Dubins and Reeds–Shepp paths

Benjamin Loison

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Introduction

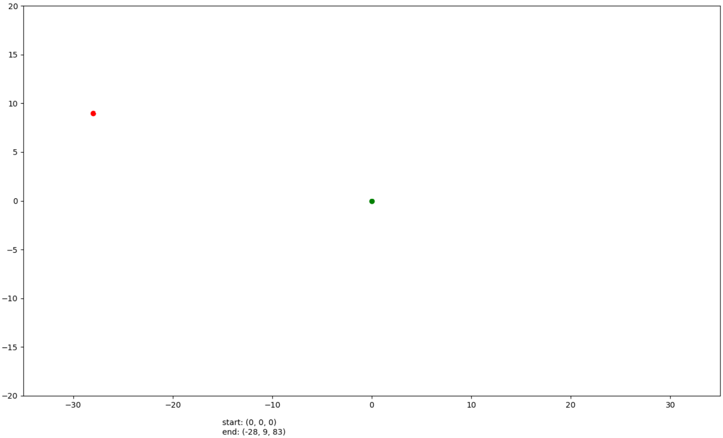

Figure: The shortest path in the case of a human being

Introduction

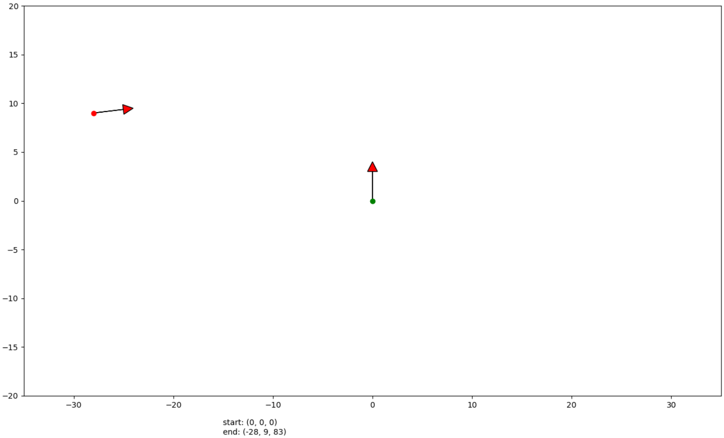

Figure: A shortest path in the case of an oriented human being

Introduction

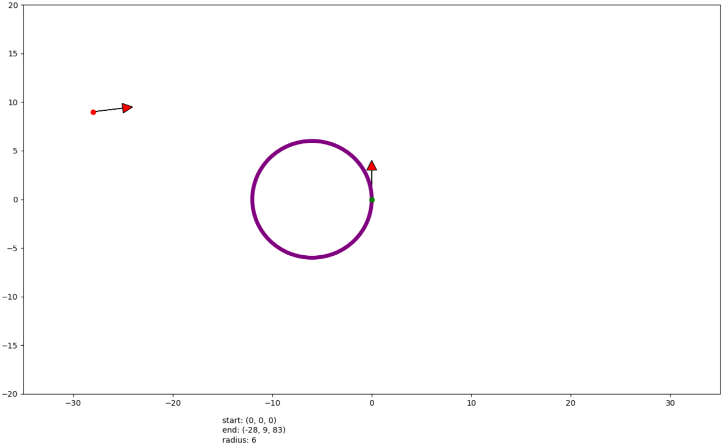

Figure: Robot's minimal turning circle

Dubins' paths

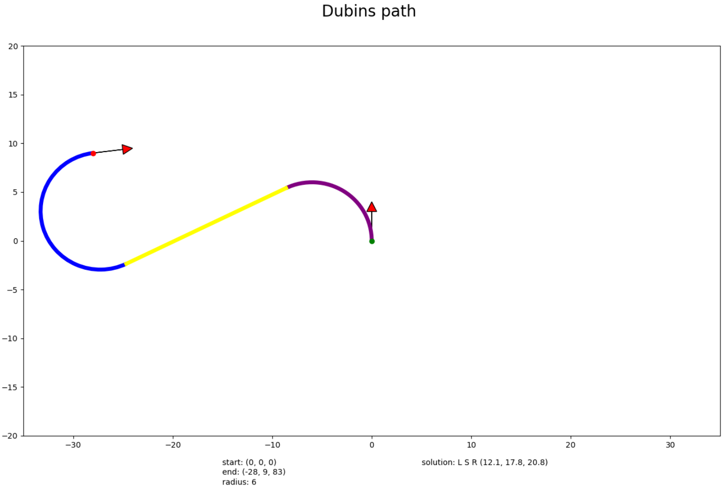

Figure: Dubins' path describing a shortest path

Dubins' paths

CSC
RSR
LSL
LSR
RSL

(a) All combinaisons of **CSC** paths

CCC
RLR
LRL

(b) All combinaisons of **CCC** paths

C: means "curve" (i.e. turning left or right)
S: means "straight"
R: means "right" (i.e. turning right)
L: means "left" (i.e. turning left)

