

# Dubins and Reeds–Shepp paths

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# Table of contents

Dubins' paths

Implementation details

Reeds-Shepp paths

Reeds-Shepp paths simplifications

# Introduction

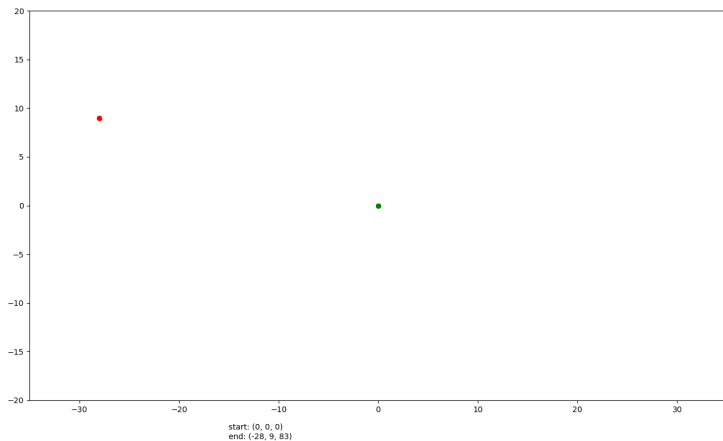


Figure: The shortest path in the case of a human being

# Introduction

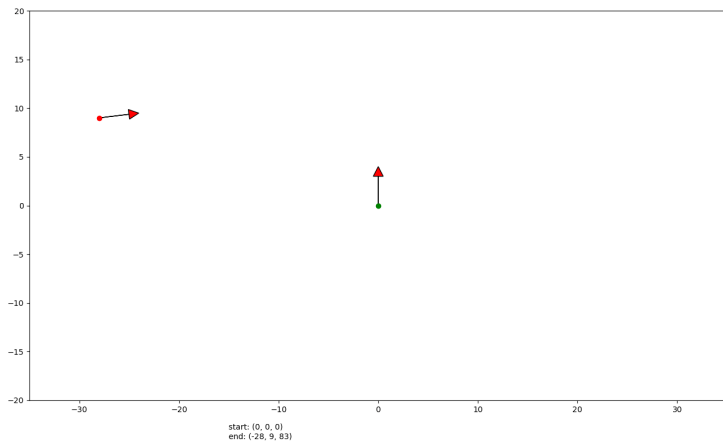


Figure: A shortest path in the case of an oriented human being

# Introduction

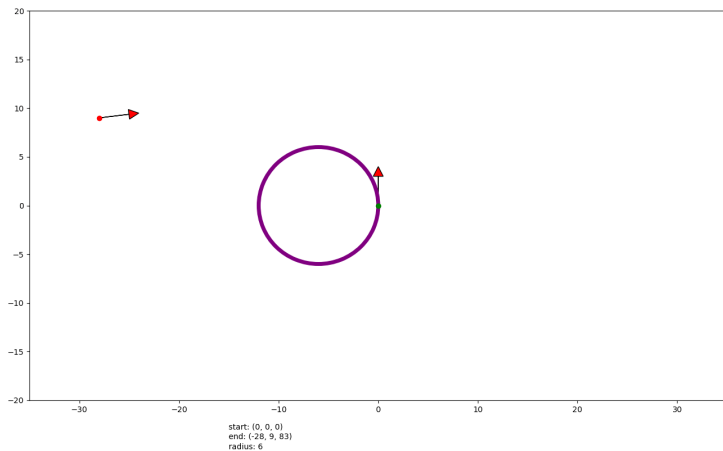


Figure: Robot's minimal turning circle

# Dubins' paths

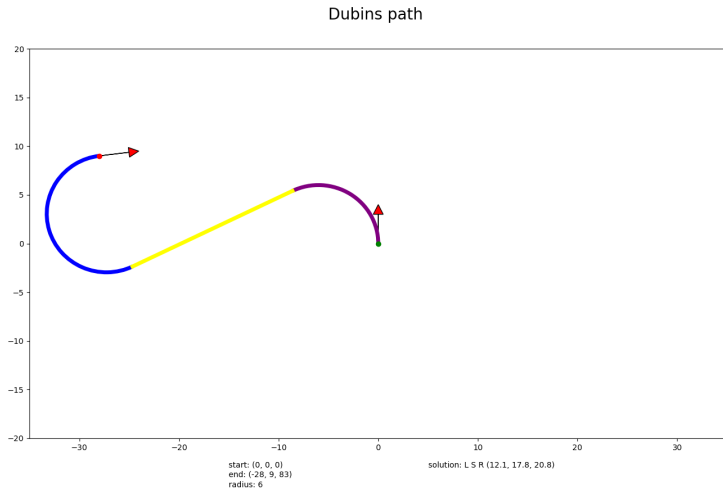


Figure: Dubins' path describing a shortest path

# Dubins' paths

<b>CSC</b>
RSR
LSL
LSR
RSL

(a) All combinaisons of **CSC** paths

<b>CCC</b>
RLR
LRL

(b) All combinaisons of **CCC** paths

C: means "curve" (i.e. turning left or right)

