



# MATRI: A Multi-Aspect and Transitive Trust Inference Model

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Joint work with

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# Roadmap



- Background and Motivations
- Modeling Multi-Aspect
- Incorporating Trust Bias
- Incorporating Trust Transitivity
- Empirical Evaluations
- Conclusions



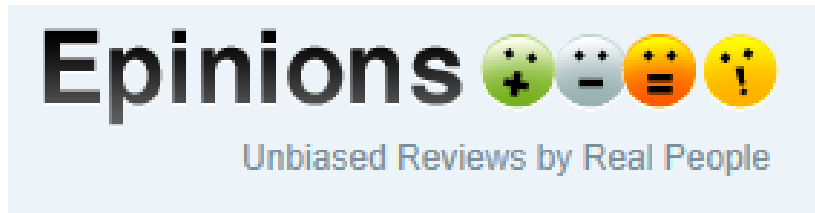
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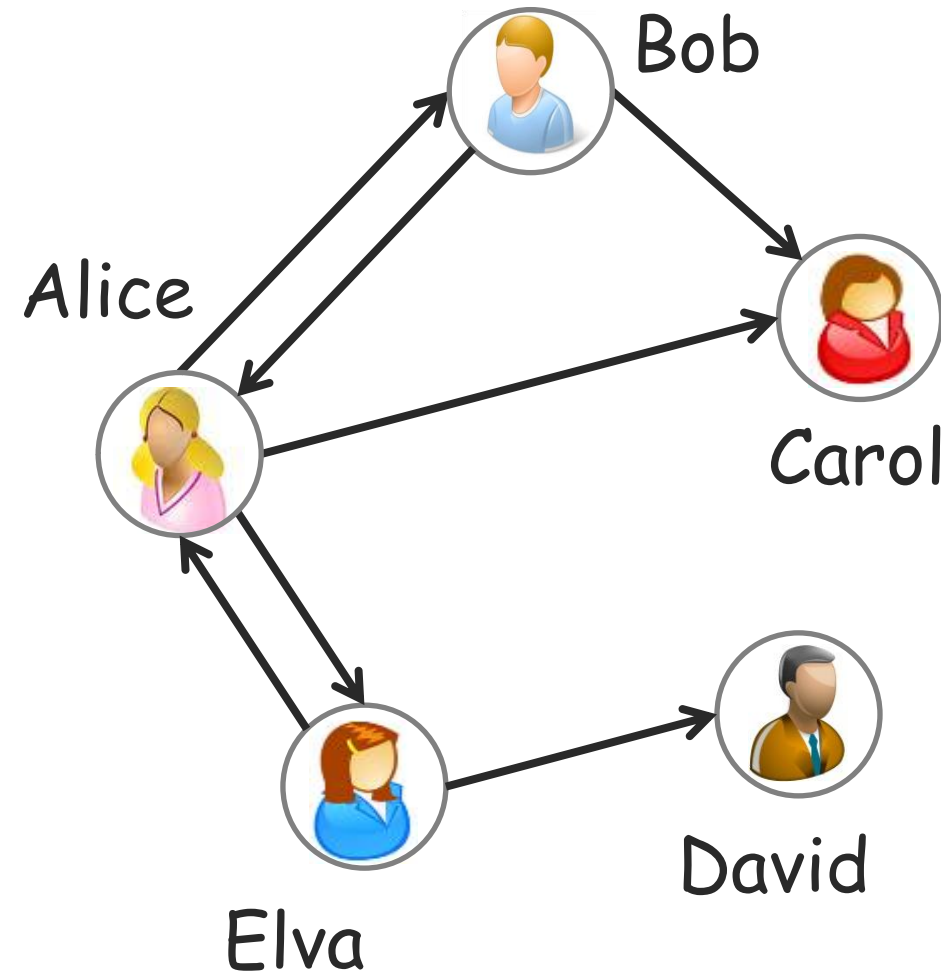
# Trust



"**Trust** is the subjective probability by which an individual (**trustor**), expects that another individual (**trustee**) will perform well on a given action."



