

Dubins and Reeds–Shepp paths

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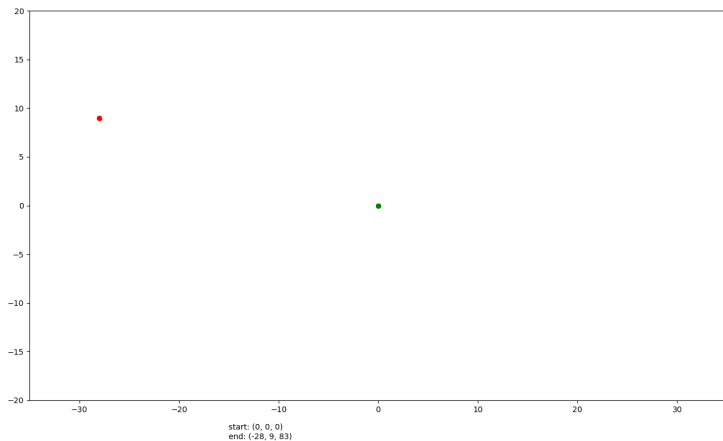


Figure: The shortest path in the case of a human being

Introduction

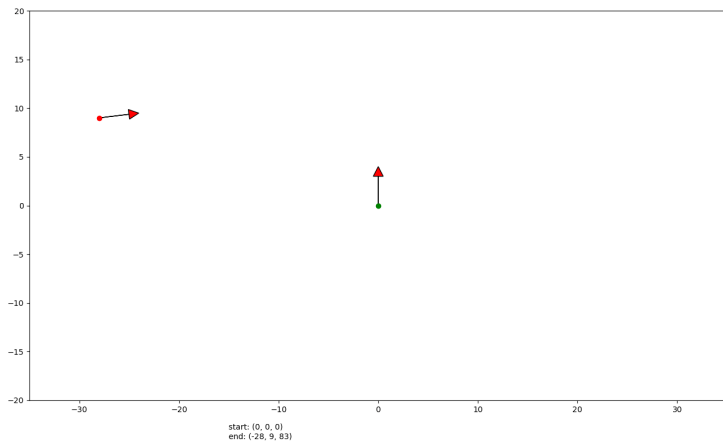


Figure: A shortest path in the case of an oriented human being

Introduction

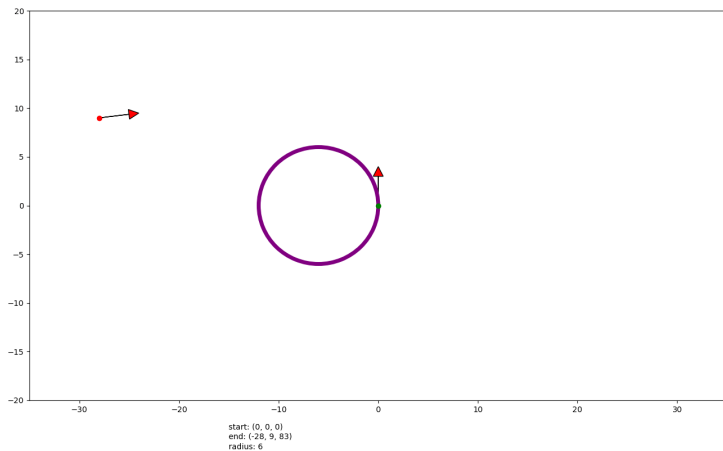


Figure: Robot's minimal turning circle

Dubins' paths

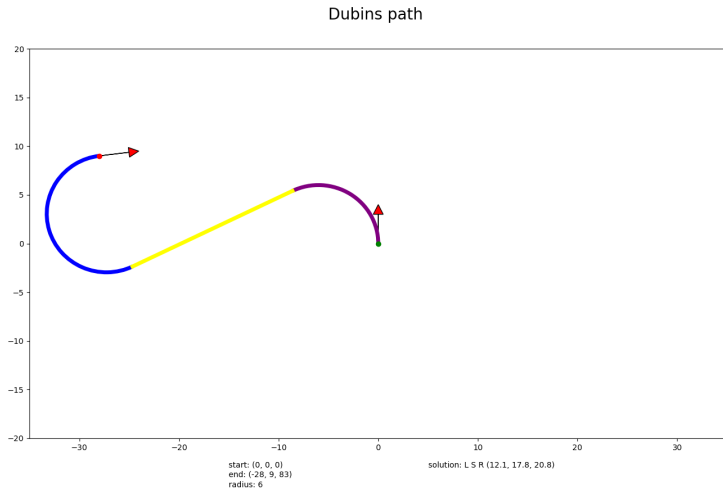


Figure: Dubins' path describing a shortest path

Dubins' paths

| |
|------------|
| CSC |
| RSR |
| LSL |
| LSR |
| RSL |

(a) All combinaisons of **CSC** paths

| |
|------------|
| CCC |
| RLR |
| LRL |

(b) All combinaisons of **CCC** paths

C: means "curve" (i.e. turning left or right)

S: means "straight"

R: means "right" (i.e. turning right)

L: means "left" (i.e. turning left)