S: means "straight"

R: means "right" (i.e. turning right)

L: means "left" (i.e. turning left)

# Dubins' paths

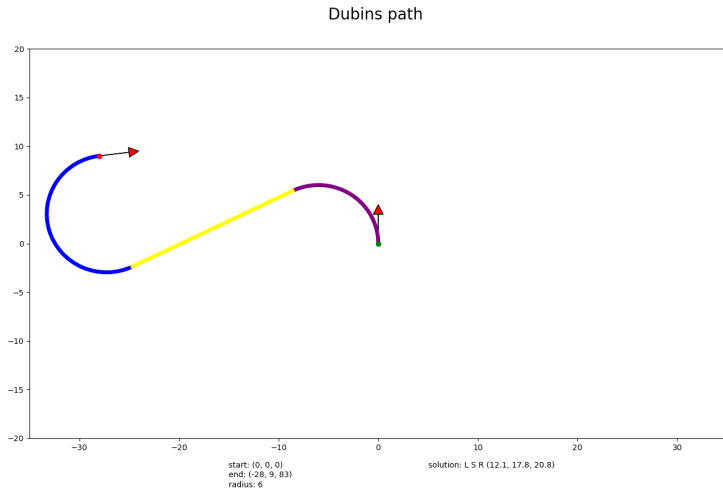


Figure: Dubins' path LSR describing the shortest path



# Implementation details

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## 1. Constant complexity

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2. Use of a table on angles

# Implementation details

1. Constant complexity
2. Use of a table on angles
3. Outer and inner tangents

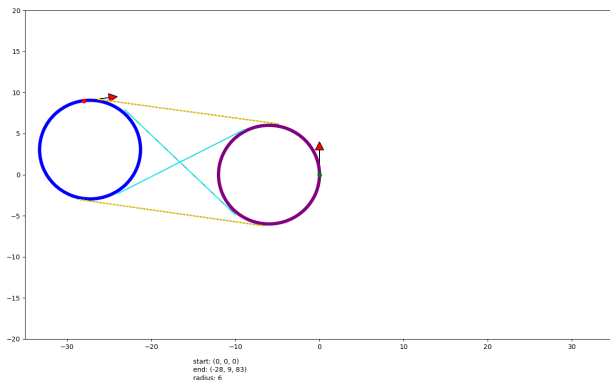


Figure: Same radius internal and external circles tangents

# Implementation details

- ▶  $\dot{x} = \cos \theta$
- ▶  $\dot{y} = \sin \theta$
- ▶  $\dot{\theta} = \frac{1}{r_{min \text{ turning}}}$
- ▶  $x_{new} = x_{prev} + \delta * \cos(\theta)$
- ▶  $y_{new} = y_{prev} + \delta * \sin(\theta)$
- ▶  $\theta_{new} = \theta_{prev} + \frac{\delta}{r_{min \text{ turning}}}$

With  $\delta$  a small value, typically below 0.05.

# Reeds-Shepp paths

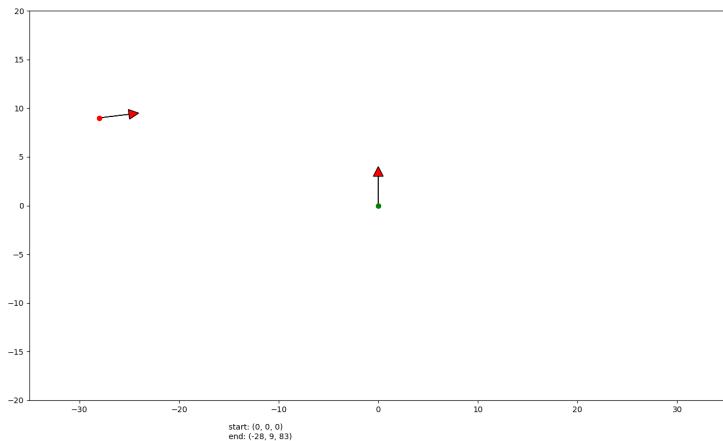


Figure: What is a shortest path for a robot able to go backward ?

