Dubins and Reeds–Shepp paths

Benjamin Loison

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Introduction

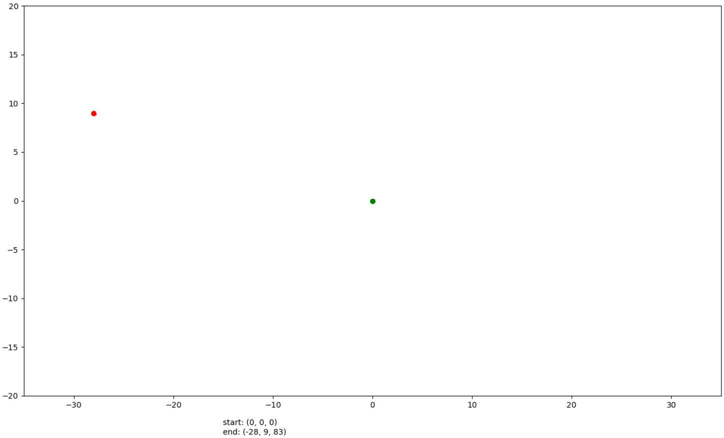

Figure: The shortest path in the case of a human being

Introduction

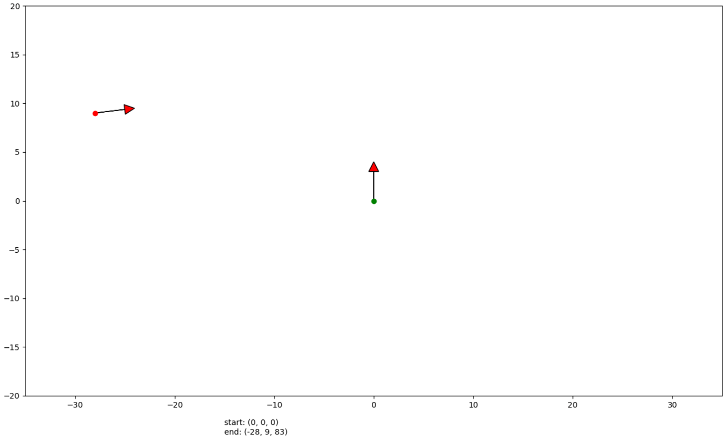

Figure: A shortest path in the case of an oriented human being

Introduction

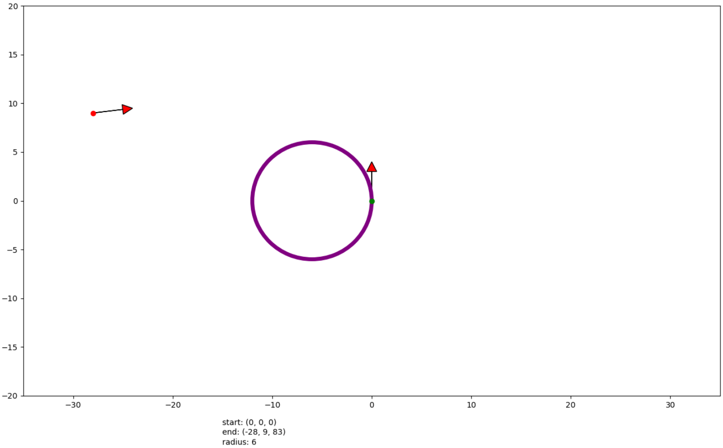

Figure: Robot's minimal turning circle

Dubins' paths

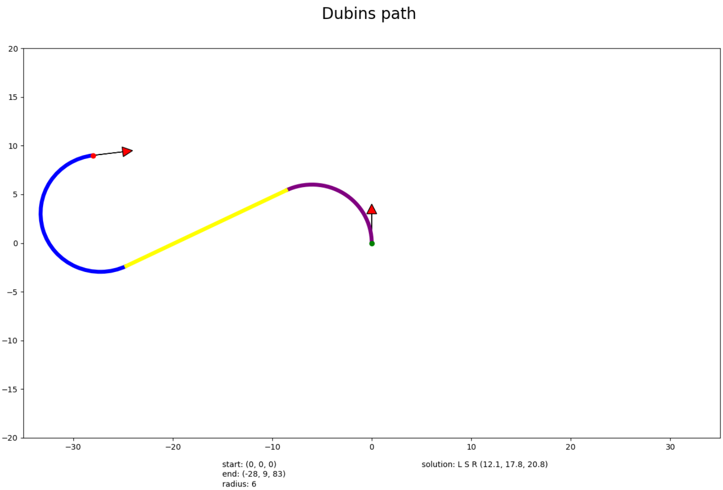

Figure: Dubins' path descriebing a shortest path

Dubins' paths

CSC
RSR
LSL
LSR
RSL

(a) All combinaisons of **CSC** paths

CCC
RLR
LRL

(b) All combinaisons of **CCC** paths

C: means "curve" (i.e. turning left or right)
S: means "straight"
R: means "right" (i.e. turning right)
L: means "left" (i.e. turning left)