Dubins' paths

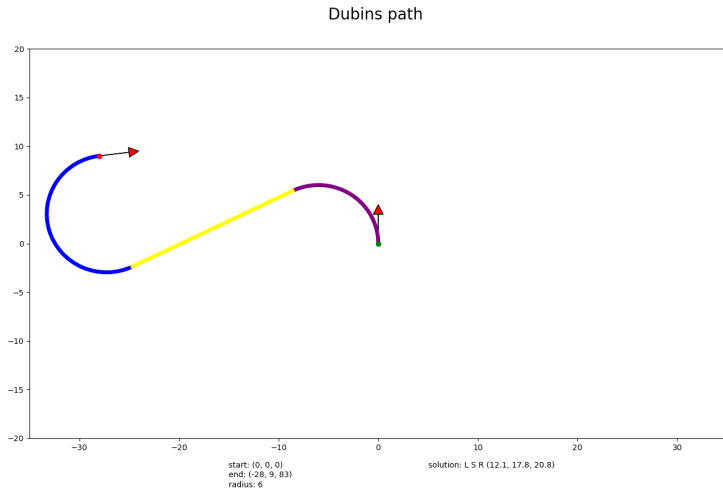


Figure: Dubins' path LSR describing the shortest path

Implementation details

Implementation details

1. Constant complexity

Implementation details

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2. Use of a table on angles

Implementation details

1. Constant complexity
2. Use of a table on angles
3. Outer and inner tangents

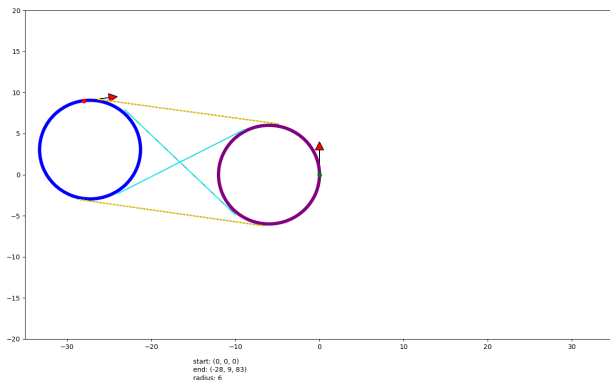


Figure: Same radius internal and external circles tangents

Implementation details

- ▶ $\dot{x} = \cos \theta$
- ▶ $\dot{y} = \sin \theta$
- ▶ $\dot{\theta} = \frac{1}{r_{min \text{ turning}}}$
- ▶ $x_{new} = x_{prev} + \delta * \cos(\theta)$
- ▶ $y_{new} = y_{prev} + \delta * \sin(\theta)$
- ▶ $\theta_{new} = \theta_{prev} + \frac{\delta}{r_{min \text{ turning}}}$

With δ a small value, typically below 0.05.

Reeds-Shepp paths

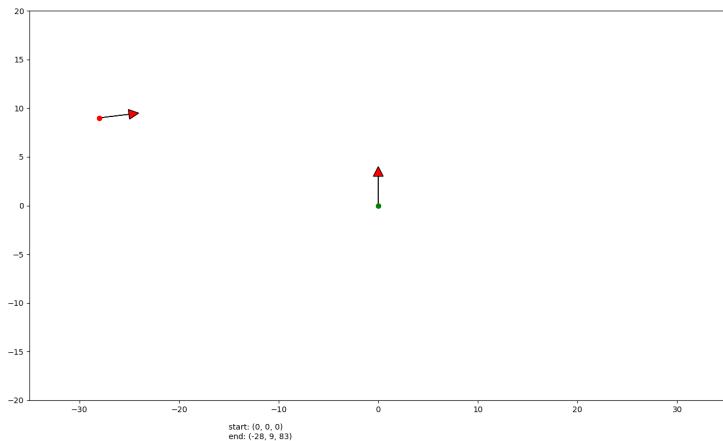


Figure: What is a shortest path for a robot able to go backward ?