# Reeds-Shepp paths

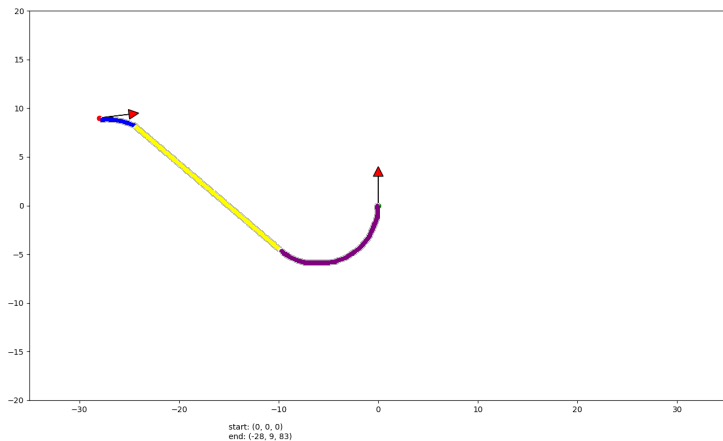


Figure: A shortest Reeds-Shepp path for a robot able to go backward

# Reeds-Shepp paths

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+ r^- l^+$	$C^+ C^- C^+$	$C \mid C \mid C$	(8.3), two roots
$l^- r^+ l^-$	$C^- C^+ C^-$	$C \mid C \mid C$	(8.3), two roots
$r^+ l^- r^+$	$C^+ C^- C^+$	$C \mid C \mid C$	(8.3), two roots
$r^- l^+ r^-$	$C^- C^+ C^-$	$C \mid C \mid C$	(8.3), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C \mid CC$	(8.4), two roots
$l^- r^+ l^+$	$C^- C^+ C^+$	$C \mid CC$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^- C^-$	$C \mid CC$	(8.4), two roots
$r^- l^+ r^+$	$C^- C^+ C^+$	$C \mid CC$	(8.4), two roots
$l^- r^- l^+$	$C^- C^- C^+$	$CC \mid C$	(8.4), two roots
$l^+ r^+ l^-$	$C^+ C^+ C^-$	$CC \mid C$	(8.4), two roots
$r^- l^- r^+$	$C^- C^- C^+$	$CC \mid C$	(8.4), two roots
$r^+ l^+ r^-$	$C^+ C^+ C^-$	$CC \mid C$	(8.4), two roots
$l^+ r_0^+ l_{-2} r^-$	$C^+ C_0^+ C_{-2} C^-$	$CC_0 \mid C_{-2} C$	(8.7), two roots
$l^- r_{-2} l_0^+ r^+$	$C^- C_{-2} C_0^+ C^+$	$CC_{-2} \mid C_0 C$	(8.7), two roots
$r^+ l_0^+ r_{-2} l^-$	$C^+ C_0^+ C_{-2} C^-$	$CC_0 \mid C_{-2} C$	(8.7), two roots
$r^- l_{-2} r_0^+ l^+$	$C^- C_{-2} C_0^+ C^+$	$CC_{-2} \mid C_0 C$	(8.7), two roots
$l^+ r_{-2} l_{-2} r^+$	$C^+ C_{-2} C_{-2} C^+$	$C \mid C_{-2} C_{-2} \mid C$	(8.8)
$l^- r_0^+ l_0^+ r^-$	$C^- C_0^+ C_0^+ C^-$	$C \mid C_0 C_0 \mid C$	(8.8)
$r^+ l_{-2} r_{-2} l^+$	$C^+ C_{-2} C_{-2} C^+$	$C \mid C_{-2} C_{-2} \mid C$	(8.8)
$r^- l_0^+ r_0^+ l^-$	$C^- C_0^+ C_0^+ C^-$	$C \mid C_0 C_0 \mid C$	(8.8)
$l^+ r_{-2} s^- l^-$	$C^+ C_{-2} s^- C^-$	$C \mid C_{-2} s C$	(8.9)
$l^- r_0 s^+ l^+$	$C^- C_0 s^+ C^+$	$C \mid C_0 s C$	(8.9)
$r^+ l_{-2} s^- r^-$	$C^+ C_{-2} s^- C^-$	$C \mid C_{-2} s C$	(8.9)
$r^- l_0 s^+ r^+$	$C^- C_0 s^+ C^+$	$C \mid C_0 s C$	(8.9)
$l^- s^- r_{-2} l^+$	$C^- s^- C_{-2} C^+$	$C s C_{-2} \mid C$	(8.9)
$l^+ s^+ r_0 l^-$	$C^+ s^+ C_0 C^-$	$C s C_0 \mid C$	(8.9)
$r^- s^- l_{-2} r^+$	$C^- s^- C_{-2} C^+$	$C s C_{-2} \mid C$	(8.9)
$r^+ s^+ l_0 r^-$	$C^+ s^+ C_0 C^-$	$C s C_0 \mid C$	(8.9)
$l^+ r_{-2} s^- r^-$	$C^+ C_{-2} s^- C^-$	$C \mid C_{-2} s C$	(8.10)
$l^- r_0 s^+ r^+$	$C^- C_0 s^+ C^+$	$C \mid C_0 s C$	(8.10)
$r^+ l_{-2} s^- l^-$	$C^+ C_{-2} s^- C^-$	$C \mid C_{-2} s C$	(8.10)
$r^- l_0 s^+ l^+$	$C^- C_0 s^+ C^+$	$C \mid C_0 s C$	(8.10)
$r^- s^- r_{-2} l^+$	$C^- s^- C_{-2} C^+$	$C s C_{-2} \mid C$	(8.10)
$r^+ s^+ r_0 l^-$	$C^+ s^+ C_0 C^-$	$C s C_0 \mid C$	(8.10)
$l^- s^- l_{-2} r^+$	$C^- s^- C_{-2} C^+$	$C s C_{-2} \mid C$	(8.10)
$l^+ s^+ l_0 r^-$	$C^+ s^+ C_0 C^-$	$C s C_0 \mid C$	(8.10)
$l^+ s^+ r^+$	$C^+ s^+ C^+$	$C s C$	(8.2)
$l^- s^- r^-$	$C^- s^- C^-$	$C s C$	(8.2)
$r^+ s^+ l^+$	$C^+ s^+ C^+$	$C s C$	(8.2)
$r^- s^- l^-$	$C^- s^- C^-$	$C s C$	(8.2)
$l^+ s^+ l^+$	$C^+ s^+ C^+$	$C s C$	(8.1)
$l^- s^- l^-$	$C^- s^- C^-$	$C s C$	(8.1)
$r^+ s^+ r^+$	$C^+ s^+ C^+$	$C s C$	(8.1)
$r^- s^- r^-$	$C^- s^- C^-$	$C s C$	(8.1)
$l^+ r_{-2} s^- l_{-2} r^+$	$C^+ C_{-2} s^- C_{-2} C^+$	$C \mid C_{-2} s C_{-2} \mid C$	(8.11), two roots
$l^- r_0 s^+ l_0^+ r^-$	$C^- C_0 s^+ C_0^+ C^-$	$C \mid C_0 s C_0 \mid C$	(8.11), two roots
$r^+ l_{-2} s^- r_{-2} l^+$	$C^+ C_{-2} s^- C_{-2} C^+$	$C \mid C_{-2} s C_{-2} \mid C$	(8.11), two roots
$r^- l_0 s^+ r_0^+ l^-$	$C^- C_0 s^+ C_0^+ C^-$	$C \mid C_0 s C_0 \mid C$	(8.11), two roots