Dubins' paths

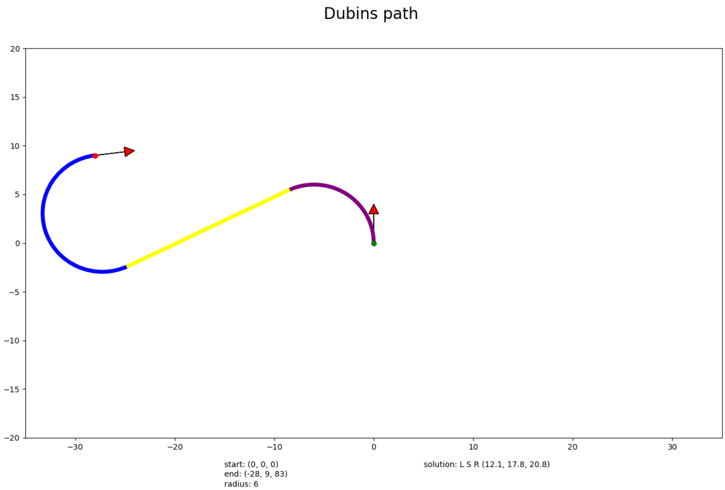

Figure: Dubins' path LSR descriebing the shortest path

Implementation details

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1. Constant complexity

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2. Use of a table on angles

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2. Use of a table on angles
3. Outer and inner tangents

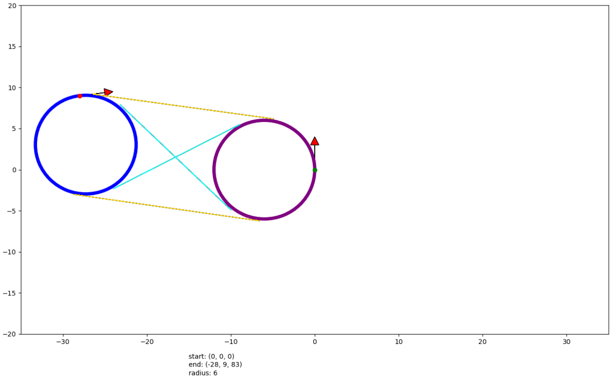

Figure: Same radius internal and external circles tangents

Implementation details

- $\dot{x} = \cos\,\theta$
- $\dot{y} = \sin\,\theta$
- $\dot{\theta} = \frac{1}{r_{min\ \ turning}}$
- $x_{new} = x_{prev} + \delta * cos(\theta)$
- $y_{new} = y_{prev} + \delta * sin(\theta)$
- $\theta_{new} = \theta_{prev} + \frac{\delta}{r_{min\ \ turning}}$

With δ a small value, typically below 0.05.

Reeds-Shepp paths

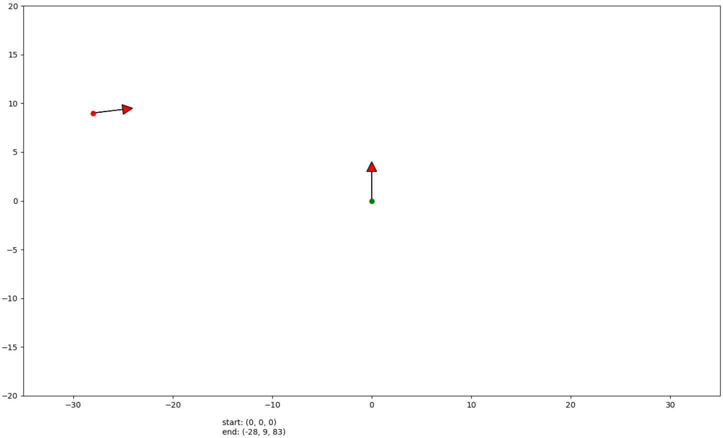

Figure: What is a shortest path for a robot able to go backward ?

