Let $S(x) = \dfrac{1}{1+e^{-x}}$. We have $0 < S(x) \le 1$, and $S'(x) = S(x)(1 - S(x)) \ge 0$. Hence, $S(x)$ is monotonically increasing. Next, we have $0 \le S'(x) \le 2$. Then, based on the Lagrange mean value theorem, we have that $S(x)$ is Lipchitz function.

Next, since $S''(x) = S(x)(1 - S(x))(1 - 2S(x))$, we have $| S''(x) | \le 6$. Then $S'(x)$ is Lipchitz function.

Finally, let $f = \sum\sum [S(\tilde{r}_{ui} - r_{vi}) - S(r_{ui} - r_{vi})]^2 + \lambda \sum p_u^2$,

Then

$$\frac{\partial f}{\partial \tilde{r}_{ui}} = 2\sum\sum [S(\tilde{r}_{ui} - r_{vi}) - S(r_{ui} - r_{vi})] S'(\tilde{r}_{ui} - r_{vi})$$

$$=2\sum [S(\tilde{r}_{ui} - r_{vi}) - S(r_{ui} - r_{vi})] S(\tilde{r}_{ui} - r_{vi})(1 - S(\tilde{r}_{ui} - r_{vi})),$$

$$\frac{\partial^2 f}{\partial \tilde{r}_{ui} \partial \tilde{r}_{uj}} = 2S(\tilde{r}_{uj} - r_{vj})[1 - S(\tilde{r}_{uj} - r_{vj})]\{S(\tilde{r}_{uj} - r_{vj})(1 - S(\tilde{r}_{uj} - r_{vj}))$$

$$+[S(\tilde{r}_{uj} - r_{vj}) - S(r_{uj} - r_{vj})][1 - 2S(\tilde{r}_{uj} - r_{vj})]\}.$$

As we can see from the above equations, $\nabla f = \left(\dfrac{\partial f}{\partial \tilde{r}_{ui}} \right)$ is Frechet derivative. Thus, it is also G-differentiable, and $\nabla^2 f = \left(\dfrac{\partial^2 f}{\partial \tilde{r}_{ui} \partial \tilde{r}_{uj}} \right)$ satisfies $\left\| \nabla^2 f \right\| \le M < \infty$. Consequently, we have ∇f is Lipschitz function (via Theorem 3.2.4 (p.70) in [1]).

[1] Ortega J. M., Rheinboldt W. C. Iterative Solution of Nonlinear Equations in Several Variables. Academic Press, New York, 1970.

[2] http://moon.nju.edu.cn/people/jingweixu/static/proof.pdf