Figure: All Reeds-Shepp paths



## Reeds-Shepp paths: timeflip simplification

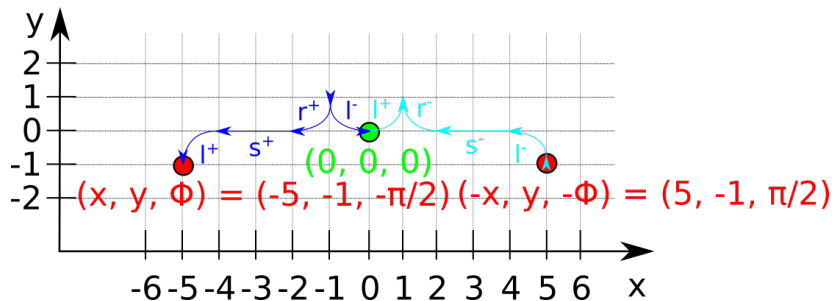


Figure: Timeflip simplification:  $l^- r^+ s^+ l^+$  goes from the  $(0, 0, 0)$  to  $(x, y, \phi)$  and  $l^+ r^- s^- l^-$  goes from  $(0, 0, 0)$  to  $(-x, y, -\phi)$

# Reeds-Shepp paths: timeflip simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+ r^- l^+$	$C^+ C^- C^+$	$C \mid C \mid C$	(8.3), two roots
$\bar{l}^- r^+ l^-$	$C^- C^+ C^-$	$C \mid C \mid C$	(8.3), two roots
$r^+ l^- r^+$	$C^+ C^- C^+$	$C \mid C \mid C$	(8.3), two roots
$r^- l^+ r^-$	$C^- C^+ C^-$	$C \mid C \mid C$	(8.3), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C \mid C C$	(8.4), two roots
$\bar{l}^- r^+ l^+$	$C^- C^+ C^+$	$C \mid C C$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^- C^-$	$C \mid C C$	(8.4), two roots
$r^- l^+ r^+$	$C^- C^+ C^+$	$C \mid C C$	(8.4), two roots
$\bar{l}^- r^+ l^+$	$C^- C^+ C^+$	$C C \mid C$	(8.4), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C C \mid C$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^- C^-$	$C C \mid C$	(8.4), two roots
$r^- l^+ r^+$	$C^- C^+ C^+$	$C C \mid C$	(8.4), two roots
$l^+ r^+ l^- r^-$	$C^+ C_+^+ C_- C^-$	$C C_+ \mid C_- C$	(8.7), two roots
$\bar{l}^- r^- l^+ r^+$	$C^- C_- C_+^+ C^+$	$C C_- \mid C_+ C$	(8.7), two roots
$r^+ l^+ r_- l^-$	$C^+ C_+^+ C_- C^-$	$C C_+ \mid C_- C$	(8.7), two roots
$r^- l^- r^+ r^+$	$C^- C_- C_+^+ C^+$	$C C_- \mid C_+ C$	(8.7), two roots
$l^+ r_- l^+ r^+$	$C^+ C_- C_+ C^+$	$C \mid C_+ C_+ \mid C$	(8.8)
$\bar{l}^- r^+ l^+ r^-$	$C^- C_+ C_+ C^-$	$C \mid C_+ C_+ \mid C$	(8.8)
$r^+ l_- r_- l^+$	$C^+ C_- C_- C^+$	$C \mid C_+ C_+ \mid C$	(8.8)
$r^- l^+ r^+ l^-$	$C^- C_+ C_+ C^-$	$C \mid C_+ C_+ \mid C$	(8.8)
$l^+ r_{\pi/2} s^- l^-$	$C^+ C_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} S C$	(8.9)
$\bar{l}^- r_{\pi/2} s^+ l^+$	$C^- C_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} S C$	(8.9)
$r^+ l_{\pi/2} s^- r^-$	$C^+ C_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} S C$	(8.9)