# Trust Inference



→ : Trust

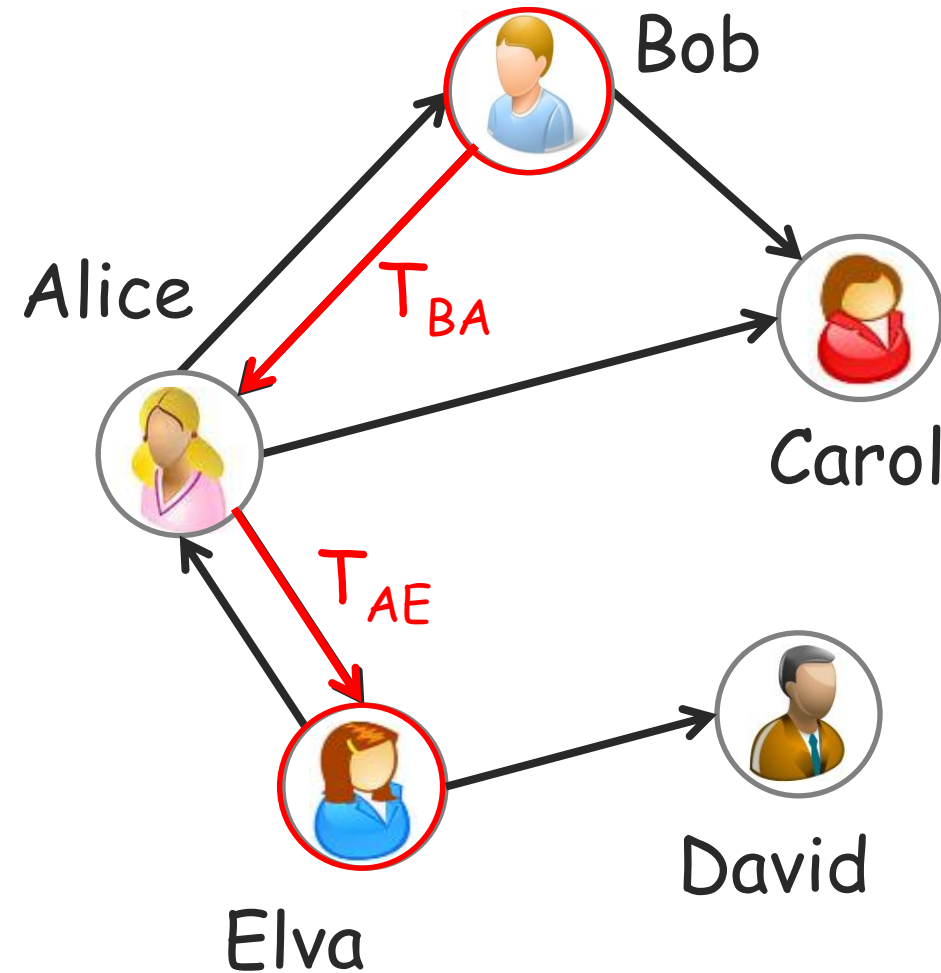
How to infer the unknown trust relationships?

E.g., to what extent should Bob trust Elva?

Trust Properties:  
Transitivity, Multi-Aspect,  
Trust Bias



# P1: Trust Transitivity



Bob  $\rightarrow$  Elva ( $T_{BE}$ )?

Trust transitivity  
(or trust propagation):

$$T_{BE} = T_{BA} * T_{AE}$$



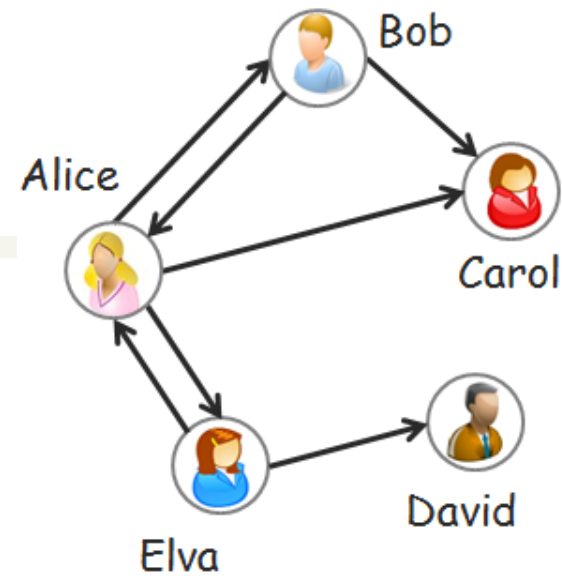
# P2: Multi-Aspect

## Truster Preferences

Delivering time Product price		
1	1	Alice
1	0	Bob
1	0	Carol
0	1	David
0	1	Elva
factors		

