Exercises, Chapter 1

Casella, Berger 2nd Edition

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1.1 For each of the following experiments, describe the sample space:

(a) Toss a coin four times.

(b) Count the number of insect-damaged leaves on a plant.

(c) Measure the lifetime (in hours) of a particular brand of lightbulb.

(d) Record the weights of 10-day-old rats.

(e) Observe the proportion of defectives in a shipment of electronic components.

Solution

(a) The sample space is $S = \{aaaa\}$ where $a \epsilon \{H, T\}$

(b) The sample space is $S = \{0, 1, 2....\}$

(c) The sample space is $S = [0, \infty)$

(d) The sample space is $S = (0, \infty)$. Suppose we know no rats weigh more than 200 lbs. Then we could say $S = (0, 200]$

(e) If n is the number of components in the shipment, the $S = \{0/n, 1/n, ...n/n\}$

1.6 Two penies, one with probability $P(head) = u$ and one with $P(head) = w$ are to be tossed together independently Define:

$$p_0 = P(0 \ heads \ occur)$$

$$p_1 = P(1 \ head \ occurs)$$

$$p_2 = P(2 \ heads \ occur)$$

Can u and w be chosen such that $p_0 = p_1 = p_2$? Prove your answer.

Solution: Start by computing the probability of each of p_0, p_1, and p_2 in terms of u and w:

$$p_0 = (1 - u)(1 - w) = 1 - u - w - uw$$

$$p_1 = u(1 - w) + (1 - u)(w) = u + w - 2uw$$

$$p_2 = uw$$

A solution to this set of equations subject to the constrain $p_0 = p_1 = p_2$ has no real solutions:

The system:

(1.6.1) $1 - u - w + uw = a$

(1.6.2) $u + w - 2uw = a$

(1.6.3) $uw = a$

Yeilds the solution $3a = 1$ or $a = 1/3$. Then

$$1 - u - w + uw = \frac{1}{3}$$

(1.6.4) $u = \frac{\frac{4}{3} - w}{(1-w)}$

Combining (1.6.3) and (1.6.4), we get

$$w = \frac{1}{3u}$$

$$w = \frac{1}{3\left(\frac{(\frac{4}{3}-w)}{(1-w)}\right)}$$

$$3w^2 + 4w + 1 = 0,\ (3w+1)(w+1) = 0,\ w = -1, w = -\frac{1}{3}$$

Similarly solving for u, we get:

$$u = \frac{1}{3}w,\ u = -\frac{1}{3}, u = -\frac{1}{9}$$

Thus no solution of u, w produces a valid probability.