Dubins' paths

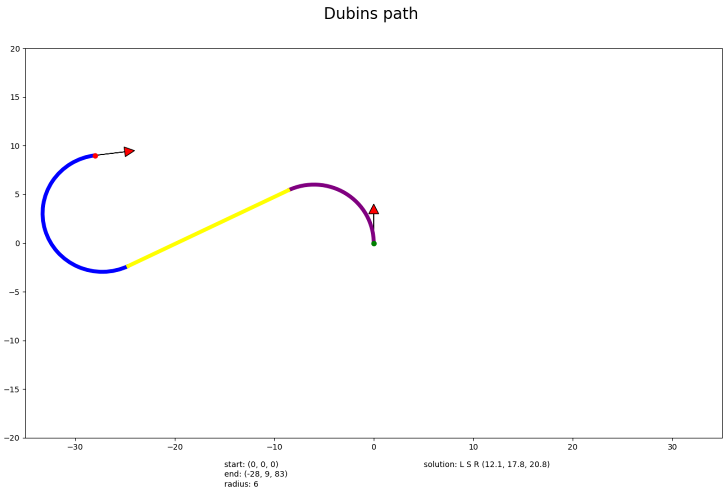

Figure: Dubins' path LSR descriebing the shortest path

Implementation details

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1. Constant complexity

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2. Use of a table on angles

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1. Constant complexity
2. Use of a table on angles
3. Outer and inner tangents

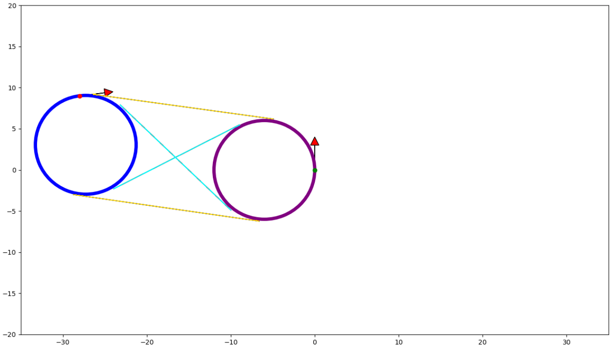

Figure: Same radius internal and external circles tangents

Implementation details

- $\dot{x} = \cos\,\theta$
- $\dot{y} = \sin\,\theta$
- $\dot{\theta} = \frac{1}{r_{min\ \ turning}}$
- $x_{new} = x_{prev} + \delta * cos(\theta)$
- $y_{new} = y_{prev} + \delta * sin(\theta)$
- $\theta_{new} = \theta_{prev} + \frac{\delta}{r_{min\ \ turning}}$

With δ a small value, typically below 0.05.

Reeds-Shepp paths

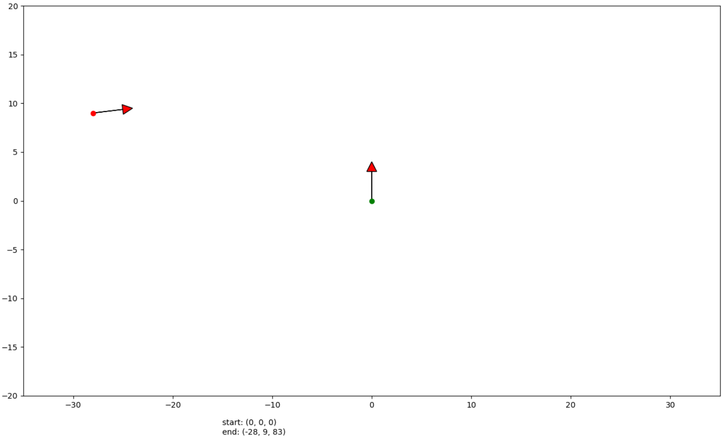

Figure: What is a shortest path for a robot able to go backward ?