Reeds-Shepp paths

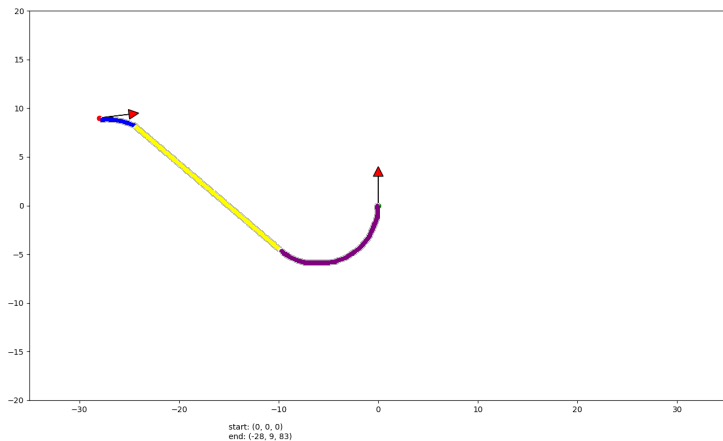


Figure: A shortest Reeds-Shepp path for a robot able to go backward

Reeds-Shepp paths

| explicit | (1.1) form | (1.2) form | Section 8 formula |
|-----------------------------|-----------------------------|---------------------------------|-------------------|
| $l^+ r^- l^+$ | $C^+ C^- C^+$ | $C \mid C \mid C$ | (8.3), two roots |
| $l^- r^+ l^-$ | $C^- C^+ C^-$ | $C \mid C \mid C$ | (8.3), two roots |
| $r^+ l^- r^+$ | $C^+ C^- C^+$ | $C \mid C \mid C$ | (8.3), two roots |
| $r^- l^+ r^-$ | $C^- C^+ C^-$ | $C \mid C \mid C$ | (8.3), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C \mid CC$ | (8.4), two roots |
| $l^- r^+ l^+$ | $C^- C^+ C^+$ | $C \mid CC$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^- C^-$ | $C \mid CC$ | (8.4), two roots |
| $r^- l^+ r^+$ | $C^- C^+ C^+$ | $C \mid CC$ | (8.4), two roots |
| $l^- r^- l^+$ | $C^- C^- C^+$ | $CC \mid C$ | (8.4), two roots |
| $l^+ r^+ l^-$ | $C^+ C^+ C^-$ | $CC \mid C$ | (8.4), two roots |
| $r^- l^- r^+$ | $C^- C^- C^+$ | $CC \mid C$ | (8.4), two roots |
| $r^+ l^+ r^-$ | $C^+ C^+ C^-$ | $CC \mid C$ | (8.4), two roots |
| $l^+ r_0^+ l_{-2} r^-$ | $C^+ C_0^+ C_{-2} C^-$ | $CC_0 \mid C_{-2} C$ | (8.7), two roots |
| $l^- r_{-2} l_0^+ r^+$ | $C^- C_{-2} C_0^+ C^+$ | $CC_{-2} \mid C_0 C$ | (8.7), two roots |
| $r^+ l_0^+ r_{-2} l^-$ | $C^+ C_0^+ C_{-2} C^-$ | $CC_0 \mid C_{-2} C$ | (8.7), two roots |
| $r^- l_{-2} r_0^+ l^+$ | $C^- C_{-2} C_0^+ C^+$ | $CC_{-2} \mid C_0 C$ | (8.7), two roots |
| $l^+ r_{-2} l_{-2} r^+$ | $C^+ C_{-2} C_{-2} C^+$ | $C \mid C_{-2} C_{-2} \mid C$ | (8.8) |
| $l^- r_0^+ l_0^+ r^-$ | $C^- C_0^+ C_0^+ C^-$ | $C \mid C_0 C_0 \mid C$ | (8.8) |
| $r^+ l_{-2} r_{-2} l^+$ | $C^+ C_{-2} C_{-2} C^+$ | $C \mid C_{-2} C_{-2} \mid C$ | (8.8) |
| $r^- l_0^+ r_0^+ l^-$ | $C^- C_0^+ C_0^+ C^-$ | $C \mid C_0 C_0 \mid C$ | (8.8) |
| $l^+ r_{-2} s^- l^-$ | $C^+ C_{-2} s^- C^-$ | $C \mid C_{-2} s C$ | (8.9) |
| $l^- r_0 s^+ l^+$ | $C^- C_0 s^+ C^+$ | $C \mid C_0 s C$ | (8.9) |
| $r^+ l_{-2} s^- r^-$ | $C^+ C_{-2} s^- C^-$ | $C \mid C_{-2} s C$ | (8.9) |
| $r^- l_0 s^+ r^+$ | $C^- C_0 s^+ C^+$ | $C \mid C_0 s C$ | (8.9) |
| $l^- s^- r_{-2} l^+$ | $C^- s^- C_{-2} C^+$ | $C s C_{-2} \mid C$ | (8.9) |
| $l^+ s^+ r_0 l^-$ | $C^+ s^+ C_0 C^-$ | $C s C_0 \mid C$ | (8.9) |
| $r^- s^- l_{-2} r^+$ | $C^- s^- C_{-2} C^+$ | $C s C_{-2} \mid C$ | (8.9) |
| $r^+ s^+ l_0 r^-$ | $C^+ s^+ C_0 C^-$ | $C s C_0 \mid C$ | (8.9) |
| $l^+ r_{-2} s^- r^-$ | $C^+ C_{-2} s^- C^-$ | $C \mid C_{-2} s C$ | (8.10) |
| $l^- r_0 s^+ r^+$ | $C^- C_0 s^+ C^+$ | $C \mid C_0 s C$ | (8.10) |
| $r^+ l_{-2} s^- l^-$ | $C^+ C_{-2} s^- C^-$ | $C \mid C_{-2} s C$ | (8.10) |
| $r^- l_0 s^+ l^+$ | $C^- C_0 s^+ C^+$ | $C \mid C_0 s C$ | (8.10) |
| $r^- s^- r_{-2} l^+$ | $C^- s^- C_{-2} C^+$ | $C s C_{-2} \mid C$ | (8.10) |
| $r^+ s^+ r_0 l^-$ | $C^+ s^+ C_0 C^-$ | $C s C_0 \mid C$ | (8.10) |
| $l^- s^- l_{-2} r^+$ | $C^- s^- C_{-2} C^+$ | $C s C_{-2} \mid C$ | (8.10) |
| $l^+ s^+ l_0 r^-$ | $C^+ s^+ C_0 C^-$ | $C s C_0 \mid C$ | (8.10) |
| $l^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C s C$ | (8.2) |
| $l^- s^- r^-$ | $C^- s^- C^-$ | $C s C$ | (8.2) |
| $r^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C s C$ | (8.2) |
| $r^- s^- l^-$ | $C^- s^- C^-$ | $C s C$ | (8.2) |
| $l^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C s C$ | (8.1) |
| $l^- s^- l^-$ | $C^- s^- C^-$ | $C s C$ | (8.1) |
| $r^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C s C$ | (8.1) |
| $r^- s^- r^-$ | $C^- s^- C^-$ | $C s C$ | (8.1) |
| $l^+ r_{-2} s^- l_{-2} r^+$ | $C^+ C_{-2} s^- C_{-2} C^+$ | $C \mid C_{-2} s C_{-2} \mid C$ | (8.11), two roots |
| $l^- r_0 s^+ l_0^+ r^-$ | $C^- C_0 s^+ C_0^+ C^-$ | $C \mid C_0 s C_0 \mid C$ | (8.11), two roots |
| $r^+ l_{-2} s^- r_{-2} l^+$ | $C^+ C_{-2} s^- C_{-2} C^+$ | $C \mid C_{-2} s C_{-2} \mid C$ | (8.11), two roots |
| $r^- l_0 s^+ r_0^+ l^-$ | $C^- C_0 s^+ C_0^+ C^-$ | $C \mid C_0 s C_0 \mid C$ | (8.11), two roots |