## Trustee Capabilities

Delivering time Product price		
0.5	0.5	Alice
1	0	Bob
1	0	Carol
0	1	David
0	1	Elva
factors		



Bob  $\rightarrow$  Elva ( $T_{BE}$ )?



# P3: Trust Bias

Overall avg. rating: 0.5



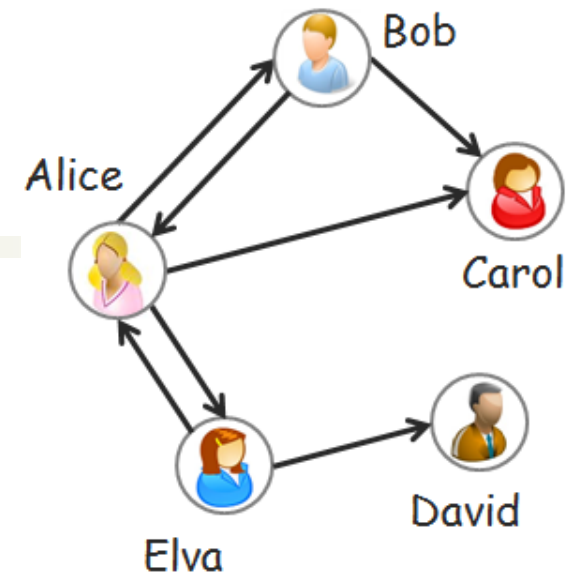
Alice Bob Carol David Elva

Truster  
bias:

0.2	0.4	-0.3	-0.1	0.1
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Trustee  
bias:

-0.1	0.2	0.1	0.2	-0.2
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Bob  $\rightarrow$  Elva ( $T_{BE}$ )?

$$T_{BE} = 0.4 - 0.2 + 0.5 = 0.7$$





# This Paper



- Q1: how to characterize multi-aspect trust directly from trust ratings?
- Q2: how to incorporate trust bias?
- Q3: how to incorporate trust transitivity?



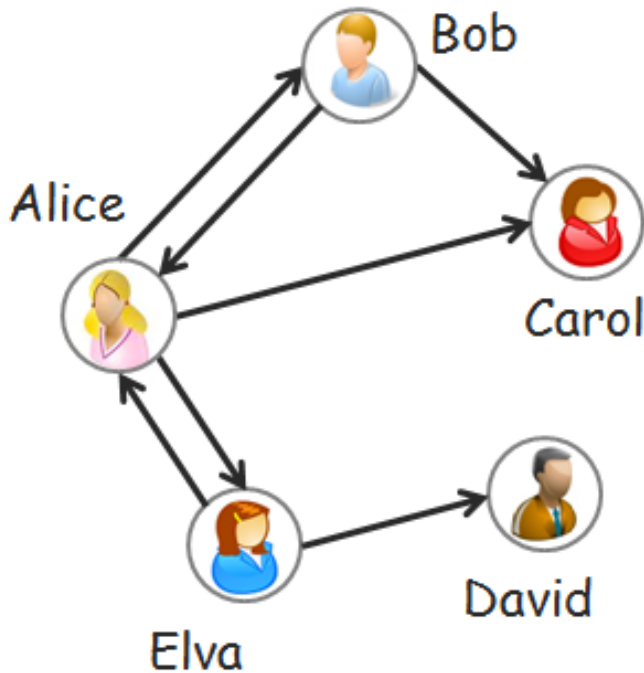
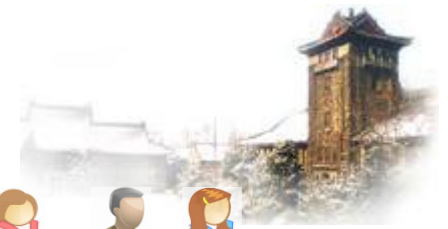
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# Modeling Multi-Aspect



