S*T*U2!	C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₂₂ C*	CSC 10/2 C CSC 10/2 C CSC 10/2 C CSC	(8.10) (8.10) (8.10) (8.2)
r*s*r* _{x2} l* f*s*l _{x2} r* f*s*l _{x2} r*	C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₁₂ C*	CSC 112 C CSC 112 C CSC 112 C	(8.10) (8.10) (8.10)
r*s*r* ₁₀₂ r* f*s*l* ₂₀₂ r* f*s*l* ₂₀₂ r* f*s*r*	C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₂₂ C* C*S*C* ₂₂ C*	CSC 10/2 C CSC 10/2 C CSC 10/2 C CSC	(8.10) (8.10) (8.10) (8.2)
T*S*T**2! T*S*T**2?** T*S*T**2?** I*S*T** 	C*S*C*C C*S*C*C C*S*C*C C*S*C*C	CSC _{W2} C CSC _{W2} C CSC _{W2} C CSC CSC	(8.10) (8.10) (8.10) (8.2) (8.2)
r*s*r*2! !'s"!=2?* !*s"!=2?* !*s"!=? !'s"r" !"s"r" !"s"r" !"s"!"	C*S*C*2C* C*S*C*2C* C*S*C*2C* C*S*C*C* C*S*C* C*S*C* C*S*C* C*S*C*	CSC ₂₇₂ IC CSC ₂₇₂ IC CSC ₂₇₂ IC CSC CSC CSC CSC CSC	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2)
r*s*r* ₂₂ ! i*s*l* ₂₂ r* i*s*l* ₂₂ r* !*s*r* !*s*r* !*s*r* !*s*!*	C*S*C** C*S*C* C*S*C* C*S*C* C*S*C* C*S*C* C*S*C*	CSC 272 C CSC 272 C CSC 272 C CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2) (8.1)
**************************************	C*S*C*2C- C*S*C*2C- C*S*C*2C- C*S*C* C*S*C* C*S*C* C*S*C* C*S*C* C*S*C*	CSC w2 C CSC w2 C CSC w2 C CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2) (8.1)
**************************************	C*S*C***C-C*C*C*C*C*C*C*C*C*C*C*C*C*C*C*	CSC w2+C CSC w2+C CSC w2+C CSC CSC CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2) (8.1) (8.1)
**************************************	C*S*C*2C- C*S*C*2C- C*S*C*2C- C*S*C* C*S*C* C*S*C* C*S*C* C*S*C* C*S*C*	CSC w2 C CSC w2 C CSC w2 C CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2) (8.1)
**************************************	C*S*C***C-C*C*C*C*C*C*C*C*C*C*C*C*C*C*C*	CSC w2+C CSC w2+C CSC w2+C CSC CSC CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.2) (8.1) (8.1)
**************************************	C'51 C'32 C'32 C'51 C'51 C'51 C'51 C'51 C'51 C'51 C'51	CSC 221C CSC 221C CSC 221C CSC CSC CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.1) (8.1) (8.1) (8.1)
75 1 22 1 15 1 22 1 15 1 22 1 15 1 22 1 15 1 22 1 15 1 22 1 15 1 22 1 15 1 2	C'S' C' ₁₀ C- C'S' C' ₁₀ C- C'S' C' C'C'S' C' C'C' C'	CSC = 21 C CSC = 21 C CSC = 25 C CSC	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.1) (8.1) (8.1) (8.1), two roots (8.11), two roots
**************************************	C'51 C'32 C'32 C'51 C'51 C'51 C'51 C'51 C'51 C'51 C'51	CSC 221C CSC 221C CSC 221C CSC CSC CSC CSC CSC CSC CSC CSC CSC C	(8.10) (8.10) (8.10) (8.2) (8.2) (8.2) (8.2) (8.1) (8.1) (8.1) (8.1)

Reeds-Shepp paths: case $I_t^+ s_u^+ I_v^+$

- $(u,t) := R(x \sin \phi, y 1 + \cos \phi)$
- \triangleright $v := M(\phi t)$

With:

- $(r, \theta) := R(x, y)$ for the polar transform $r \cos \theta = x$ and $r \sin \theta = y$ with $r \le 0$ and $-\pi \le \theta < \pi$
- $lack \phi = M(heta)$ if $\phi \equiv \theta \mod 2\pi$ and $-\pi \le \phi < \pi$

Dubins and Reeds-Shepp paths in reality

▶ All kinds of sources for error in the real-world

Dubins and Reeds-Shepp paths in reality

- ▶ All kinds of sources for error in the real-world
- ▶ Paths found in the absence of obstacles however can use a rapidly exploring random tree and Dubins' pseudo distance or Reeds-Shepp distance

Sources

- ▶ Optimal paths for a car that goes both forwards and backwards (1990), J. A. Reeds and L. A. Shepp
- ▶ Planning algorithms: Reeds-Shepp curves (2006), Steven M. LaValle
- ► A Comprehensive, Step-by-Step Tutorial to Computing Dubins' Paths (2013), Andy G's Blog