Figure: All Reeds-Shepp paths

Reeds-Shepp paths: timeflip simplification

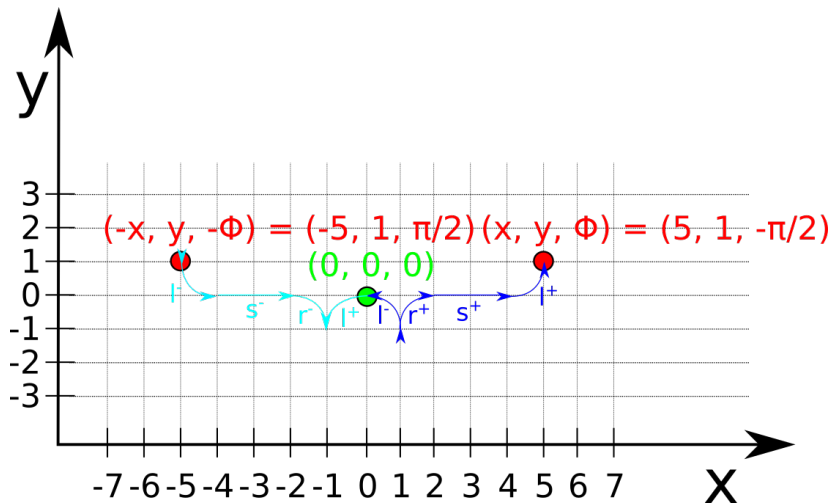


Figure: Timeflip simplification: $l^- r^+ s^+ l^+$ goes from the $(0, 0, 0)$ to (x, y, ϕ) and $l^+ r^- s^- l^-$ goes from $(0, 0, 0)$ to $(-x, y, -\phi)$