$\mathbf{T} =$

	Alice	Bob	Carol	David	Elva	
Alice	/	1	1	?	1	Alice
Bob	0.5	/	1	?	?	Bob
Carol	?	?	/	?	?	Carol
David	?	?	?	/	?	David
Elva	0.5	?	?	1	/	Elva

item rating

trustees → item

↓ user

$$\min_{\mathbf{F}, \mathbf{G}} \sum_{(i,j) \in \mathcal{K}} (\mathbf{T}(i,j) - \mathbf{F}(i,:) \mathbf{G}(j,:))'^2 + \lambda \|\mathbf{F}\|_{fro}^2 + \lambda \|\mathbf{G}\|_{fro}^2$$



# Modeling Multi-Aspect

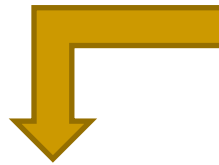
$$\min_{\mathbf{F}, \mathbf{G}} \sum_{(i,j) \in \mathcal{K}} (\mathbf{T}(i,j) - \mathbf{F}(i,:) \mathbf{G}(j,:))'^2 + \lambda \|\mathbf{F}\|_{fro}^2 + \lambda \|\mathbf{G}\|_{fro}^2$$

$\mathbf{T} =$

Alice	Bob	Carol	David	Elva	
/	1	1	?	1	Alice
0.5	/	1	?	?	Bob
?	?	/	?	?	Carol
?	?	?	/	?	David
0.5	?	?	1	/	Elva

trustors

trustees



Delivering time  
Product price

$\mathbf{F} =$

1	1	Alice
1	0	Bob
1	0	Carol
0	1	David
0	1	Elva

factors

trustors

Delivering time  
Product price

$\mathbf{G} =$

0.5	0.5	Alice
1	0	Bob
1	0	Carol
0	1	David
0	1	Elva

factors

trustees



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# Incorporating Trust Bias



- Three types of trust bias:
  - Global bias ( $\mu$ ), trustor bias ( $\mathbf{x}$ ), trustee bias ( $\mathbf{y}$ )

$$\min_{\mathbf{F}, \mathbf{G}} \sum_{(i,j) \in \mathcal{K}} (\mathbf{T}(i, j) - \mathbf{F}(i, :) \mathbf{G}(j, :)^T)^2 + \lambda \|\mathbf{F}\|_{fro}^2 + \lambda \|\mathbf{G}\|_{fro}^2$$

Subject to:

$$\mathbf{F}(:, 1) = \mu \mathbf{1}, \quad \mathbf{G}(:, 1) = \alpha_1 \mathbf{1} / \sqrt{n} \quad (\text{global bias})$$

$$\mathbf{F}(:, 2) = \mathbf{x}, \quad \mathbf{G}(:, 2) = \alpha_2 \mathbf{1} / \sqrt{n} \quad (\text{trustor bias})$$

$$\mathbf{F}(:, 3) = \alpha_3 \mathbf{1} / \sqrt{n}, \quad \mathbf{G}(:, 3) = \mathbf{y} \quad (\text{trustee bias})$$

$$\min_{\mathbf{F}_0, \mathbf{G}_0, \alpha} \sum_{(i,j) \in \mathcal{K}}$$

$$(\mathbf{T}(i, j) - (\alpha' [\mu, \mathbf{x}(i), \mathbf{y}(j)]' + \mathbf{F}_0(i, :) \mathbf{G}_0(j, :)^T))^2$$

$$+ \lambda \|\mathbf{F}_0\|_{fro}^2 + \lambda \|\mathbf{G}_0\|_{fro}^2 + \lambda \|\alpha\|^2$$



# Computing Bias



$$\begin{array}{l} \text{Global Bias:} \\ \text{Trustor Bias:} \\ \text{Trustee Bias:} \end{array} \left\{ \begin{array}{l} \mu = \sum_{(i,j) \in \mathcal{K}} \mathbf{T}(i,j) / |\mathcal{K}| \\ \mathbf{x}(i) = \sum_{j, (i,j) \in \mathcal{K}} \mathbf{T}(i,j) / |\text{row}_i| - \mu \\ \mathbf{y}(j) = \sum_{i, (i,j) \in \mathcal{K}} \mathbf{T}(i,j) / |\text{col}_j| - \mu \end{array} \right.$$