$r^- l_{\pi/2} s^+ r^+$	$C^- C_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} S C$	(8.9)
$l^+ s^- r_{\pi/2} l^+$	$C^+ S^- C_{-\pi/2} C^+$	$C S C_{\pi/2} \mid C$	(8.9)
$\bar{l}^- s^+ r_{\pi/2} l^-$	$C^- S^+ C_{\pi/2} C^-$	$C S C_{\pi/2} \mid C$	(8.9)
$r^+ s^- l_{\pi/2} r^+$	$C^+ S^- C_{-\pi/2} C^+$	$C S C_{\pi/2} \mid C$	(8.9)
$r^- s^+ l_{\pi/2} r^-$	$C^- S^+ C_{\pi/2} C^-$	$C S C_{\pi/2} \mid C$	(8.9)
$l^+ r_{\pi/2} s^- r^-$	$C^+ C_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} S C$	(8.10)
$\bar{l}^- r_{\pi/2} s^+ r^+$	$C^- C_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} S C$	(8.10)
$r^+ l_{\pi/2} s^- l^-$	$C^+ C_{-\pi/2} S^- C^-$	$C \mid C_{\pi/2} S C$	(8.10)
$r^- l_{\pi/2} s^+ l^+$	$C^- C_{\pi/2} S^+ C^+$	$C \mid C_{\pi/2} S C$	(8.10)
$r^- s^- r_{\pi/2} l^+$	$C^- S^- C_{-\pi/2} C^+$	$C S C_{\pi/2} \mid C$	(8.10)
$r^+ s^+ r_{\pi/2} l^-$	$C^+ S^+ C_{\pi/2} C^-$	$C S C_{\pi/2} \mid C$	(8.10)
$\bar{l}^- s^- l_{\pi/2} r^+$	$C^- S^- C_{-\pi/2} C^+$	$C S C_{\pi/2} \mid C$	(8.10)
$l^+ s^+ l_{\pi/2} r^-$	$C^+ S^+ C_{\pi/2} C^-$	$C S C_{\pi/2} \mid C$	(8.10)
$l^+ s^+ r^+$	$C^+ S^+ C^+$	$C S C$	(8.2)
$\bar{l}^- s^- r^-$	$C^- S^- C^-$	$C S C$	(8.2)
$r^+ s^+ l^+$	$C^+ S^+ C^+$	$C S C$	(8.2)
$r^- s^- l^-$	$C^- S^- C^-$	$C S C$	(8.2)
$l^+ s^+ l^+$	$C^+ S^+ C^+$	$C S C$	(8.1)
$\bar{l}^- s^- l^-$	$C^- S^- C^-$	$C S C$	(8.1)
$r^+ s^+ r^+$	$C^+ S^+ C^+$	$C S C$	(8.1)
$r^- s^- r^-$	$C^- S^- C^-$	$C S C$	(8.1)
$l^+ r_{\pi/2} s^- l_{\pi/2} r^+$	$C^+ C_{-\pi/2} S^- C_{-\pi/2} C^+$	$C \mid C_{\pi/2} S C_{\pi/2} \mid C$	(8.11), two roots
$\bar{l}^- r_{\pi/2} s^+ l_{\pi/2} r^-$	$C^- C_{\pi/2} S^+ C_{\pi/2} C^-$	$C \mid C_{\pi/2} S C_{\pi/2} \mid C$	(8.11), two roots
$r^+ l_{\pi/2} s^- r_{\pi/2} l^+$	$C^+ C_{-\pi/2} S^- C_{-\pi/2} C^+$	$C \mid C_{\pi/2} S C_{\pi/2} \mid C$	(8.11), two roots
$r^- l_{\pi/2} s^+ r_{\pi/2} l^-$	$C^- C_{\pi/2} S^+ C_{\pi/2} C^-$	$C \mid C_{\pi/2} S C_{\pi/2} \mid C$	(8.11), two roots