Reeds-Shepp paths: timeflip simplification

| explicit | (1.1) form | (1.2) form | Section 8 formula |
|---|---|---|-------------------|
| $l^+ r^- l^+$ | $C^+ C^- C^+$ | $C \mid C \mid C$ | (8.3), two roots |
| $\bar{l}^- r^+ l^-$ | $C^- C^+ C^-$ | $C \mid C \mid C$ | (8.3), two roots |
| $r^+ l^- r^+$ | $C^+ C^- C^+$ | $C \mid C \mid C$ | (8.3), two roots |
| $\bar{r}^- l^+ \bar{r}^-$ | $C^- C^+ C^-$ | $C \mid C \mid C$ | (8.3), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C \mid C C$ | (8.4), two roots |
| $\bar{l}^- r^+ l^+$ | $C^- C^+ C^+$ | $C \mid C C$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^- C^-$ | $C \mid C C$ | (8.4), two roots |
| $\bar{r}^- l^+ r^+$ | $C^- C^+ C^+$ | $C \mid C C$ | (8.4), two roots |
| $\bar{l}^- r^+ l^+$ | $C^- C^+ C^+$ | $C C \mid C$ | (8.4), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C C \mid C$ | (8.4), two roots |
| $\bar{l}^- r^+ l^+$ | $C^- C^+ C^+$ | $C C \mid C$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^- C^-$ | $C C \mid C$ | (8.4), two roots |
| $\bar{r}^- l^+ r^+$ | $C^- C^+ C^+$ | $C C \mid C$ | (8.4), two roots |
| $l^+ r_{\alpha 1}^- l_{\alpha 2}^-$ | $C^+ C_{\alpha 1}^- C_{\alpha 2}^-$ | $C C_{\alpha 1} \mid C_{\alpha 2} C$ | (8.7), two roots |
| $\bar{l}^- r_{\alpha 1}^+ l_{\alpha 2}^+$ | $C^- C_{\alpha 1}^+ C_{\alpha 2}^+$ | $C C_{\alpha 1} \mid C_{\alpha 2} C$ | (8.7), two roots |
| $r^+ l_{\alpha 1}^- r_{\alpha 2}^-$ | $C^+ C_{\alpha 1}^- C_{\alpha 2}^-$ | $C C_{\alpha 1} \mid C_{\alpha 2} C$ | (8.7), two roots |
| $\bar{r}^- l_{\alpha 1}^+ r_{\alpha 2}^+$ | $C^- C_{\alpha 1}^+ C_{\alpha 2}^+$ | $C C_{\alpha 1} \mid C_{\alpha 2} C$ | (8.7), two roots |
| $l^+ r_{\alpha 1}^- l_{\alpha 2}^+$ | $C^+ C_{\alpha 1}^- C_{\alpha 2}^+$ | $C \mid C_{\alpha 1} C_{\alpha 2} \mid C$ | (8.8) |
| $\bar{l}^- r_{\alpha 1}^+ l_{\alpha 2}^-$ | $C^- C_{\alpha 1}^+ C_{\alpha 2}^-$ | $C \mid C_{\alpha 1} C_{\alpha 2} \mid C$ | (8.8) |
| $r^+ l_{\alpha 1}^- r_{\alpha 2}^+$ | $C^+ C_{\alpha 1}^- C_{\alpha 2}^+$ | $C \mid C_{\alpha 1} C_{\alpha 2} \mid C$ | (8.8) |
| $\bar{r}^- l_{\alpha 1}^+ r_{\alpha 2}^-$ | $C^- C_{\alpha 1}^+ C_{\alpha 2}^-$ | $C \mid C_{\alpha 1} C_{\alpha 2} \mid C$ | (8.8) |
| $l^+ r_{\alpha 1} s^- l^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C \mid C_{\alpha 1} s C$ | (8.9) |
| $\bar{l}^- r_{\alpha 1} s^+ l^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C \mid C_{\alpha 1} s C$ | (8.9) |
| $r^+ l_{\alpha 1} s^- r^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C \mid C_{\alpha 1} s C$ | (8.9) |
| $\bar{r}^- l_{\alpha 1} s^+ r^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C \mid C_{\alpha 1} s C$ | (8.9) |
| $\bar{l}^- r_{\alpha 1} s^+ l^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C S C_{\alpha 1} \mid C$ | (8.9) |
| $l^+ r_{\alpha 1} s^- l^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C S C_{\alpha 1} \mid C$ | (8.9) |
| $\bar{l}^- r_{\alpha 1} s^+ l^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C S C_{\alpha 1} \mid C$ | (8.9) |
| $r^+ l_{\alpha 1} s^- r^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C S C_{\alpha 1} \mid C$ | (8.9) |
| $\bar{r}^- l_{\alpha 1} s^+ r^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C S C_{\alpha 1} \mid C$ | (8.9) |
| $l^+ r_{\alpha 1} s^- r^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C \mid C_{\alpha 1} s C$ | (8.10) |
| $\bar{l}^- r_{\alpha 1} s^+ r^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C \mid C_{\alpha 1} s C$ | (8.10) |
| $r^+ l_{\alpha 1} s^- l^-$ | $C^+ C_{\alpha 1} s^- C^-$ | $C \mid C_{\alpha 1} s C$ | (8.10) |
| $\bar{r}^- l_{\alpha 1} s^+ l^+$ | $C^- C_{\alpha 1} s^+ C^+$ | $C \mid C_{\alpha 1} s C$ | (8.10) |
| $\bar{r}^- s^- r_{\alpha 1} l^+$ | $C^- s^- C_{\alpha 1}^+ C^+$ | $C S C_{\alpha 1} \mid C$ | (8.10) |
| $r^+ s^+ r_{\alpha 1} l^-$ | $C^+ s^+ C_{\alpha 1}^- C^-$ | $C S C_{\alpha 1} \mid C$ | (8.10) |
| $\bar{l}^- s^- l_{\alpha 1} r^+$ | $C^- s^- C_{\alpha 1}^+ C^+$ | $C S C_{\alpha 1} \mid C$ | (8.10) |
| $l^+ s^+ l_{\alpha 1} r^-$ | $C^+ s^+ C_{\alpha 1}^- C^-$ | $C S C_{\alpha 1} \mid C$ | (8.10) |
| $l^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C S C$ | (8.2) |
| $\bar{l}^- s^- r^-$ | $C^- s^- C^-$ | $C S C$ | (8.2) |
| $r^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C S C$ | (8.2) |
| $\bar{r}^- s^- l^-$ | $C^- s^- C^-$ | $C S C$ | (8.2) |
| $l^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C S C$ | (8.1) |
| $\bar{l}^- s^- l^-$ | $C^- s^- C^-$ | $C S C$ | (8.1) |
| $r^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C S C$ | (8.1) |
| $\bar{r}^- s^- r^-$ | $C^- s^- C^-$ | $C S C$ | (8.1) |
| $l^+ r_{\alpha 1} s^- l_{\alpha 2} r^+$ | $C^+ C_{\alpha 1} s^- C_{\alpha 2}^+ C^+$ | $C \mid C_{\alpha 1} s C_{\alpha 2} \mid C$ | (8.11), two roots |
| $\bar{l}^- r_{\alpha 1} s^+ l_{\alpha 2} \bar{r}^-$ | $C^- C_{\alpha 1} s^+ C_{\alpha 2}^- C^-$ | $C \mid C_{\alpha 1} s C_{\alpha 2} \mid C$ | (8.11), two roots |
| $r^+ l_{\alpha 1} s^- r_{\alpha 2} l^+$ | $C^+ C_{\alpha 1} s^- C_{\alpha 2}^+ C^+$ | $C \mid C_{\alpha 1} s C_{\alpha 2} \mid C$ | (8.11), two roots |
| $\bar{r}^- l_{\alpha 1} s^+ r_{\alpha 2} \bar{l}^-$ | $C^- C_{\alpha 1} s^+ C_{\alpha 2}^- C^-$ | $C \mid C_{\alpha 1} s C_{\alpha 2} \mid C$ | (8.11), two roots |

