



Predicting Service Life of a Bridge Using Markov Chain

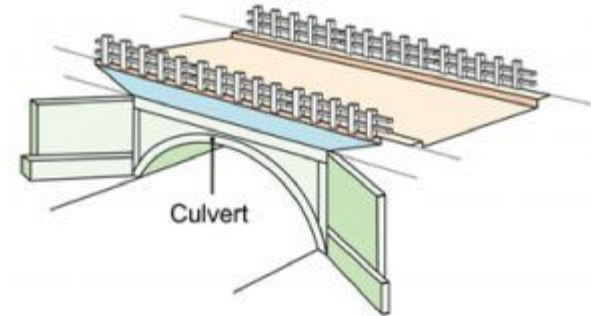
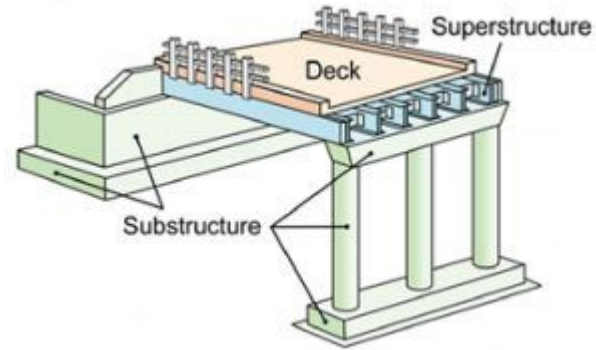
(A case study of Pennsylvania State Highways)

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March 2023

Introduction

- Bridges across the country are inspected at least every 2 years
- Bridges are rated based on the condition of
 - Superstructure: supporting part of the bridge
 - **Substructure**: part of bridge supporting the superstructure
 - Culvert: curved/rectangular structure below the roadway



Bridge Rating System

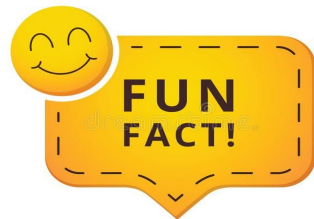
- The Federal Highway Administration (FHWA) have a scale for condition rating of bridges used by all bridge managers in the country.
- The scale is from 0 to 9.
 - 9 - bridge is in excellent condition (newly built)
 - 3 - bridge needs serious repair or replacement
 - 0 - bridge has failed and has to be out of service
- There are many classifications of bridges by materials (Steel, Concrete, Masonry, etc). This project focuses on Concrete bridges

Objective

The objective of this project is to use Markov chain to predict the service life of **concrete** bridge **substructures** based on the current condition or rating

Data Collection & Assumptions

- Data for Pennsylvania bridges was obtained from U.S. Department of Transportation database for inspection years of 2020 and 2022
- For 2020:
 - Data size is **22,965 rows** (or distinct bridges)
 - Data reduced to **10,923** after cleaning
 - Assume that only bridges with substructure ratings above 3 are in use
 - Removed 2,759 nulls in bridge substructure rating column
- For 2022:
 - Data size is **23,202 rows** (or distinct bridges)
 - Data reduced to **11,079** after cleaning
 - Assume that only bridges with substructure ratings above 3 are in use
 - Removed 2,983 nulls in bridge substructure rating column
- After joining both data, final set was **10,923 rows/bridges** and 5 columns. This means new bridges were removed
- Codes were written in Python



Pittsburgh is the city with the most bridges, 446, in the **World**



What is a Markov Chain?

- Markov Chain is a discrete stochastic (or random) process in which the probability of the future depends on the present
- Examples
 - Next Word Prediction: Google Predicts the next word in your sentence based on your previous entry within Gmail
 - Flood Prediction: Using historic flood data in a region to generate a current state and then predict when the next flood will likely happen



Model Development

- Assume that we only use bridges that are functional, having the standard rating \mathbf{S} between 4 and 9
- There are two sides to the model:
 - 1) Expected rating (\mathbf{E}) of the bridge in the next t years, given the current condition \mathbf{P}_0

$$\mathbf{E}(t) = \mathbf{P}_0^T \mathbf{P}^t \mathbf{S}^T$$

- 2) The time t (in years) it will take for the bridge rating \mathbf{R} to drop to a particular (low) rating \mathbf{r} , given the current condition \mathbf{P}_0

Model Development: Transition Probability Matrix, P

$$P = \begin{bmatrix} p_{1,1} & p_{1,2} & 0 & 0 & 0 & 0 \\ 0 & p_{2,2} & p_{2,3} & 0 & 0 & 0 \\ 0 & 0 & p_{3,3} & p_{3,4} & 0 & 0 \\ 0 & 0 & 0 & p_{4,4} & p_{4,5} & 0 \\ 0 & 0 & 0 & 0 & p_{5,5} & p_{5,6} \\ 0 & 0 & 0 & 0 & 0 & p_{6,6} \end{bmatrix}$$