Reeds-Shepp paths

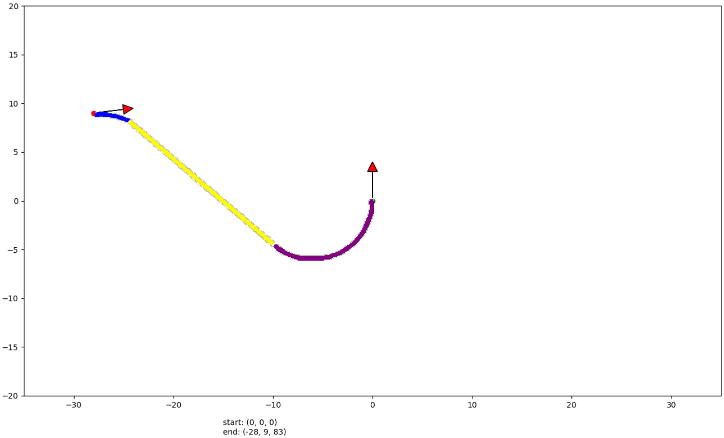

Figure: A shortest Reeds-Shepp path for a robot able to go backward

Reeds-Shepp paths

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$l^-r^+l^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$r^+l^-r^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$r^-l^+r^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$l^+r^-l^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$l^-r^+l^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$r^+l^-r^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$r^-l^+r^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$l^-r^-l^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$l^+r^+l^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$r^-l^-r^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$r^+l^+r^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$l^+r^+_u l^-_{-u} r^-$	$C^+C^+_u C^-_{-u} C^-$	$CC_u \mid C_u C$	(8.7), two roots
$l^-r^-_{-u} l^+_u r^+$	$C^-C^-_{-u} C^+_u C^+$	$CC_u \mid C_u C$	(8.7), two roots
$r^+l^+_u r^-_{-u} l^-$	$C^+C^+_u C^-_{-u} C^-$	$CC_u \mid C_u C$	(8.7), two roots
$r^-l^-_{-u} r^+_u l^+$	$C^-C^-_{-u} C^+_u C^+$	$CC_u \mid C_u C$	(8.7), two roots
$l^+r^-_{-u} l^-_{-u} r^+$	$C^+C^-_{-u} C^-_{-u} C^+$	$C \mid C_u C_u \mid C$	(8.8)
$l^-r^+_u l^+_u r^-$	$C^-C^+_u C^+_u C^-$	$C \mid C_u C_u \mid C$	(8.8)
$r^+l^-_{-u} r^-_{-u} l^+$	$C^+C^-_{-u} C^-_{-u} C^+$	$C \mid C_u C_u \mid C$	(8.8)
$r^-l^+_u r^+_u l^-$	$C^-C^+_u C^+_u C^-$	$C \mid C_u C_u \mid C$	(8.8)
$l^+r^-_{-\pi/2} s^- l^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} SC$	(8.9)
$l^-r^+_{\pi/2} s^+ l^+$	$C^-C^+_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} SC$	(8.9)
$r^+l^-_{-\pi/2} s^- r^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} SC$	(8.9)
$r^-l^+_{\pi/2} s^+ r^+$	$C^-C^+_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} SC$	(8.9)
$l^-s^-r^-_{-\pi/2} l^+$	$C^-S^-C^-_{-\pi/2} C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+s^+r^+_{\pi/2} l^-$	$C^+S^+C^+_{\pi/2} C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$r^-s^-l^-_{-\pi/2} r^+$	$C^-S^-C^-_{-\pi/2} C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$r^+s^+l^+_{\pi/2} r^-$	$C^+S^+C^+_{\pi/2} C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+r^-_{-\pi/2} s^- r^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} SC$	(8.10)
$l^-r^+_{\pi/2} s^+ r^+$	$C^-C^+_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} SC$	(8.10)
$r^+l^-_{-\pi/2} s^- l^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} SC$	(8.10)
$r^-l^+_{\pi/2} s^+ l^+$	$C^-C^+_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} SC$	(8.10)
$r^-s^-r^-_{-\pi/2} l^+$	$C^-S^-C^-_{-\pi/2} C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$r^+s^+r^+_{\pi/2} l^-$	$C^+S^+C^+_{\pi/2} C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^-s^-l^-_{-\pi/2} r^+$	$C^-S^-C^-_{-\pi/2} C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+l^+_{\pi/2} r^-$	$C^+S^+C^+_{\pi/2} C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
$l^-s^-r^-$	$C^-S^-C^-$	CSC	(8.2)
$r^+s^+l^+$	$C^+S^+C^+$	CSC	(8.2)
$r^-s^-l^-$	$C^-S^-C^-$	CSC	(8.2)
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
$l^-s^-l^-$	$C^-S^-C^-$	CSC	(8.1)
$r^+s^+r^+$	$C^+S^+C^+$	CSC	(8.1)
$r^-s^-r^-$	$C^-S^-C^-$	CSC	(8.1)
$l^+r^-_{-\pi/2} s^- l^-_{-\pi/2} r^+$	$C^+C^-_{-\pi/2} S^- C^-_{-\pi/2} C^+$	$C \mid C_{\pi/2} SC_{\pi/2} \mid C$	(8.11), two roots
$l^-r^+_{\pi/2} s^+ l^+_{\pi/2} r^-$	$C^-C^+_{\pi/2} S^+ C^+_{\pi/2} C^-$	$C \mid C_{\pi/2} SC_{\pi/2} \mid C$	(8.11), two roots
$r^+l^-_{-\pi/2} s^- r^-_{-\pi/2} l^+$	$C^+C^-_{-\pi/2} S^- C^-_{-\pi/2} C^+$	$C \mid C_{\pi/2} SC_{\pi/2} \mid C$	(8.11), two roots
$r^-l^+_{\pi/2} s^+ r^+_{\pi/2} l^-$	$C^-C^+_{\pi/2} S^+ C^+_{\pi/2} C^-$	$C \mid C_{\pi/2} SC_{\pi/2} \mid C$	(8.11), two roots