Figure: All Reeds-Shepp paths with timeflip simplification

Reeds-Shepp paths: reflect simplification

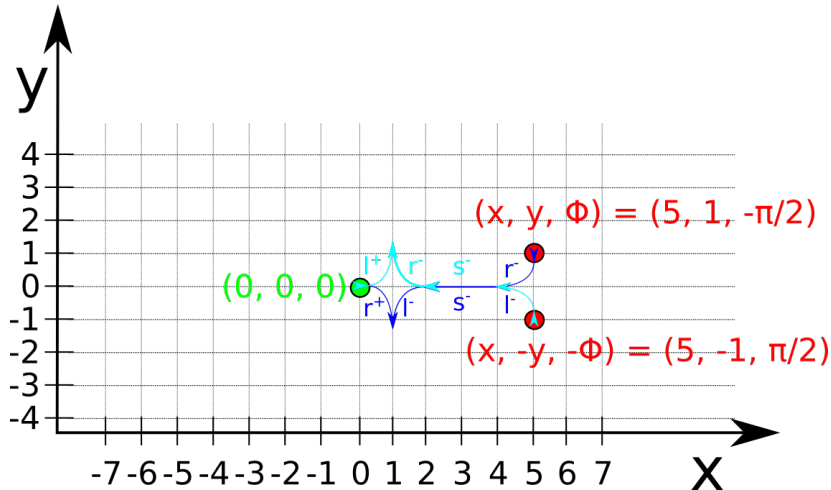


Figure: Reflect simplification: $r^+l^-s^-r^-$ goes from the $(0, 0, 0)$ to (x, y, ϕ) and $l^+r^-s^-l^-$ goes from $(0, 0, 0)$ to $(x, -y, -\phi)$

Reeds-Shepp paths: reflect simplification

| explicit | (1.1) form | (1.2) form | Section 8 formula |
|---------------------------------|---------------------------------|---------------------------|-------------------|
| $l^+ r^- l^+$ | $C^+ C^- C^+$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $l^+ r^- l^-$ | $C^- C^+ C^-$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $r^+ l^- r^+$ | $C^+ C^- C^+$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $r^+ l^- r^-$ | $C^- C^+ C^-$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r^- l^+$ | $C^- C^+ C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^- C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^+$ | $C^- C^+ C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r^- l^+$ | $C^- C^- C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r^- l^-$ | $C^+ C^+ C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^+$ | $C^- C^- C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^+ C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r_{12}^+ l^- r^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.7), two roots |
| $l^+ r_{12}^+ l^+ r^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.7), two roots |
| $r^+ l_{12}^+ r_{12}^- l^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.7), two roots |
| $r^+ l_{12}^+ r_{12}^+ l^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.7), two roots |
| $l^+ r_{12}^+ l^- r^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.8) |
| $l^+ r_{12}^+ l^+ r^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.8) |
| $r^+ l_{12}^+ r_{12}^- l^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.8) |
| $r^+ l_{12}^+ r_{12}^+ l^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.8) |
| $l^+ r_{12}^+ l^- r^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.9) |
| $l^+ r_{12}^+ l^+ r^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.9) |
| $r^+ l_{12}^+ r_{12}^- l^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.9) |
| $r^+ l_{12}^+ r_{12}^+ l^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.9) |
| $l^+ r_{12}^+ l^- r^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.10) |
| $l^+ r_{12}^+ l^+ r^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.10) |
| $r^+ l_{12}^+ r_{12}^- l^-$ | $C^+ C_{12}^+ C_{12}^- C^-$ | $C C_{12}^+ C_{12}^- C$ | (8.10) |
| $r^+ l_{12}^+ r_{12}^+ l^+$ | $C^- C_{12}^- C_{12}^+ C^+$ | $C C_{12}^+ C_{12}^- C$ | (8.10) |
| $r^- s^- r_{12}^+ l^+$ | $C^- S^- C_{12}^+ C^+$ | $C S C_{12}^+ C$ | (8.10) |
| $r^+ s^+ r_{12}^+ l^+$ | $C^+ S^+ C_{12}^+ C^+$ | $C S C_{12}^+ C$ | (8.10) |
| $l^+ s^+ l_{12}^+ r^+$ | $C^- S^- C_{12}^+ C^+$ | $C S C_{12}^+ C$ | (8.10) |
| $l^+ s^+ l_{12}^+ r^-$ | $C^+ S^+ C_{12}^+ C^-$ | $C S C_{12}^+ C$ | (8.10) |
| $l^+ s^+ r^+$ | $C^+ S^+ C^+$ | $C S C$ | (8.2) |
| $l^+ s^+ r^-$ | $C^- S^- C^-$ | $C S C$ | (8.2) |
| $r^+ s^+ l^+$ | $C^+ S^+ C^+$ | $C S C$ | (8.2) |
| $r^+ s^+ l^-$ | $C^- S^- C^-$ | $C S C$ | (8.2) |
| $l^+ s^+ l^+$ | $C^+ S^+ C^+$ | $C S C$ | (8.1) |
| $l^+ s^+ l^-$ | $C^- S^- C^-$ | $C S C$ | (8.1) |
| $r^+ s^+ r^+$ | $C^+ S^+ C^+$ | $C S C$ | (8.1) |
| $r^+ s^+ r^-$ | $C^- S^- C^-$ | $C S C$ | (8.1) |
| $l^+ r_{12}^+ s^+ l_{12}^+ r^+$ | $C^+ C_{12}^+ S^+ C_{12}^- C^+$ | $C C_{12}^+ S C_{12}^- C$ | (8.11), two roots |
| $l^+ r_{12}^+ s^+ l_{12}^+ r^-$ | $C^- C_{12}^- S^+ C_{12}^+ C^-$ | $C C_{12}^+ S C_{12}^- C$ | (8.11), two roots |
| $r^+ l_{12}^+ s^+ r_{12}^+ l^+$ | $C^+ C_{12}^+ S^+ C_{12}^- C^+$ | $C C_{12}^+ S C_{12}^- C$ | (8.11), two roots |
| $r^+ l_{12}^+ s^+ r_{12}^+ l^-$ | $C^- C_{12}^- S^+ C_{12}^+ C^-$ | $C C_{12}^+ S C_{12}^- C$ | (8.11), two roots |