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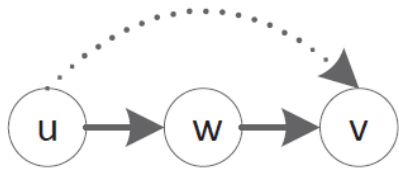




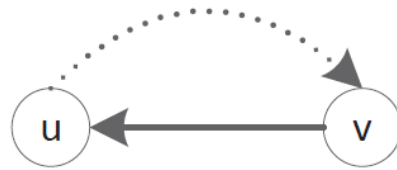
# Incorporating Trust Transitivity



## ■ Four types of trust propagation

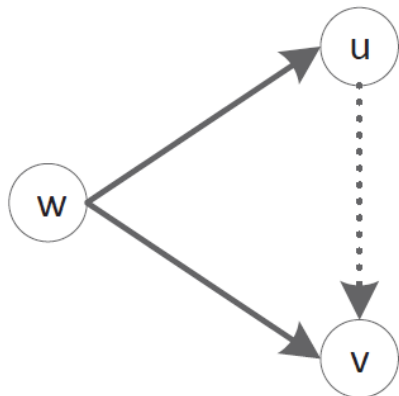


(a) Direct propagation

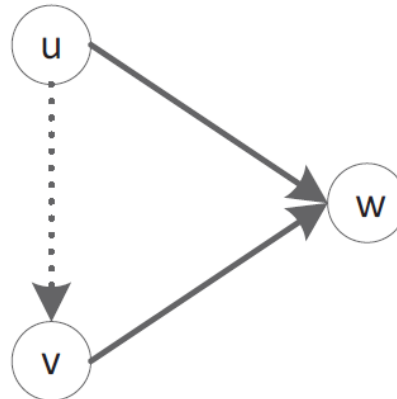


(b) Transpose trust

$\longrightarrow$  : known trust  
 $\cdots\longrightarrow$  : inferred trust



(c) Co-citation



(d) Trust coupling

(a)  $T * T$   
(b)  $T'$   
(c)  $T' * T$   
(d)  $T * T'$

$(i,j)$

$\longrightarrow$   $Z_{ij}$



# Computing Propagation



Propagation:  $(\mathbf{z}_{ij})$

$$\left\{ \begin{array}{lcl} \mathbf{T}^t(i, j) & = & \mathbf{L}(i, :)(\mathbf{R}'\mathbf{L})^{t-1}\mathbf{R}(j, :)' \\ (\mathbf{T}')^t(i, j) & = & \mathbf{R}(i, :)(\mathbf{L}'\mathbf{R})^{t-1}\mathbf{L}(j, :)' \\ (\mathbf{T}'\mathbf{T})^t(i, j) & = & \mathbf{R}(i, :)((\mathbf{L}'\mathbf{L})(\mathbf{R}'\mathbf{R}))^{t-1}(\mathbf{L}'\mathbf{L})\mathbf{R}(j, :)' \\ (\mathbf{T}\mathbf{T}')^t(i, j) & = & \mathbf{L}(i, :)((\mathbf{R}'\mathbf{R})(\mathbf{L}'\mathbf{L}))^{t-1}(\mathbf{R}'\mathbf{R})\mathbf{L}(j, :)' \end{array} \right.$$



# Our Final Model: MaTrI



Trust bias

Trust transitivity

$$\min_{\mathbf{F}_0, \mathbf{G}_0, \alpha, \beta} \sum_{(i,j) \in \mathcal{K}}$$

$$(\mathbf{T}(i, j) - (\alpha' [\mu, \mathbf{x}(i), \mathbf{y}(j)]') + \beta' \mathbf{z}_{ij})$$

$$+ (\mathbf{F}_0(i, :) \mathbf{G}_0(j, :))' )^2 + \lambda \|\mathbf{F}_0\|_{fro}^2 + \lambda \|\mathbf{G}_0\|_{fro}^2 + \lambda \|\alpha\|^2 + \lambda \|\beta\|^2$$