Figure: All Reeds-Shepp paths

Reeds-Shepp paths: timeflip simplification

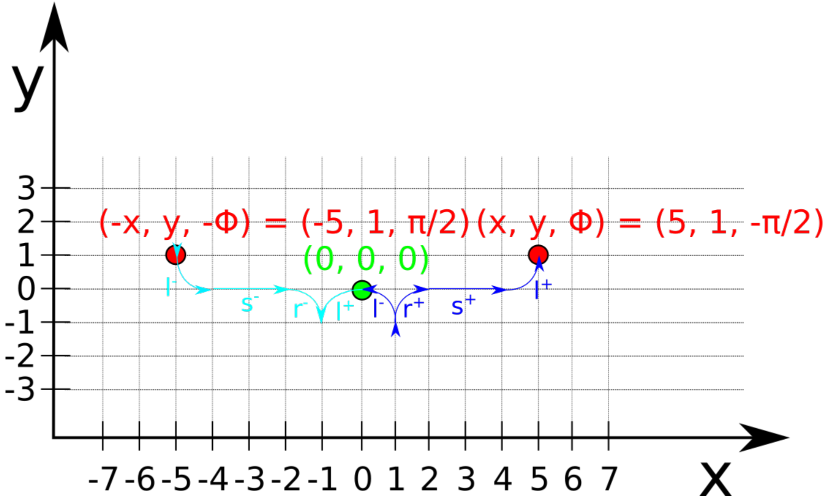

Figure: Timeflip simplification: $l^- r^+ s^+ l^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+ r^- s^- l^-$ goes from (0, 0, 0) to $(-x, y, -\phi)$