Figure: All Reeds-Shepp paths with **timeflip** and **reflect** simplifications

Reeds-Shepp paths: reverse simplification

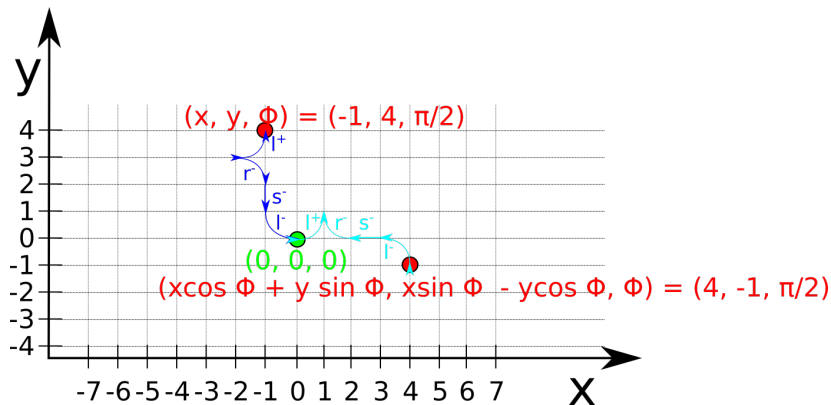


Figure: Reverse simplification: $l^- s^- r^- l^+$ goes from the $(0, 0, 0)$ to (x, y, ϕ) and $l^+ r^- s^- l^-$ goes from $(0, 0, 0)$ to $(x \cos \phi + y \sin \phi, x \sin \phi - y \cos \phi, \phi)$