Figure: All Reeds-Shepp paths with timeflip simplification

## Reeds-Shepp paths: reflect simplification

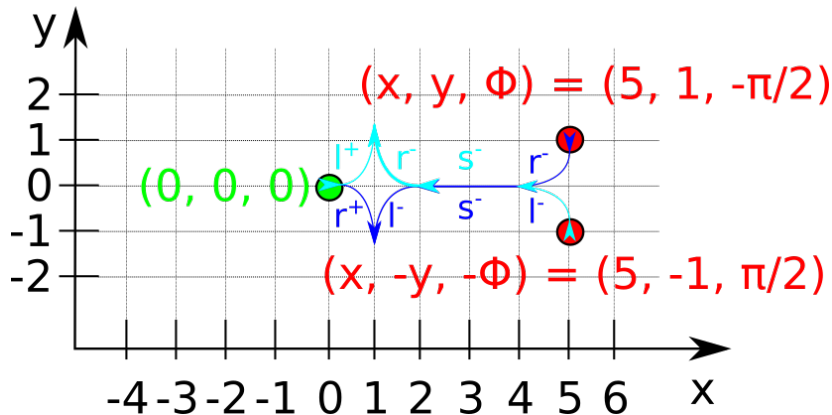


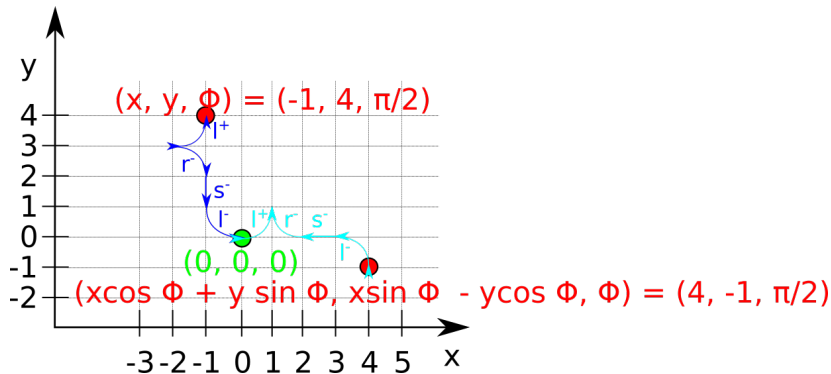
Figure: Reflect simplification:  $r^+l^-s^-r^-$  goes from the  $(0, 0, 0)$  to  $(x, y, \phi)$  and  $l^+r^-s^-l^-$  goes from  $(0, 0, 0)$  to  $(x, -y, -\phi)$