Reeds-Shepp paths

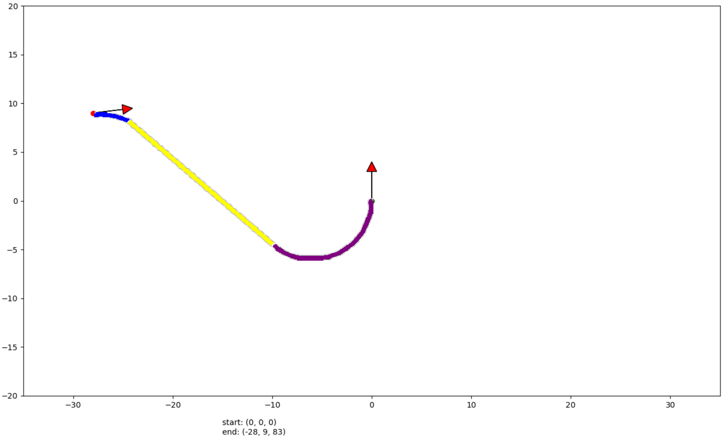

Figure: A shortest Reeds-Shepp path for a robot able to go backward

Reeds-Shepp paths

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$l^-r^+l^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$r^+l^-r^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$r^-l^+r^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$l^+r^-l^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$l^-r^+l^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$r^+l^-r^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$r^-l^+r^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$l^-r^-l^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$l^+r^+l^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$r^-l^-r^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$r^+l^+r^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$l^+r^+_ul^-_{-u}r^-$	$C^+C^+_uC^-_{-u}C^-$	$CC_u \mid C_uC$	(8.7), two roots
$l^-r^-_{-u}l^+_ur^+$	$C^-C^-_{-u}C^+_uC^+$	$CC_u \mid C_uC$	(8.7), two roots
$r^+l^+_ur^-_{-u}l^-$	$C^+C^+_uC^-_{-u}C^-$	$CC_u \mid C_uC$	(8.7), two roots
$r^-l^-_{-u}r^+_ul^+$	$C^-C^-_{-u}C^+_uC^+$	$CC_u \mid C_uC$	(8.7), two roots
$l^+r^-_{-u}l^-_{-u}r^+$	$C^+C^-_{-u}C^-_{-u}C^+$	$C \mid C_uC_u \mid C$	(8.8)
$l^-r^+_ul^+_ur^-$	$C^-C^+_uC^+_uC^-$	$C \mid C_uC_u \mid C$	(8.8)
$r^+l^-_{-u}r^-_{-u}l^+$	$C^+C^-_{-u}C^-_{-u}C^+$	$C \mid C_uC_u \mid C$	(8.8)
$r^-l^+_ur^+_ul^-$	$C^-C^+_uC^+_uC^-$	$C \mid C_uC_u \mid C$	(8.8)
$l^+r^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
$l^-r^+_{\pi/2}s^+l^+$	$C^-C^+_{\pi/2}S^+C^+$	$C \mid C_{\pi/2}SC$	(8.9)
$r^+l^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
$r^-l^+_{\pi/2}s^+r^+$	$C^-C^+_{\pi/2}S^+C^+$	$C \mid C_{\pi/2}SC$	(8.9)
$l^-s^-r^-_{-\pi/2}l^+$	$C^-S^-C^-_{-\pi/2}C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$r^-s^-l^-_{-\pi/2}r^+$	$C^-S^-C^-_{-\pi/2}C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$r^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+r^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
$l^-r^+_{\pi/2}s^+r^+$	$C^-C^+_{\pi/2}S^+C^+$	$C \mid C_{\pi/2}SC$	(8.10)
$r^+l^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
$r^-l^+_{\pi/2}s^+l^+$	$C^-C^+_{\pi/2}S^+C^+$	$C \mid C_{\pi/2}SC$	(8.10)
$r^-s^-r^-_{-\pi/2}l^+$	$C^-S^-C^-_{-\pi/2}C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$r^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^-s^-l^-_{-\pi/2}r^+$	$C^-S^-C^-_{-\pi/2}C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
$l^-s^-r^-$	$C^-S^-C^-$	CSC	(8.2)
$r^+s^+l^+$	$C^+S^+C^+$	CSC	(8.2)
$r^-s^-l^-$	$C^-S^-C^-$	CSC	(8.2)
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
$l^-s^-l^-$	$C^-S^-C^-$	CSC	(8.1)
$r^+s^+r^+$	$C^+S^+C^+$	CSC	(8.1)
$r^-s^-r^-$	$C^-S^-C^-$	CSC	(8.1)
$l^+r^-_{-\pi/2}s^-l^-_{-\pi/2}r^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$l^-r^+_{\pi/2}s^+l^+_{\pi/2}r^-$	$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$r^+l^-_{-\pi/2}s^-r^-_{-\pi/2}l^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$r^-l^+_{\pi/2}s^+r^+_{\pi/2}l^-$	$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots

Figure: All Reeds-Shepp paths

Reeds-Shepp paths: timeflip simplification

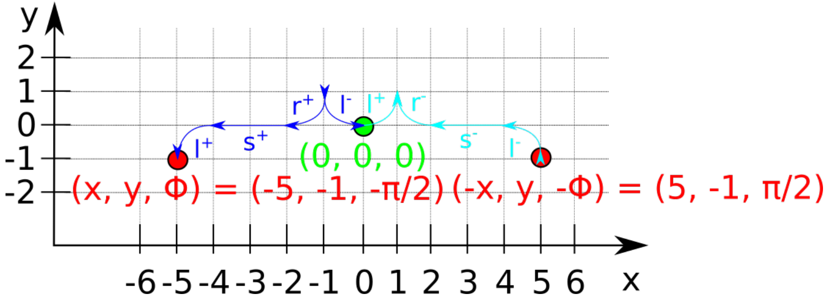

Figure: Timeflip simplification: $l^- r^+ s^+ l^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+ r^- s^- l^-$ goes from (0, 0, 0) to $(-x, y, -\phi)$

Reeds-Shepp paths: timeflip simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
~~$l^-r^+l^-$~~	~~$C^-C^+C^-$~~	~~$C \mid C \mid C$~~	~~(8.3), two roots~~
$r^+l^-r^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
~~$r^-l^+r^-$~~	~~$C^-C^+C^-$~~	~~$C \mid C \mid C$~~	~~(8.3), two roots~~
$l^+r^-l^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
~~$l^-r^+l^+$~~	~~$C^-C^+C^+$~~	~~$C \mid CC$~~	~~(8.4), two roots~~
$r^+l^-r^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
~~$r^-l^+r^+$~~	~~$C^-C^+C^+$~~	~~$C \mid CC$~~	~~(8.4), two roots~~
~~$l^-r^-l^+$~~	~~$C^-C^-C^+$~~	~~$CC \mid C$~~	~~(8.4), two roots~~
$l^+r^+l^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
~~$r^-l^-r^+$~~	~~$C^-C^-C^+$~~	~~$CC \mid C$~~	~~(8.4), two roots~~
$r^+l^+r^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$l^+r_u^+l_{-u}^-r^-$	$C^+C_u^+C_{-u}^-C^-$	$CC_u \mid C_uC$	(8.7), two roots
~~$l^-r_{-u}^-l_u^+r^+$~~	~~$C^-C_{-u}^-C_u^+C^+$~~	~~$CC_u \mid C_uC$~~	~~(8.7), two roots~~
$r^+l_u^+r_{-u}^-l^-$	$C^+C_u^+C_{-u}^-C^-$	$CC_u \mid C_uC$	(8.7), two roots
~~$r^-l_{-u}^-r_u^+l^+$~~	~~$C^-C_{-u}^-C_u^+C^+$~~	~~$CC_u \mid C_uC$~~	~~(8.7), two roots~~
$l^+r_{-u}^-l_{-u}^-r^+$	$C^+C_{-u}^-C_{-u}^-C^+$	$C \mid C_uC_u \mid C$	(8.8)
~~$l^-r_u^+l_u^+r^-$~~	~~$C^-C_u^+C_u^+C^-$~~	~~$C \mid C_uC_u \mid C$~~	~~(8.8)~~
$r^+l_{-u}^-r_{-u}^-l^+$	$C^+C_{-u}^-C_{-u}^-C^+$	$C \mid C_uC_u \mid C$	(8.8)
~~$r^-l_u^+r_u^+l^-$~~	~~$C^-C_u^+C_u^+C^-$~~	~~$C \mid C_uC_u \mid C$~~	~~(8.8)~~
$l^+r_{-\pi/2}^-s^-l^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
~~$l^-r_{\pi/2}^+s^+l^+$~~	~~$C^-C_{\pi/2}^+S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.9)~~
$r^+l_{-\pi/2}^-s^-r^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
~~$r^-l_{\pi/2}^+s^+r^+$~~	~~$C^-C_{\pi/2}^+S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$l^-s^-r_{-\pi/2}^-l^+$~~	~~$C^-S^-C_{-\pi/2}^-C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.9)~~
$l^+s^+r_{\pi/2}^+l^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.9)
~~$r^-s^-l_{-\pi/2}^-r^+$~~	~~$C^-S^-C_{-\pi/2}^-C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.9)~~
$r^+s^+l_{\pi/2}^+r^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+r_{-\pi/2}^-s^-r^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
~~$l^-r_{\pi/2}^+s^+r^+$~~	~~$C^-C_{\pi/2}^+S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.10)~~
$r^+l_{-\pi/2}^-s^-l^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
~~$r^-l_{\pi/2}^+s^+l^+$~~	~~$C^-C_{\pi/2}^+S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-s^-r_{-\pi/2}^-l^+$~~	~~$C^-S^-C_{-\pi/2}^-C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.10)~~
$r^+s^+r_{\pi/2}^+l^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.10)
~~$l^-s^-l_{-\pi/2}^-r^+$~~	~~$C^-S^-C_{-\pi/2}^-C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.10)~~
$l^+s^+l_{\pi/2}^+r^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
~~$l^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$r^+s^+l^+$	$C^+S^+C^+$	CSC	(8.2)
~~$r^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
~~$l^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$r^+s^+r^+$	$C^+S^+C^+$	CSC	(8.1)
~~$r^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$l^+r_{-\pi/2}^-s^-l_{-\pi/2}^-r^+$	$C^+C_{-\pi/2}^-S^-C_{-\pi/2}^-C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
~~$l^-r_{\pi/2}^+s^+l_{\pi/2}^+r^-$~~	~~$C^-C_{\pi/2}^+S^+C_{\pi/2}^+C^-$~~	~~$C \mid C_{\pi/2}SC_{\pi/2} \mid C$~~	~~(8.11), two roots~~
$r^+l_{-\pi/2}^-s^-r_{-\pi/2}^-l^+$	$C^+C_{-\pi/2}^-S^-C_{-\pi/2}^-C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
~~$r^-l_{\pi/2}^+s^+r_{\pi/2}^+l^-$~~	~~$C^-C_{\pi/2}^+S^+C_{\pi/2}^+C^-$~~	~~$C \mid C_{\pi/2}SC_{\pi/2} \mid C$~~	~~(8.11), two roots~~