Reeds-Shepp paths: timeflip simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C\|C\|C$	(8.3), two roots
~~$l^-r^+l^-$~~	~~$C^-C^+C^-$~~	~~$C\|C\|C$~~	~~(8.3), two roots~~
$r^+l^-r^+$	$C^+C^-C^+$	$C\|C\|C$	(8.3), two roots
~~$r^-l^+r^-$~~	~~$C^-C^+C^-$~~	~~$C\|C\|C$~~	~~(8.3), two roots~~
$l^+r^-l^-$	$C^+C^-C^-$	$C\|CC$	(8.4), two roots
~~$l^-r^+l^+$~~	~~$C^-C^+C^+$~~	~~$C\|CC$~~	~~(8.4), two roots~~
$r^+l^-r^-$	$C^+C^-C^-$	$C\|CC$	(8.4), two roots
~~$r^-l^+r^+$~~	~~$C^-C^+C^+$~~	~~$C\|CC$~~	~~(8.4), two roots~~
~~$l^-r^-l^+$~~	~~$C^-C^-C^+$~~	~~$CC\|C$~~	~~(8.4), two roots~~
$l^+r^+l^-$	$C^+C^+C^-$	$CC\|C$	(8.4), two roots
~~$r^-l^-r^+$~~	~~$C^-C^-C^+$~~	~~$CC\|C$~~	~~(8.4), two roots~~
$r^+l^+r^-$	$C^+C^+C^-$	$CC\|C$	(8.4), two roots
$l^+r^+_ul^-_ur^-$	$C^+C^+_uC^-_uC^-$	$CC_u\|C_uC$	(8.7), two roots
~~$l^-r^-_ul^+_ur^+$~~	~~$C^-C^-_uC^+_uC^+$~~	~~$CC_u\|C_uC$~~	~~(8.7), two roots~~
$r^+l^+_ur^-_ul^-$	$C^+C^+_uC^-_uC^-$	$CC_u\|C_uC$	(8.7), two roots
~~$r^-l^-_ur^+_ul^+$~~	~~$C^-C^-_uC^+_uC^+$~~	~~$CC_u\|C_uC$~~	~~(8.7), two roots~~
$l^+r^-_ul^-_ur^+$	$C^+C^-_uC^-_uC^+$	$C\|C_uC_u\|C$	(8.8)
~~$l^-r^+_ul^+_ur^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C\|C_uC_u\|C$~~	~~(8.8)~~
$r^+l^-_ur^-_ul^+$	$C^+C^-_uC^-_uC^+$	$C\|C_uC_u\|C$	(8.8)
~~$r^-l^+_ur^+_ul^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C\|C_uC_u\|C$~~	~~(8.8)~~
$l^+r^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\|C_{\pi/2}SC$	(8.9)
~~$l^-r^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.9)~~
$r^+l^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\|C_{\pi/2}SC$	(8.9)
~~$r^-l^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.9)~~
~~$l^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
$l^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\|C$	(8.9)
~~$r^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
$r^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\|C$	(8.9)
$l^+r^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\|C_{\pi/2}SC$	(8.10)
~~$l^-r^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.10)~~
$r^+l^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\|C_{\pi/2}SC$	(8.10)
~~$r^-l^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
$r^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\|C$	(8.10)
~~$l^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
$l^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\|C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
~~$l^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$r^+s^+l^+$	$C^+S^+C^+$	CSC	(8.2)
~~$r^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
~~$l^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$r^+s^+r^+$	$C^+S^+C^+$	CSC	(8.1)
~~$r^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$l^+r^-_{-\pi/2}s^-l^-_{-\pi/2}r^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C\|C_{\pi/2}SC_{\pi/2}\|C$	(8.11), two roots
~~$l^-r^+_{\pi/2}s^+l^+_{\pi/2}r^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C\|C_{\pi/2}SC_{\pi/2}\|C$~~	~~(8.11), two roots~~
$r^+l^-_{-\pi/2}s^-r^-_{-\pi/2}l^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C\|C_{\pi/2}SC_{\pi/2}\|C$	(8.11), two roots
~~$r^-l^+_{\pi/2}s^+r^+_{\pi/2}l^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C\|C_{\pi/2}SC_{\pi/2}\|C$~~	~~(8.11), two roots~~

Figure: All Reeds-Shepp paths with timeflip simplification

Reeds-Shepp paths: reflect simplification

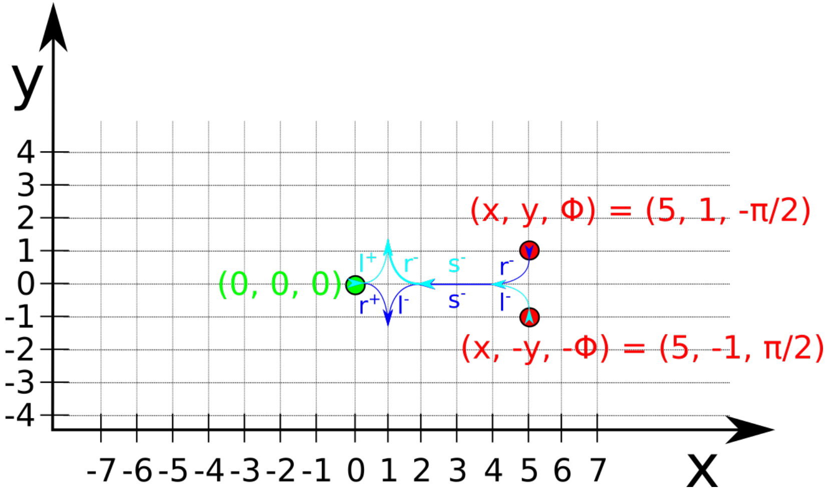

Figure: Reflect simplification: $r^+l^-s^-r^-$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+r^-s^-l^-$ goes from (0, 0, 0) to $(x, -y, -\phi)$