- Assumptions:
 - The bridge condition rating would not drop more than 1 within a two year rating period
 - No bridge maintenance was done within this two-year period
- Used the Percentage Probability Method (PPM) to develop the transition matrix

$$p_{i,i} = \frac{n_{i,i}}{n_i} \qquad p_{i,j} = 1 - p_{i,i}$$

- Number of bridges *rated* 9 in 2020 and maintained *rating* 9 in 2022, $n_{9,9} = 207$
- Total number of bridges in *rated* 9 in 2020, $n_9 = 330$
- $p_{1,1} = 207/330 = \mathbf{0.6273}$ and $p_{1,2} = 1 - 0.6273 = \mathbf{0.3727}$

Model Development: Transition Probability Matrix, P


2020 Substructure_rating	Count of 2020 rating	2022 Substructure_rating	Count of 2022 rating	p _(i,i)
9	330	9.0	207	0.6273
8	1383	8.0	1252	0.9053
7	3543	7.0	3406	0.9613
6	2631	6.0	2495	0.9483
5	2001	5.0	1870	0.9345
4	395	4.0	358	0.9063

$$p_{i,i} = \frac{n_{i,i}}{n_i}$$

$$P = \begin{bmatrix} 0.6273 & 0.3727 & 0 & 0 & 0 & 0 \\ 0 & 0.9053 & 0.0947 & 0 & 0 & 0 \\ 0 & 0 & 0.9613 & 0.0387 & 0 & 0 \\ 0 & 0 & 0 & 0.9483 & 0.517 & 0 \\ 0 & 0 & 0 & 0 & 0.9345 & 0.0655 \\ 0 & 0 & 0 & 0 & 0 & 0.9063 \end{bmatrix}$$

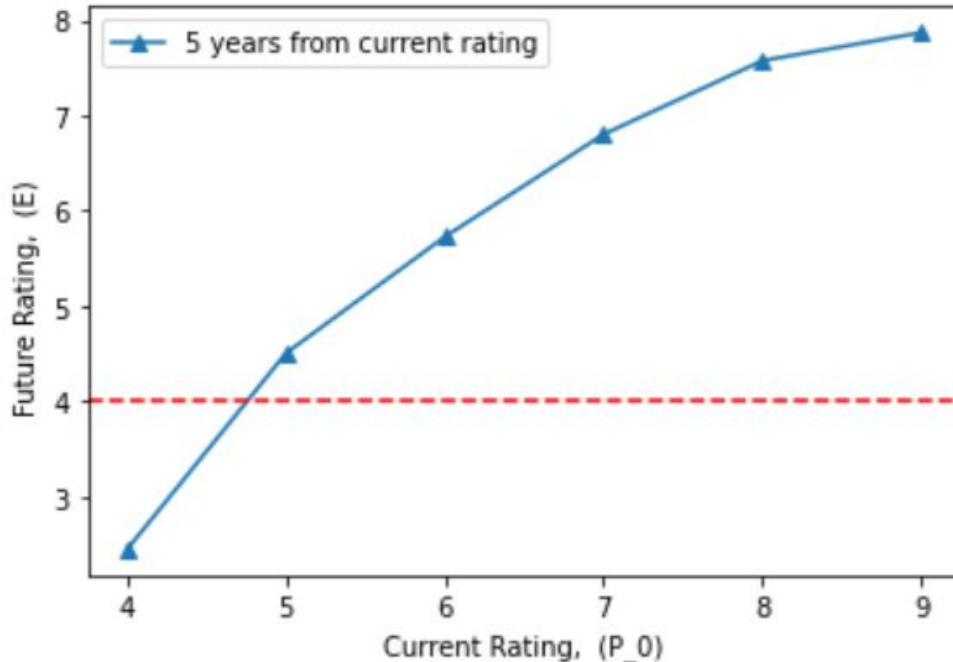
Model: Expected (future) rating, E

$$E(t) = P_0^T P^t S^T$$

$$E(t) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \times \begin{bmatrix} 0.6273 & 0.3727 & 0 & 0 & 0 & 0 \\ 0 & 0.9053 & 0.0947 & 0 & 0 & 0 \\ 0 & 0 & 0.9613 & 0.0387 & 0 & 0 \\ 0 & 0 & 0 & 0.9483 & 0.517 & 0 \\ 0 & 0 & 0 & 0 & 0.9345 & 0.0655 \\ 0 & 0 & 0 & 0 & 0 & 0.9063 \end{bmatrix}^t \times (9 \ 8 \ 7 \ 6 \ 5 \ 4)^T$$