Figure: All Reeds-Shepp paths with timeflip simplification

Reeds-Shepp paths: reflect simplification

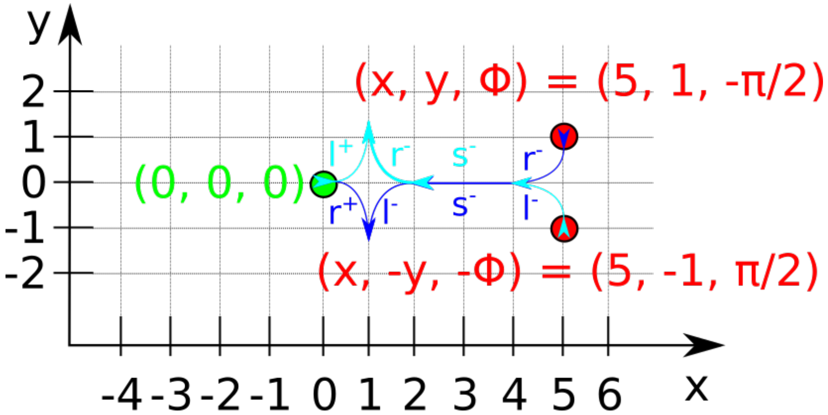

Figure: Reflect simplification: $r^+l^-s^-r^-$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+r^-s^-l^-$ goes from (0, 0, 0) to $(x, -y, -\phi)$