Multi-Aspect



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# Experiments



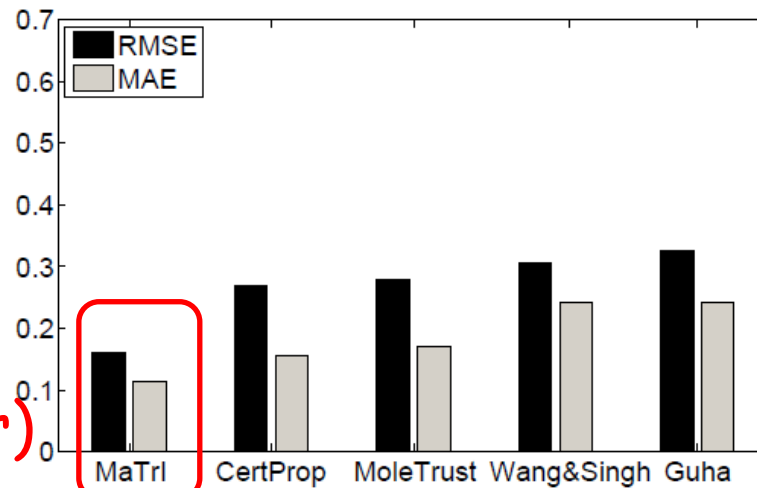
- Datasets
  - Advogato  
([http://www.trustlet.org/wiki/Advogato\\_dataset](http://www.trustlet.org/wiki/Advogato_dataset))
  - PGP (Pretty Good Privacy)
- Effectiveness: how accurate is the proposed MATRI for trust inference?
- Efficiency: how fast is the proposed MATRI?



# Effectiveness Results

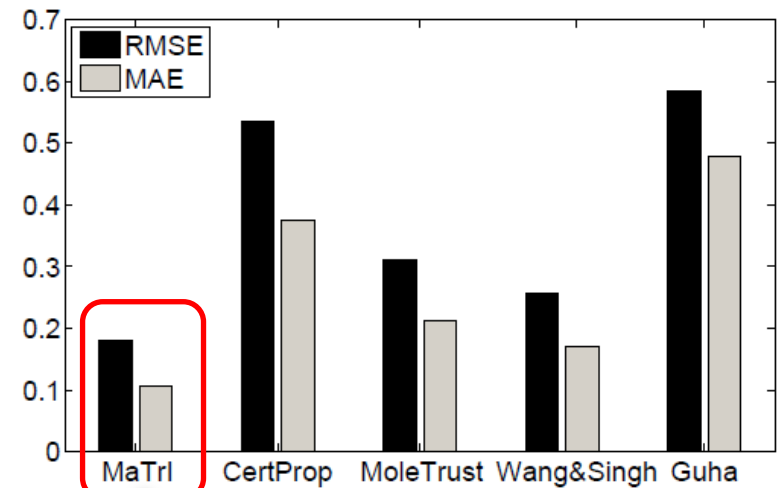


Comparisons with trust propagation models.



(a) *advogato* data set

Our method



(b) *PGP* data set

Our method



# Effectiveness Results



Comparisons with related methods. Smaller is better.

RMSE/MAE	advogato	PGP
SVD	0.629 / 0.579	0.447 / 0.306
HCD	0.269 / 0.219	0.314 / 0.216
KBV	0.179 / 0.125	0.217 / 0.133
MATRI	0.159 / 0.113	0.181 / 0.105

Our method

HCD: C. Hsieh et al., Low rank modeling of signed networks. *KDD 2012*.  
KBV: Y. Koren et al., Matrix factorization techniques for recommender systems. *Computer 2009*