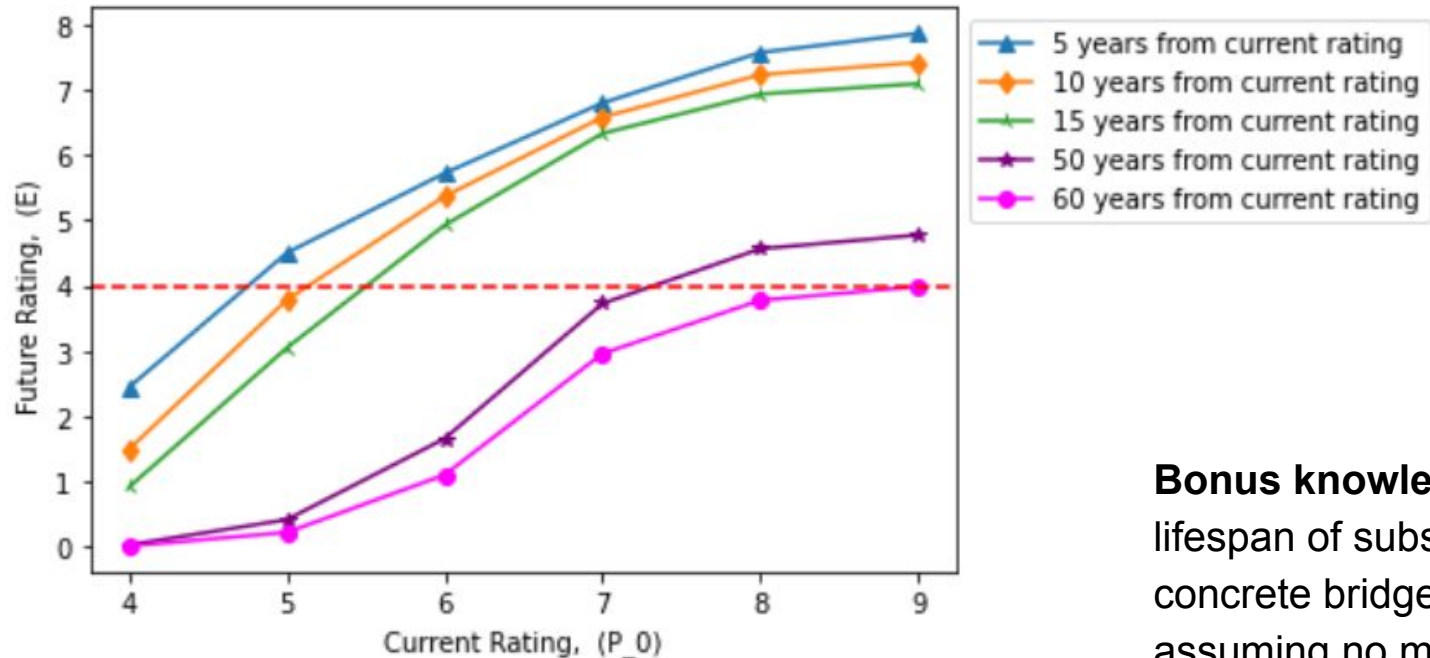
- If we use $t=10$, then the *Expected or future rate*, $E=7.42$
- Interpretation: The substructure of a concrete bridge that is rated 9 will drop to an average rating of **7.42** in **10** years

Model Simulation I



- Interpretation: In the next 5 years, a bridge substructure currently rated (say) 9 will drop to 7.8. Similarly, rating of 5 will drop to 4.5
- A future rating threshold of 4 was set to flag when a bridge is due to major repair or replacement
- This threshold is important because according to the FHWA rating, any bridge below 4 is due to major repair or replacement

Model Simulation II



Bonus knowledge: The average lifespan of substructure of a concrete bridge is 60 years, assuming no maintenance was done.

Conclusion

- Over time we see a bridge rating drops, regardless of the current rating. This reflect reality
-assuming no maintenance was done
- It takes 60 years before a newly built bridge would need major repair or replacement of its substructure (pillars and foundation)

Limitations

- Model was implemented with data for 2 years time period
- Bridge maintenance was not accounted for

Future Works

- Develop transition matrix with a combination of multiple years
- Extend the model to predict the service life by age of the bridge
- Compare results from Markov Chain model with other methods:
 - regression-based optimization
 - nonlinear optimization

Reference

- Srikanth, Ishwarya & Arockiasamy, Madasamy. (2020). Deterioration models for prediction of remaining useful life of timber and concrete bridges: A review. *Journal of Traffic and Transportation Engineering (English Edition)*. 7. 10.1016/j.jtte.2019.09.005.
- Jiang, Yi, and Kumares C. Sinha. "Bridge service life prediction model using the Markov chain." *Transportation research record* 1223 (1989): 24-30
- U.S. Department of Transportation, Federal Highway Administration: [dataset](#) and [data dictionary](#)
- Bridge Condition and ratings [linked here](#)