Reeds-Shepp paths: reflect simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
~~$l^-r^+l^-$~~	~~$C^-C^+C^-$~~	~~$C \mid C \mid C$~~	~~(8.3), two roots~~
~~$r^+l^-r^+$~~	~~$C^+C^-C^+$~~	~~$C \mid C \mid C$~~	~~(8.3), two roots~~
~~$r^-l^+r^-$~~	~~$C^-C^+C^-$~~	~~$C \mid C \mid C$~~	~~(8.3), two roots~~
$l^+r^-l^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
~~$l^-r^+l^+$~~	~~$C^-C^+C^+$~~	~~$C \mid CC$~~	~~(8.4), two roots~~
~~$r^+l^-r^-$~~	~~$C^+C^-C^-$~~	~~$C \mid CC$~~	~~(8.4), two roots~~
~~$r^-l^+r^+$~~	~~$C^-C^+C^+$~~	~~$C \mid CC$~~	~~(8.4), two roots~~
~~$l^-r^-l^+$~~	~~$C^-C^-C^+$~~	~~$CC \mid C$~~	~~(8.4), two roots~~
$l^+r^+l^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
~~$r^-l^-r^+$~~	~~$C^-C^-C^+$~~	~~$CC \mid C$~~	~~(8.4), two roots~~
~~$r^+l^+r^-$~~	~~$C^+C^+C^-$~~	~~$CC \mid C$~~	~~(8.4), two roots~~
$l^+r^+_ul^-_{-u}r^-$	$C^+C^+_uC^-_{-u}C^-$	$CC_u \mid C_uC$	(8.7), two roots
~~$l^-r^-_{-u}l^+_ur^+$~~	~~$C^-C^-_{-u}C^+_uC^+$~~	~~$CC_u \mid C_uC$~~	~~(8.7), two roots~~
~~$r^+l^+_ur^-_{-u}l^-$~~	~~$C^+C^+_uC^-_{-u}C^-$~~	~~$CC_u \mid C_uC$~~	~~(8.7), two roots~~
~~$r^-l^-_{-u}r^+_ul^+$~~	~~$C^-C^-_{-u}C^+_uC^+$~~	~~$CC_u \mid C_uC$~~	~~(8.7), two roots~~
$l^+r^-_{-u}l^-_{-u}r^+$	$C^+C^-_{-u}C^-_{-u}C^+$	$C \mid C_uC_u \mid C$	(8.8)
~~$l^-r^+_ul^+_ur^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C \mid C_uC_u \mid C$~~	~~(8.8)~~
~~$r^+l^-_{-u}r^-_{-u}l^+$~~	~~$C^+C^-_{-u}C^-_{-u}C^+$~~	~~$C \mid C_uC_u \mid C$~~	~~(8.8)~~
~~$r^-l^+_ur^+_ul^-$~~	~~$C^-C^+_uC^+_uC^-$~~	~~$C \mid C_uC_u \mid C$~~	~~(8.8)~~
$l^+r^-_{-\pi/2}s^-l^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
~~$l^-r^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^+l^-_{-\pi/2}s^-r^-$~~	~~$C^+C^-_{-\pi/2}S^-C^-$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$r^-l^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.9)~~
~~$l^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.9)~~
$l^+s^+r^+_{\pi/2}l^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.9)
~~$r^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.9)~~
~~$r^+s^+l^+_{\pi/2}r^-$~~	~~$C^+S^+C^+_{\pi/2}C^-$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.9)~~
$l^+r^-_{-\pi/2}s^-r^-$	$C^+C^-_{-\pi/2}S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
~~$l^-r^+_{\pi/2}s^+r^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^+l^-_{-\pi/2}s^-l^-$~~	~~$C^+C^-_{-\pi/2}S^-C^-$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-l^+_{\pi/2}s^+l^+$~~	~~$C^-C^+_{\pi/2}S^+C^+$~~	~~$C \mid C_{\pi/2}SC$~~	~~(8.10)~~
~~$r^-s^-r^-_{-\pi/2}l^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.10)~~
~~$r^+s^+r^+_{\pi/2}l^-$~~	~~$C^+S^+C^+_{\pi/2}C^-$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.10)~~
~~$l^-s^-l^-_{-\pi/2}r^+$~~	~~$C^-S^-C^-_{-\pi/2}C^+$~~	~~$CSC_{\pi/2} \mid C$~~	~~(8.10)~~
$l^+s^+l^+_{\pi/2}r^-$	$C^+S^+C^+_{\pi/2}C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
~~$l^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
~~$r^+s^+l^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.2)~~
~~$r^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.2)~~
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
~~$l^-s^-l^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
~~$r^+s^+r^+$~~	~~$C^+S^+C^+$~~	~~CSC~~	~~(8.1)~~
~~$r^-s^-r^-$~~	~~$C^-S^-C^-$~~	~~CSC~~	~~(8.1)~~
$l^+r^-_{-\pi/2}s^-l^-_{-\pi/2}r^+$	$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
~~$l^-r^+_{\pi/2}s^+l^+_{\pi/2}r^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C \mid C_{\pi/2}SC_{\pi/2} \mid C$~~	~~(8.11), two roots~~
~~$r^+l^-_{-\pi/2}s^-r^-_{-\pi/2}l^+$~~	~~$C^+C^-_{-\pi/2}S^-C^-_{-\pi/2}C^+$~~	~~$C \mid C_{\pi/2}SC_{\pi/2} \mid C$~~	~~(8.11), two roots~~
~~$r^-l^+_{\pi/2}s^+r^+_{\pi/2}l^-$~~	~~$C^-C^+_{\pi/2}S^+C^+_{\pi/2}C^-$~~	~~$C \mid C_{\pi/2}SC_{\pi/2} \mid C$~~	~~(8.11), two roots~~