# Reeds-Shepp paths: reflect simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+ r^- l^+$	$C^+ C^- C^+$	$C^+ C^+ C^+$	(8.3), two roots
$l^+ r^- l^-$	$C^- C^+ C^-$	$C^+ C^+ C^+$	(8.3), two roots
$r^+ l^- r^+$	$C^+ C^- C^+$	$C^+ C^+ C^+$	(8.3), two roots
$r^+ l^- r^-$	$C^- C^+ C^-$	$C^+ C^+ C^+$	(8.3), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r^- l^+$	$C^- C^+ C^+$	$C^+ C^+ C^+$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^- C^-$	$C^+ C^+ C^+$	(8.4), two roots
$r^+ l^- r^+$	$C^- C^+ C^+$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r^- l^+$	$C^- C^- C^+$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r^- l^-$	$C^+ C^+ C^-$	$C^+ C^+ C^+$	(8.4), two roots
$r^+ l^- r^+$	$C^- C^- C^+$	$C^+ C^+ C^+$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^+ C^-$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r_{12}^+ l^- r^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.7), two roots
$l^+ r_{12}^+ l^+ r^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.7), two roots
$r^+ l_{12}^+ r_{12}^- l^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.7), two roots
$r^+ l_{12}^+ r_{12}^+ l^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.7), two roots
$l^+ r_{12}^+ l^- r^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.8)
$l^+ r_{12}^+ l^+ r^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.8)
$r^+ l_{12}^+ r_{12}^- l^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.8)
$r^+ l_{12}^+ r_{12}^+ l^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.8)
$l^+ r_{12}^+ l^- r^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.9)
$l^+ r_{12}^+ l^+ r^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.9)
$r^+ l_{12}^+ r_{12}^- l^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.9)
$r^+ l_{12}^+ r_{12}^+ l^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.9)
$l^+ r_{12}^+ l^- r^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.10)
$l^+ r_{12}^+ l^+ r^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.10)
$r^+ l_{12}^+ r_{12}^- l^-$	$C^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ C_{12}^- C$	(8.10)
$r^+ l_{12}^+ r_{12}^+ l^+$	$C^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ C_{12}^- C$	(8.10)
$r^- s^- r_{12}^+ l^+$	$C^- S^- C_{12}^+ C^+$	$C S C_{12}^+ C$	(8.10)
$r^+ s^+ r_{12}^+ l^+$	$C^+ S^+ C_{12}^+ C^+$	$C S C_{12}^+ C$	(8.10)
$l^+ s^+ l_{12}^+ r^+$	$C^- S^- C_{12}^+ C^+$	$C S C_{12}^+ C$	(8.10)
$l^+ s^+ l_{12}^+ r^-$	$C^+ S^+ C_{12}^+ C^+$	$C S C_{12}^+ C$	(8.10)
$l^+ s^+ r^+$	$C^+ S^+ C^+$	$C S C$	(8.2)
$l^+ s^+ r^-$	$C^- S^- C^-$	$C S C$	(8.2)
$r^+ s^+ l^+$	$C^+ S^+ C^+$	$C S C$	(8.2)
$r^+ s^+ l^-$	$C^- S^- C^-$	$C S C$	(8.2)
$l^+ s^+ l^+$	$C^+ S^+ C^+$	$C S C$	(8.1)
$l^+ s^+ l^-$	$C^- S^- C^-$	$C S C$	(8.1)
$r^+ s^+ r^+$	$C^+ S^+ C^+$	$C S C$	(8.1)
$r^+ s^+ r^-$	$C^- S^- C^-$	$C S C$	(8.1)
$l^+ r_{12}^+ l_{12}^- l_{12}^+ r^+$	$C^+ C_{12}^+ S^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ S^- C_{12}^- C_{12}^+ C$	(8.11), two roots
$l^+ r_{12}^+ l_{12}^- l_{12}^+ r^-$	$C^- C_{12}^- S^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ S^- C_{12}^- C_{12}^+ C$	(8.11), two roots
$r^+ l_{12}^+ r_{12}^- r_{12}^+ l^+$	$C^+ C_{12}^+ S^- C_{12}^- C_{12}^+ C^+$	$C C_{12}^+ S^- C_{12}^- C_{12}^+ C$	(8.11), two roots
$r^+ l_{12}^+ r_{12}^- r_{12}^+ l^-$	$C^- C_{12}^- S^+ C_{12}^+ C_{12}^- C^-$	$C C_{12}^+ S^- C_{12}^- C_{12}^+ C$	(8.11), two roots

Figure: All Reeds-Shepp paths with **timeflip** and **reflect** simplifications

## Reeds-Shepp paths: reverse simplification



**Figure:** Reverse simplification:  $I^-s^-r^-I^+$  goes from the  $(0, 0, 0)$  to  $(x, y, \phi)$  and  $I^+r^-s^-I^-$  goes from  $(0, 0, 0)$  to  $(x \cos \phi + y \sin \phi, x \sin \phi - y \cos \phi, \phi)$