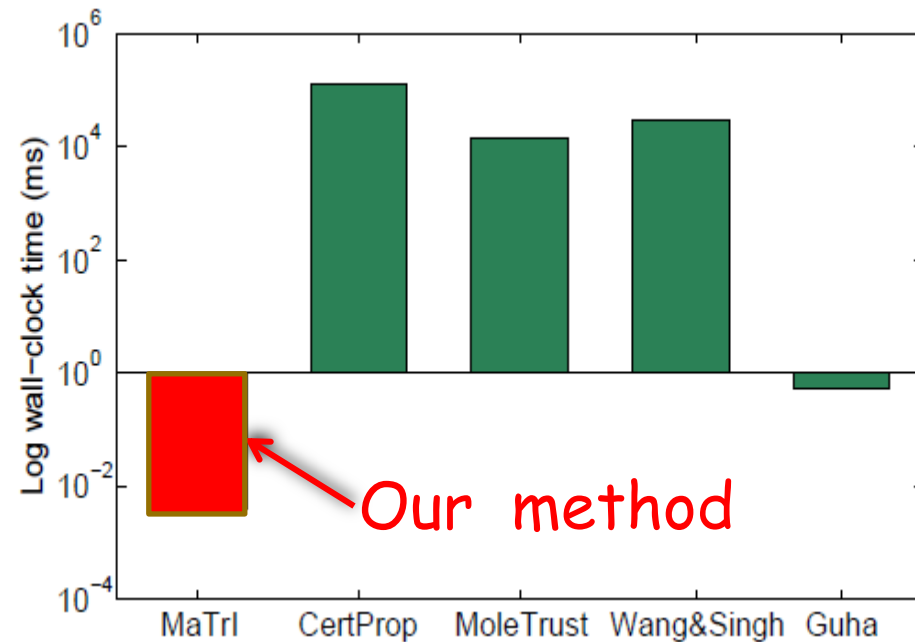
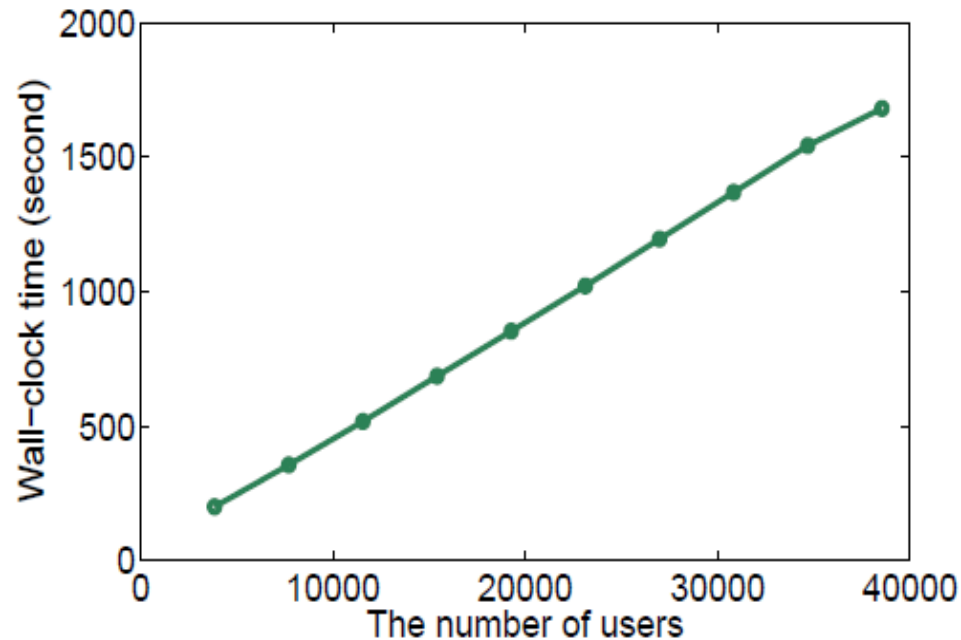


# Efficiency Results



Pre-computational time:  $O(m+n)$

Online response time:  $O(1)$







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# Conclusions



## An Integral Trust-Inference Model

- Q1: how to characterize multi-aspect?
- A1: analogy to recommendation problem
- Q2: how to incorporate trust bias?
- A2: treat bias as specified factors
- Q3: how to incorporate trust transitivity?
- A3: propagation through factorization

## Empirical Evaluations

- Effectiveness: >10% improvement
- Efficiency:
  - linear in pre-computation
  - constant online response



Thanks!

Q&A