Reeds-Shepp paths: reflect simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C\mid C\mid C$	(8.3), two roots
~~$l^-r^+l^-$~~	~~$C^-C^+C^-$~~	~~$C\mid C\mid C$~~	~~(8.3), two roots~~
~~$r^+l^-r^+$~~	~~$C^+C^-C^+$~~	~~$C\mid C\mid C$~~	~~(8.3), two roots~~
~~$r^-l^+r^-$~~	~~$C^-C^+C^-$~~	~~$C\mid C\mid C$~~	~~(8.3), two roots~~
$l^+r^-l^-$	$C^+C^-C^-$	$C\mid CC$	(8.4), two roots
~~$l^-r^+l^+$~~	~~$C^-C^+C^+$~~	~~$C\mid CC$~~	~~(8.4), two roots~~
~~$r^+l^-r^-$~~	~~$C^+C^-C^-$~~	~~$C\mid CC$~~	~~(8.4), two roots~~
~~$r^-l^+r^+$~~	~~$C^-C^+C^+$~~	~~$C\mid CC$~~	~~(8.4), two roots~~
~~$l^-r^-l^+$~~	~~$C^-C^-C^+$~~	~~$CC\mid C$~~	~~(8.4), two roots~~
$l^+r^+l^-$	$C^+C^+C^-$	$CC\mid C$	(8.4), two roots
~~$r^-l^-r^+$~~	~~$C^-C^-C^+$~~	~~$CC\mid C$~~	~~(8.4), two roots~~
~~$r^+l^+r^-$~~	~~$C^+C^+C^-$~~	~~$CC\mid C$~~	~~(8.4), two roots~~
$l^+r^+_ul^-_{-u}r^-$	$C^+C^+_uC^-_{-u}C^-$	$CC_u\mid C_uC$	(8.7), two roots
~~$l^-r^-_{-u}l^+_ur^+$~~	~~$C^-C^-_{-u}C^+_uC^+$~~	~~$CC_u\mid C_uC$~~	~~(8.7), two roots~~
~~$r^+l^+_ur^-_{-u}l^-$~~	~~$C^+C^+_uC^-_{-u}C^-$~~	~~$CC_u\mid C_uC$~~	~~(8.7), two roots~~
~~$r^-l^-_{-u}r^+_ul^+$~~	~~$C^-C^-_{-u}C^+_uC^+$~~	~~$CC_u\mid C_uC$~~	~~(8.7), two roots~~
$l^+r^-_{-u}l^-_{-u}r^+$	$C^+C^-_{-u}C^-_{-u}C^+$	$C\mid C_uC_u\mid C$	(8.8)
~~$l^-r^+_ul^+_ur^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C\mid C_uC_u\mid C$~~	~~(8.8)~~
~~$r^+l^-_{-u}r^-_{-u}l^+$~~	~~$C^+C^-_{-u}C^-_{-u}C^+$~~	~~$C\mid C_uC_u\mid C$~~	~~(8.8)~~
~~$r^-l^+_ur^+_ul^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C\mid C_uC_u\mid C$~~	~~(8.8)~~
$l^+r^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\mid C_{\pi/2}SC$	(8.9)
~~$l^-r^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^+l^-_{-\pi/2}s^-r^-$~~	~~$C^+C^-_{-\pi/2}S^-C^-$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^-l^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$l^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.9)~~
$l^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\mid C$	(8.9)
~~$r^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.9)~~
~~$r^+s^+l^+_{\pi/2}r^-$~~	~~$C^+S^+C^+_{\pi/2}C^-$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.9)~~
$l^+r^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C\mid C_{\pi/2}SC$	(8.10)
~~$l^-r^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^+l^-_{-\pi/2}s^-l^-$~~	~~$C^+C^-_{-\pi/2}S^-C^-$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-l^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C\mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.10)~~
~~$r^+s^+r^+_{\pi/2}l^-$~~	~~$C^+S^+C^+_{\pi/2}C^-$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.10)~~
~~$l^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2}\mid C$~~	~~(8.10)~~
$l^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2}\mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
~~$l^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
~~$r^+s^+l^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.2)~~
~~$r^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
~~$l^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
~~$r^+s^+r^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.1)~~
~~$r^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$l^+r^-_{-\pi/2}s^-l^-_{-\pi/2}r^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C\mid C_{\pi/2}SC_{\pi/2}\mid C$	(8.11), two roots
~~$l^-r^+_{\pi/2}s^+l^+_{\pi/2}r^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C\mid C_{\pi/2}SC_{\pi/2}\mid C$~~	~~(8.11), two roots~~
~~$r^+l^-_{-\pi/2}s^-r^-_{-\pi/2}l^+$~~	~~$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$~~	~~$C\mid C_{\pi/2}SC_{\pi/2}\mid C$~~	~~(8.11), two roots~~
~~$r^-l^+_{\pi/2}s^+r^+_{\pi/2}l^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C\mid C_{\pi/2}SC_{\pi/2}\mid C$~~	~~(8.11), two roots~~

