The proof that the derivative of Eq. (12) is Lipschitz continuous:

Let $S(x)=\dfrac{1}{1+e^{-x}}$. We have $0<S(x)\le 1$, and $S'(x)=S(x)(1-S(x))\ge 0$. Hence, $S(x)$ is monotonically increasing. Next, we have $0\le S'(x)\le 2$. Then, based on the Lagrange mean value theorem, we have that $S(x)$ is Lipchitz function.
Next, since $S''(x)=S(x)(1-S(x))(1-2S(x))$, we have $|S''(x)|\le 6$. Then $S'(x)$ is Lipchitz function.
Finally, let $f=\sum\sum[S(r_{ui}-r_{vi})-S(r_{ui}-r_{vi})]^2+\lambda\sum p_u^2$,

Then

$$\frac{\partial f}{\partial r_{ui}}=2\sum\sum[S(r_{ui}-r_{vi})-S(r_{ui}-r_{vi})]S'(r_{ui}-r_{vi})$$

$$=2\sum[S(r_{ui}-r_{vi})-S(r_{ui}-r_{vi})]S(r_{ui}-r_{vi})(1-S(r_{ui}-r_{vi})),$$

$$\frac{\partial^2 f}{\partial r_{ui}\partial r_{uj}}=2S(r_{uj}-r_{vj})[1-S(r_{uj}-r_{vj})]\{S(r_{uj}-r_{vj})(1-S(r_{uj}-r_{vj}))$$

$$+[S(r_{uj}-r_{vj})-S(r_{uj}-r_{vj})][1-2S(r_{uj}-r_{vj})]\}.$$

As we can see from the above equations, $\nabla f=\left(\dfrac{\partial f}{\partial r_{ui}}\right)$ is Frechet derivative, and $\nabla^2 f=\left(\dfrac{\partial^2 f}{\partial r_{ui}\partial r_{uj}}\right)$ satisfies $\left\|\nabla^2 f\right\|\le M<\infty$. Consequently, we have ∇f is Lipschitz function (via Theorem 3.2.4 (p.70) in [1]).

[1] Ortega J. M., Rheinboldt W. C. Iterative Solution of Nonlinear Equations in Several Variables. Academic Press, New York, 1970.