Figure: All Reeds-Shepp paths with timeflip and reflect simplifications

Reeds-Shepp paths: reverse simplification

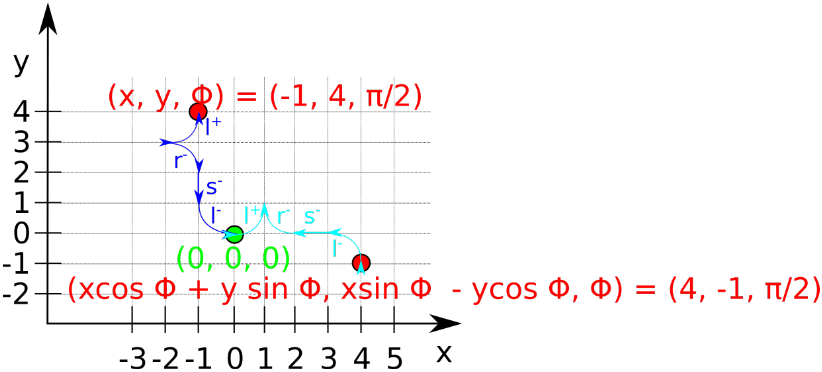

Figure: Reverse simplification: $l^-s^-r^-l^+$ goes from the (0, 0, 0) to (x, y, ϕ) and $l^+r^-s^-l^-$ goes from (0, 0, 0) to $(x\cos\phi + y\sin\phi,\ x\sin\phi - y\cos\phi,\ \phi)$