# Reeds-Shepp paths: reverse simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+ r^- l^+$	$C^+ C^- C^+$	$C^+ C^+ C^+$	(8.3), two roots
$l^- r^+ l^-$	$C^- C^+ C^-$	$C^+ C^+ C^+$	(8.3), two roots
$r^+ l^- r^+$	$C^+ C^- C^+$	$C^+ C^+ C^+$	(8.3), two roots
$r^- l^+ r^-$	$C^- C^+ C^-$	$C^+ C^+ C^+$	(8.3), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C^+ C^+ C^+$	(8.4), two roots
$l^- r^+ l^+$	$C^- C^+ C^+$	$C^+ C^+ C^+$	(8.4), two roots
$r^+ l^- r^-$	$C^+ C^- C^-$	$C^+ C^+ C^+$	(8.4), two roots
$r^- l^+ r^+$	$C^- C^+ C^+$	$C^+ C^+ C^+$	(8.4), two roots
$l^- r^+ l^+$	$C^- C^+ C^+$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r^- l^-$	$C^+ C^- C^-$	$C^+ C^+ C^+$	(8.4), two roots
$l^+ r^+ l^- r^-$	$C^+ C_1^+ C_{-2}^- C^-$	$C C_{-1}^+ C_{-2}^- C^-$	(8.7), two roots
$l^- r^- l^+ r^+$	$C^- C_{-1}^- C_2^+ C^+$	$C C_{-1}^+ C_{-2}^- C^-$	(8.7), two roots
$r^+ l^+ r^- l^-$	$C^+ C_1^+ C_{-2}^- C^-$	$C C_{-1}^+ C_{-2}^- C^-$	(8.7), two roots
$r^- l^- r^+ r^+$	$C^- C_{-1}^- C_2^+ C^+$	$C C_{-1}^+ C_{-2}^- C^-$	(8.7), two roots
$l^+ r^- l^- r^+$	$C^+ C_{-2}^- C_{-1}^- C^+$	$C^+ C_{-1}^- C_{-2}^- C^+$	(8.8)
$l^- r^+ l^+ r^-$	$C^- C_1^+ C_2^+ C^-$	$C^+ C_{-1}^- C_{-2}^- C^+$	(8.8)
$r^+ l^- r^+ l^-$	$C^+ C_{-2}^- C_{-1}^- C^+$	$C^+ C_{-1}^- C_{-2}^- C^+$	(8.8)
$r^- l^+ r^- l^+$	$C^- C_1^+ C_2^+ C^-$	$C^+ C_{-1}^- C_{-2}^- C^+$	(8.8)
$l^+ r_{w/2}^+ l^-$	$C^+ C_{-w/2}^+ C^-$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$l^- r_{w/2}^- l^+$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$r^+ l_{w/2}^+ r^-$	$C^+ C_{-w/2}^+ C^-$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$r^- l_{w/2}^- r^+$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$l^+ r^- r_{w/2}^+ l^+$	$C^- C^- C_{-w/2}^+ C^+$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$l^- r^+ r_{w/2}^- l^-$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$r^+ l_{w/2}^+ r^-$	$C^+ C_{-w/2}^+ C^-$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$r^- l_{w/2}^- r^+$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.9)
$l^+ r_{w/2}^+ r^-$	$C^+ C_{-w/2}^+ C^-$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$l^- r_{w/2}^- r^+$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$r^+ l_{w/2}^+ r^-$	$C^+ C_{-w/2}^+ C^-$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$r^- l_{w/2}^- r^+$	$C^- C_{w/2}^- C^+$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$r^- r^- r_{w/2}^+ l^+$	$C^- C^- C_{-w/2}^+ C^+$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$r^+ r^+ r_{w/2}^- l^-$	$C^+ C^+ C_{w/2}^- C^-$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$l^+ r^- r_{w/2}^+ l^+$	$C^- C^- C_{-w/2}^+ C^+$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$l^- r^+ r_{w/2}^- l^-$	$C^- C^+ C_{w/2}^- C^-$	$C^+ C_{-w/2}^+ C^-$	(8.10)
$l^+ r^+ r^-$	$C^+ C^+ C^-$	$C^+ C^+ C^-$	(8.2)
$l^- r^- r^+$	$C^- C^- C^+$	$C^+ C^+ C^-$	(8.2)
$r^+ r^+ l^+$	$C^+ C^+ C^+$	$C^+ C^+ C^-$	(8.2)
$r^- r^- l^-$	$C^- C^- C^-$	$C^+ C^+ C^-$	(8.2)
$l^+ r^+ l^+$	$C^+ C^+ C^+$	$C^+ C^+ C^-$	(8.1)
$l^- r^- l^-$	$C^- C^- C^-$	$C^+ C^+ C^-$	(8.1)
$r^+ r^+ r^+$	$C^+ C^+ C^+$	$C^+ C^+ C^-$	(8.1)
$r^- r^- r^-$	$C^- C^- C^-$	$C^+ C^+ C^-$	(8.1)
$l^+ r_{w/2}^+ l_{w/2}^+ r^-$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	(8.11), two roots
$l^- r_{w/2}^- l_{w/2}^- r^+$	$C^- C_{w/2}^- C_{-w/2}^- C^+$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	(8.11), two roots
$r^+ l_{w/2}^+ r_{w/2}^+ l^-$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	(8.11), two roots
$r^- l_{w/2}^- r_{w/2}^- l^+$	$C^- C_{w/2}^- C_{-w/2}^- C^+$	$C^+ C_{w/2}^+ C_{-w/2}^+ C^-$	(8.11), two roots

Figure: Reeds-Shepp paths with timeflip, reflect, reverse simplifications

## Reeds-Shepp paths: case $I_t^+ s_u^+ I_v^+$

- ▶  $(u, t) := R(x - \sin \phi, y - 1 + \cos \phi)$
- ▶  $v := M(\phi - t)$

With:

- ▶  $(r, \theta) := R(x, y)$  for the polar transform  $r \cos \theta = x$  and  $r \sin \theta = y$  with  $r \leq 0$  and  $-\pi \leq \theta < \pi$
- ▶  $\phi = M(\theta)$  if  $\phi \equiv \theta \pmod{2\pi}$  and  $-\pi \leq \phi < \pi$

# Dubins and Reeds-Shepp paths in reality

- ▶ All kinds of sources for error in the real-world



# Dubins and Reeds-Shepp paths in reality

- ▶ All kinds of sources for error in the real-world
- ▶ Paths found in the absence of obstacles however can use a rapidly exploring random tree and Dubins' pseudo distance or Reeds-Shepp distance

# Sources

- ▶ Optimal paths for a car that goes both forwards and backwards (1990), J. A. Reeds and L. A. Shepp
- ▶ Planning algorithms: Reeds-Shepp curves (2006), Steven M. LaValle
- ▶ A Comprehensive, Step-by-Step Tutorial to Computing Dubins' Paths (2013), Andy G's Blog