Figure: All Reeds-Shepp paths with timeflip and reflect simplifications

Reeds-Shepp paths: reverse simplification

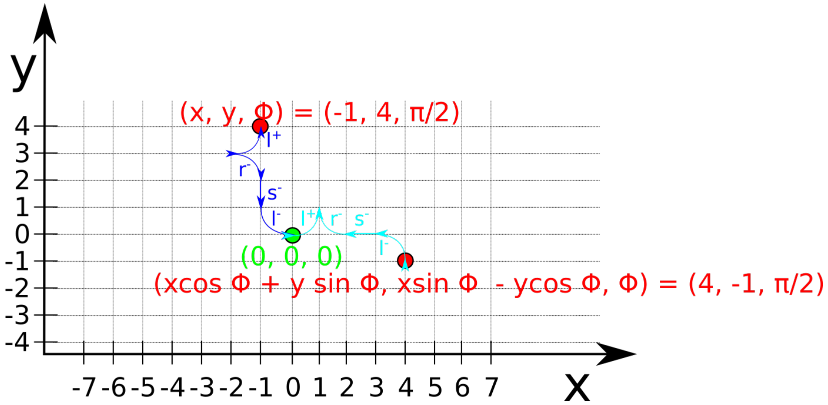

Figure: Reverse simplification: $l^- s^- r^- l^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+ r^- s^- l^-$ goes from (0, 0, 0) to ($x$cos $\phi + y$sin ϕ, $x$sin $\phi - y$cos ϕ, ϕ)

Reeds-Shepp paths: reverse simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C\|C\|C$	(8.3), two roots
~~$l^-r^+l^-$~~	~~$C^-C^+C^-$~~	~~$C\|C\|C$~~	~~(8.3), two roots~~
~~$r^+l^-r^+$~~	~~$C^+C^-C^+$~~	~~$C\|C\|C$~~	~~(8.3), two roots~~
~~$r^-l^+r^-$~~	~~$C^-C^+C^-$~~	~~$C\|C\|C$~~	~~(8.3), two roots~~
$l^+r^-l^-$	$C^+C^-C^-$	$C\|CC$	(8.4), two roots
~~$l^-r^+l^+$~~	~~$C^-C^+C^+$~~	~~$C\|CC$~~	~~(8.4), two roots~~
~~$r^+l^-r^-$~~	~~$C^+C^-C^-$~~	~~$C\|CC$~~	~~(8.4), two roots~~
~~$r^-l^+r^+$~~	~~$C^-C^+C^+$~~	~~$C\|CC$~~	~~(8.4), two roots~~
~~$l^-r^-l^+$~~	~~$C^-C^-C^+$~~	~~$CC\|C$~~	~~(8.4), two roots~~
~~$l^+r^+l^-$~~	~~$C^+C^+C^-$~~	~~$CC\|C$~~	~~(8.4), two roots~~
~~$r^-l^-r^+$~~	~~$C^-C^-C^+$~~	~~$CC\|C$~~	~~(8.4), two roots~~
~~$r^+l^+r^-$~~	~~$C^+C^+C^-$~~	~~$CC\|C$~~	~~(8.4), two roots~~
$l^+r^+_u l^-_{-u} r^-$	$C^+C^+_u C^-_{-u} C^-$	$CC_u\|C_uC$	(8.7), two roots
~~$l^-r^-_{-u} l^+_u r^+$~~	~~$C^-C^-_{-u} C^+_u C^+$~~	~~$CC_u\|C_uC$~~	~~(8.7), two roots~~
~~$r^+l^+_u r^-_{-u} l^-$~~	~~$C^+C^+_u C^-_{-u} C^-$~~	~~$CC_u\|C_uC$~~	~~(8.7), two roots~~
~~$r^-l^-_{-u} r^+_u l^+$~~	~~$C^-C^-_{-u} C^+_u C^+$~~	~~$CC_u\|C_uC$~~	~~(8.7), two roots~~
$l^+r^-_{-u} l^-_{-u} r^+$	$C^+C^-_{-u} C^-_{-u} C^+$	$C\|C_uC_u\|C$	(8.8)
~~$l^-r^+_u l^+_u r^-$~~	~~$C^-C^+_u C^+_u C^-$~~	~~$C\|C_uC_u\|C$~~	~~(8.8)~~
~~$r^+l^-_{-u} r^-_{-u} l^+$~~	~~$C^+C^-_{-u} C^-_{-u} C^+$~~	~~$C\|C_uC_u\|C$~~	~~(8.8)~~
~~$r^-l^+_u r^+_u l^-$~~	~~$C^-C^+_u C^+_u C^-$~~	~~$C\|C_uC_u\|C$~~	~~(8.8)~~
$l^+r^-_{-\pi/2} s^- l^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C\|C_{\pi/2}SC$	(8.9)
~~$l^-r^+_{\pi/2} s^+ l^+$~~	~~$C^-C^+_{\pi/2} S^+ C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^+l^-_{-\pi/2} s^- r^-$~~	~~$C^+C^-_{-\pi/2} S^- C^-$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^-l^+_{\pi/2} s^+ r^+$~~	~~$C^-C^+_{\pi/2} S^+ C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.9)~~
~~$l^-s^-r^-_{-\pi/2} l^+$~~	~~$C^-S^-C^-_{-\pi/2} C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
~~$l^+s^+r^+_{\pi/2} l^-$~~	~~$C^+S^+C^+_{\pi/2} C^-$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
~~$r^-s^-l^-_{-\pi/2} r^+$~~	~~$C^-S^-C^-_{-\pi/2} C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
~~$r^+s^+l^+_{\pi/2} r^-$~~	~~$C^+S^+C^+_{\pi/2} C^-$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.9)~~
$l^+r^-_{-\pi/2} s^- r^-$	$C^+C^-_{-\pi/2} S^- C^-$	$C\|C_{\pi/2}SC$	(8.10)