Reeds-Shepp paths: reverse simplification

explicit	(1.1) form	(1.2) form	Section 8 formula
$l^+r^-l^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$l^-r^+l^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$r^+l^-r^+$	$C^+C^-C^+$	$C \mid C \mid C$	(8.3), two roots
$r^-l^+r^-$	$C^-C^+C^-$	$C \mid C \mid C$	(8.3), two roots
$l^+r^-l^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$l^-r^+l^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$r^+l^-r^-$	$C^+C^-C^-$	$C \mid CC$	(8.4), two roots
$r^-l^+r^+$	$C^-C^+C^+$	$C \mid CC$	(8.4), two roots
$l^-r^-l^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$l^+r^+l^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$r^-l^-r^+$	$C^-C^-C^+$	$CC \mid C$	(8.4), two roots
$r^+l^+r^-$	$C^+C^+C^-$	$CC \mid C$	(8.4), two roots
$l^+r_u^+l_{-u}^-r^-$	$C^+C_u^+C_{-u}^-C^-$	$CC_u \mid C_uC$	(8.7), two roots
$l^-r_{-u}^-l_u^+r^+$	$C^-C_{-u}^-C_u^+C^+$	$CC_u \mid C_uC$	(8.7), two roots
$r^+l_u^+r_{-u}^-l^-$	$C^+C_u^+C_{-u}^-C^-$	$CC_u \mid C_uC$	(8.7), two roots
$r^-l_{-u}^-r_u^+l^+$	$C^-C_{-u}^-C_u^+C^+$	$CC_u \mid C_uC$	(8.7), two roots
$l^+r_{-u}^-l_{-u}^-r^+$	$C^+C_{-u}^-C_{-u}^-C^+$	$C \mid C_uC_u \mid C$	(8.8)
$l^-r_u^+l_u^+r^-$	$C^-C_u^+C_u^+C^-$	$C \mid C_uC_u \mid C$	(8.8)
$r^+l_{-u}^-r_{-u}^-l^+$	$C^+C_{-u}^-C_{-u}^-C^+$	$C \mid C_uC_u \mid C$	(8.8)
$r^-l_u^+r_u^+l^-$	$C^-C_u^+C_u^+C^-$	$C \mid C_uC_u \mid C$	(8.8)
$l^+r_{-\pi/2}^-s^-l^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
$l^-r_{\pi/2}^+s^+l^+$	$C^-C_{\pi/2}^+S^+C^+$	$C \mid C_{\pi/2}SC$	(8.9)
$r^+l_{-\pi/2}^-s^-r^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.9)
$r^-l_{\pi/2}^+s^+r^+$	$C^-C_{\pi/2}^+S^+C^+$	$C \mid C_{\pi/2}SC$	(8.9)
$l^-s^-r_{-\pi/2}^-l^+$	$C^-S^-C_{-\pi/2}^-C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+s^+r_{\pi/2}^+l^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$r^-s^-l_{-\pi/2}^-r^+$	$C^-S^-C_{-\pi/2}^-C^+$	$CSC_{\pi/2} \mid C$	(8.9)
$r^+s^+l_{\pi/2}^+r^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.9)
$l^+r_{-\pi/2}^-s^-r^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
$l^-r_{\pi/2}^+s^+r^+$	$C^-C_{\pi/2}^+S^+C^+$	$C \mid C_{\pi/2}SC$	(8.10)
$r^+l_{-\pi/2}^-s^-l^-$	$C^+C_{-\pi/2}^-S^-C^-$	$C \mid C_{\pi/2}SC$	(8.10)
$r^-l_{\pi/2}^+s^+l^+$	$C^-C_{\pi/2}^+S^+C^+$	$C \mid C_{\pi/2}SC$	(8.10)
$r^-s^-r_{-\pi/2}^-l^+$	$C^-S^-C_{-\pi/2}^-C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$r^+s^+r_{\pi/2}^+l^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^-s^-l_{-\pi/2}^-r^+$	$C^-S^-C_{-\pi/2}^-C^+$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+l_{\pi/2}^+r^-$	$C^+S^+C_{\pi/2}^+C^-$	$CSC_{\pi/2} \mid C$	(8.10)
$l^+s^+r^+$	$C^+S^+C^+$	CSC	(8.2)
$l^-s^-r^-$	$C^-S^-C^-$	CSC	(8.2)
$r^+s^+l^+$	$C^+S^+C^+$	CSC	(8.2)
$r^-s^-l^-$	$C^-S^-C^-$	CSC	(8.2)
$l^+s^+l^+$	$C^+S^+C^+$	CSC	(8.1)
$l^-s^-l^-$	$C^-S^-C^-$	CSC	(8.1)
$r^+s^+r^+$	$C^+S^+C^+$	CSC	(8.1)
$r^-s^-r^-$	$C^-S^-C^-$	CSC	(8.1)
$l^+r_{-\pi/2}^-s^-l_{-\pi/2}^-r^+$	$C^+C_{-\pi/2}^-S^-C_{-\pi/2}^-C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$l^-r_{\pi/2}^+s^+l_{\pi/2}^+r^-$	$C^-C_{\pi/2}^+S^+C_{\pi/2}^+C^-$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$r^+l_{-\pi/2}^-s^-r_{-\pi/2}^-l^+$	$C^+C_{-\pi/2}^-S^-C_{-\pi/2}^-C^+$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots
$r^-l_{\pi/2}^+s^+r_{\pi/2}^+l^-$	$C^-C_{\pi/2}^+S^+C_{\pi/2}^+C^-$	$C \mid C_{\pi/2}SC_{\pi/2} \mid C$	(8.11), two roots

Figure: Reeds-Shepp paths with timeflip, reflect, reverse simplifications

Reeds-Shepp paths: case $l_t^+ s_u^+ l_v^+$

- $(u, t) := R(x$ - sin ϕ, y - 1 + cos $\phi)$
- $v := M(\phi - t)$

With:

- $(r, \theta) := R(x, y)$ for the polar transform $r\cos\theta = x$ and $r\sin\theta = y$ with $r \leq 0$ and $-\pi \leq \theta < \pi$
- $\phi = M(\theta)$ if $\phi \equiv \theta \bmod 2\pi$ and $-\pi \leq \phi < \pi$

Dubins and Reeds-Shepp paths in reality

- All kinds of sources for error in the real-world

Dubins and Reeds-Shepp paths in reality

- All kinds of sources for error in the real-world
- Paths found in the absence of obstacles however can use a rapidly exploring random tree and Dubins' pseudo distance or Reeds-Shepp distance

Sources

- Optimal paths for a car that goes both forwards and backwards (1990), J. A. Reeds and L. A. Shepp
- Planning algorithms: Reeds-Shepp curves (2006), Steven M. LaValle
- A Comprehensive, Step-by-Step Tutorial to Computing Dubins' Paths (2013), Andy G's Blog