Reeds-Shepp paths: reverse simplification

| explicit | (1.1) form | (1.2) form | Section 8 formula |
|-------------------------------|---------------------------------|---------------------------------|-------------------|
| $l^+ r^- l^+$ | $C^+ C^- C^+$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $l^- r^+ l^-$ | $C^- C^+ C^-$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $r^+ l^- r^+$ | $C^+ C^- C^+$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $r^- l^+ r^-$ | $C^- C^+ C^-$ | $C^+ C^+ C^+$ | (8.3), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^- r^+ l^+$ | $C^- C^+ C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^-$ | $C^+ C^- C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^- l^+ r^+$ | $C^- C^+ C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^- r^+ l^+$ | $C^- C^- C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r^- l^-$ | $C^+ C^- C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^+ l^- r^+$ | $C^+ C^- C^+$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $r^- l^+ r^-$ | $C^- C^+ C^-$ | $C^+ C^+ C^+$ | (8.4), two roots |
| $l^+ r^+ l^- r^-$ | $C^+ C_1^+ C_{w_2}^- C^-$ | $C C_{w_1}^+ C_{w_2}^-$ | (8.7), two roots |
| $l^- r^- l^+ r^+$ | $C^- C_{w_1}^- C_1^+ C^+$ | $C C_{w_1}^- C_{w_2}^-$ | (8.7), two roots |
| $r^+ l^+ r^- l^-$ | $C^+ C_1^+ C_{w_2}^- C^-$ | $C C_{w_1}^- C_{w_2}^-$ | (8.7), two roots |
| $r^- l^- r^+ l^+$ | $C^- C_{w_1}^- C_1^+ C^+$ | $C C_{w_1}^- C_{w_2}^-$ | (8.7), two roots |
| $l^+ r^- l^- r^+$ | $C^+ C_{w_2}^- C_{w_1}^- C^+$ | $C^+ C_{w_2}^- C_{w_1}^-$ | (8.8) |
| $l^- r^+ l^+ r^-$ | $C^- C_1^+ C_2^+ C^-$ | $C^+ C_{w_1}^- C_{w_2}^-$ | (8.8) |
| $r^+ l^- r^+ l^-$ | $C^+ C_{w_2}^- C_{w_1}^- C^-$ | $C^+ C_{w_1}^- C_{w_2}^-$ | (8.8) |
| $r^- l^+ r^- l^+$ | $C^- C_1^+ C_2^+ C^-$ | $C^+ C_{w_1}^- C_{w_2}^-$ | (8.8) |
| $l^+ r_{w_2} s^- l^-$ | $C^+ C_{w_2} s^- C^-$ | $C^+ C_{w_2} s^- C^-$ | (8.9) |
| $l^- r_{w_2} s^+ l^+$ | $C^- C_{w_2} s^+ C^+$ | $C^+ C_{w_2} s^- C^-$ | (8.9) |
| $r^+ l_{w_2} s^- r^-$ | $C^+ C_{w_2} s^- C^-$ | $C^+ C_{w_2} s^- C^-$ | (8.9) |
| $r^- l_{w_2} s^+ r^+$ | $C^- C_{w_2} s^+ C^+$ | $C^+ C_{w_2} s^- C^-$ | (8.9) |
| $l^+ s^- r_{w_2} l^+$ | $C^- s^- C_{w_2}^+ C^+$ | $C s^- C_{w_2}^+ C^+$ | (8.9) |
| $l^- s^+ r_{w_2} l^-$ | $C^+ s^+ C_{w_2}^- C^-$ | $C s^- C_{w_2}^+ C^+$ | (8.9) |
| $r^- s^- l_{w_2} r^+$ | $C^- s^- C_{w_2}^+ C^+$ | $C s^- C_{w_2}^+ C^+$ | (8.9) |
| $r^+ s^+ l_{w_2} r^-$ | $C^+ s^+ C_{w_2}^- C^-$ | $C s^- C_{w_2}^+ C^+$ | (8.9) |
| $l^+ r_{w_2} s^- r^-$ | $C^+ C_{w_2} s^- C^-$ | $C^+ C_{w_2} s^- C^-$ | (8.10) |
| $l^- r_{w_2} s^+ r^+$ | $C^- C_{w_2} s^+ C^+$ | $C^+ C_{w_2} s^- C^-$ | (8.10) |
| $r^+ l_{w_2} s^- l^-$ | $C^+ C_{w_2} s^- C^-$ | $C^+ C_{w_2} s^- C^-$ | (8.10) |
| $r^- l_{w_2} s^+ l^+$ | $C^- C_{w_2} s^+ C^+$ | $C^+ C_{w_2} s^- C^-$ | (8.10) |
| $r^- s^- r_{w_2} l^+$ | $C^- s^- C_{w_2}^+ C^+$ | $C s^- C_{w_2}^+ C^+$ | (8.10) |
| $r^+ s^+ r_{w_2} l^-$ | $C^+ s^+ C_{w_2}^- C^-$ | $C s^- C_{w_2}^+ C^+$ | (8.10) |
| $l^+ s^- l_{w_2} r^+$ | $C^- s^- C_{w_2}^+ C^+$ | $C s^- C_{w_2}^+ C^+$ | (8.10) |
| $l^- s^+ l_{w_2} r^-$ | $C^+ s^+ C_{w_2}^- C^-$ | $C s^- C_{w_2}^+ C^+$ | (8.10) |
| $l^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C s^-$ | (8.2) |
| $l^- s^- r^-$ | $C^- s^- C^-$ | $C s^-$ | (8.2) |
| $r^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C s^-$ | (8.2) |
| $r^- s^- l^-$ | $C^- s^- C^-$ | $C s^-$ | (8.2) |
| $l^+ s^+ l^+$ | $C^+ s^+ C^+$ | $C s^-$ | (8.1) |
| $l^- s^- l^-$ | $C^- s^- C^-$ | $C s^-$ | (8.1) |
| $r^+ s^+ r^+$ | $C^+ s^+ C^+$ | $C s^-$ | (8.1) |
| $r^- s^- r^-$ | $C^- s^- C^-$ | $C s^-$ | (8.1) |
| $l^+ r_{w_2} s^- l_{w_2} r^+$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | (8.11), two roots |
| $l^- r_{w_2} s^+ l_{w_2} r^-$ | $C^- C_{w_2} s^+ C_{w_2}^- C^-$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | (8.11), two roots |
| $r^+ l_{w_2} s^- r_{w_2} l^-$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | (8.11), two roots |
| $r^- l_{w_2} s^+ r_{w_2} l^+$ | $C^- C_{w_2} s^+ C_{w_2}^- C^-$ | $C^+ C_{w_2} s^- C_{w_2}^+ C^+$ | (8.11), two roots |

Figure: Reeds-Shepp paths with timeflip, reflect, reverse simplifications

Reeds-Shepp paths: case $I_t^+ s_u^+ I_v^+$

- ▶ $(u, t) := R(x - \sin \phi, y - 1 + \cos \phi)$
- ▶ $v := M(\phi - t)$

With:

- ▶ $(r, \theta) := R(x, y)$ for the polar transform $r \cos \theta = x$ and $r \sin \theta = y$ with $r \leq 0$ and $-\pi \leq \theta < \pi$
- ▶ $\phi = M(\theta)$ if $\phi \equiv \theta \pmod{2\pi}$ and $-\pi \leq \phi < \pi$

Dubins and Reeds-Shepp paths in reality

- ▶ All kinds of sources for error in the real-world

Dubins and Reeds-Shepp paths in reality

- ▶ All kinds of sources for error in the real-world
- ▶ Paths found in the absence of obstacles however can use a rapidly exploring random tree and Dubins' pseudo distance or Reeds-Shepp distance

Sources

- ▶ Optimal paths for a car that goes both forwards and backwards (1990), J. A. Reeds and L. A. Shepp
- ▶ Planning algorithms: Reeds-Shepp curves (2006), Steven M. LaValle
- ▶ A Comprehensive, Step-by-Step Tutorial to Computing Dubins' Paths (2013), Andy G's Blog