~~$l^-r^+_{\pi/2} s^+ r^+$~~	~~$C^-C^+_{\pi/2} S^+ C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^+l^-_{-\pi/2} s^- l^-$~~	~~$C^+C^-_{-\pi/2} S^- C^-$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-l^+_{\pi/2} s^+ l^+$~~	~~$C^-C^+_{\pi/2} S^+ C^+$~~	~~$C\|C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-s^-r^-_{-\pi/2} l^+$~~	~~$C^-S^-C^-_{-\pi/2} C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
~~$r^+s^+r^+_{\pi/2} l^-$~~	~~$C^+S^+C^+_{\pi/2} C^-$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
~~$l^-s^-l^-_{-\pi/2} r^+$~~	~~$C^-S^-C^-_{-\pi/2} C^+$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
~~$l^+s^+l^+_{\pi/2} r^-$~~	~~$C^+S^+C^+_{\pi/2} C^-$~~	~~$CSC_{\pi/2}\|C$~~	~~(8.10)~~
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
~~$l^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
~~$r^+s^+l^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.2)~~
~~$r^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
~~$l^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
~~$r^+s^+r^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.1)~~
~~$r^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$l^+r^-_{-\pi/2} s^- l^-_{-\pi/2} r^+$	$C^+C^-_{-\pi/2} S^- C^-_{-\pi/2} C^+$	$C\|C_{\pi/2}SC_{\pi/2}\|C$	(8.11), two roots
~~$l^-r^+_{\pi/2} s^+ l^+_{\pi/2} r^-$~~	~~$C^-C^+_{\pi/2} S^+ C^+_{\pi/2} C^-$~~	~~$C\|C_{\pi/2}SC_{\pi/2}\|C$~~	~~(8.11), two roots~~
~~$r^+l^-_{-\pi/2} s^- r^-_{-\pi/2} l^+$~~	~~$C^+C^-_{-\pi/2} S^- C^-_{-\pi/2} C^+$~~	~~$C\|C_{\pi/2}SC_{\pi/2}\|C$~~	~~(8.11), two roots~~
~~$r^-l^+_{\pi/2} s^+ r^+_{\pi/2} l^-$~~	~~$C^-C^+_{\pi/2} S^+ C^+_{\pi/2} C^-$~~	~~$C\|C_{\pi/2}SC_{\pi/2}\|C$~~	~~(8.11), two roots~~

Figure: Reeds-Shepp paths with timeflip, reflect, reverse simplifications

Reeds-Shepp paths: case $l_t^+ s_u^+ l_v^+$

- $(u, t) := R(x$ - $\sin\phi$, y - 1 + $\cos\phi)$
- $v := M(\phi - t)$

With:

- $(r, \theta) := R(x, y)$ for the polar transform $r\cos\theta = x$ and $r\sin\theta = y$ with $r \leq 0$ and $-\pi \leq \theta < \pi$
- $\phi = M(\theta)$ if $\phi \equiv \theta \bmod 2\pi$ and $-\pi \leq \phi < \pi$

Dubins and Reeds-Shepp paths in reality

- All kinds of sources for error in the real-world

Dubins and Reeds-Shepp paths in reality

- All kinds of sources for error in the real-world
- Paths found in the absence of obstacles however can use a rapidly exploring random tree and Dubins' pseudo distance or Reeds-Shepp distance

Sources

- Optimal paths for a car that goes both forwards and backwards (1990), J. A. Reeds and L. A. Shepp
- Planning algorithms: Reeds-Shepp curves (2006), Steven M. LaValle
- A Comprehensive, Step-by-Step Tutorial to Computing Dubins' Paths (2013), Andy G's Blog