The proof that the derivative of Eq. (12) is Lipschitz continuous:

Let $S(x) = \frac{1}{1+e^{\ \ x}}$. We have $0 < S(x) \quad 1$, and $S\ (x) = S(x)(1 \quad S(x)) \quad 0$. Hence, $S(x)$ is monotonically increasing. Next, we have $0 \quad S\ (x) \quad 2$. Then, based on the Lagrange mean value theorem, we have that $S(x)$ is Lipchitz function.

Next, since $S\ \ (x) = S(x)(1 \quad S(x))(1 \quad 2S(x))$, we have $|\,S\ \ (x)\,| \quad 6$. Then $S\ \ (x)$ is Lipchitz function.

Finally, let $f = \qquad [S(r_{ui} \quad r_{vi}) \quad S(r_{ui} \quad r_{vi})]^2 + \qquad p_u^2$,

Then

$$\frac{\ f}{\ r_{ui}} = 2 \qquad [S(r_{ui} \quad r_{vi}) \quad S(r_{ui} \quad r_{vi})]S\ (r_{ui} \quad r_{vi})$$

$$=2 \quad [S(r_{ui} \quad r_{vi}) \quad S(r_{ui} \quad r_{vi})]S(r_{ui} \quad r_{vi})(1 \quad S(r_{ui} \quad r_{vi})),$$

$$\frac{\ ^2 f}{\ r_{ui}\ \ r_{uj}} = 2S(r_{uj} \quad r_{vj})[1 \quad S(r_{uj} \quad r_{vj})]\{S(r_{uj} \quad r_{vj})(1 \quad S(r_{uj} \quad r_{vj}))$$

$$+[S(r_{uj} \quad r_{vj}) \quad S(r_{uj} \quad r_{vj})][1 \quad 2S(r_{uj} \quad r_{vj})]\}.$$

As we can see from the above equations, $\ f = \frac{\ f}{\ r_{ui}}$ is Frechet derivative, and $\ ^2 f = \frac{\ ^2 f}{\ r_{ui}\ \ r_{uj}}$ satisfies $\left\|\ ^2 f\right\| \quad M <$. Consequently, we have $\ f$ is Lipschitz function (via Theorem 3.2.4 (p.70) in [1]).

[1] Ortega J. M., Rheinboldt W. C. Iterative Solution of Nonlinear Equations in Several Variables. Academic Press, New York, 1970.