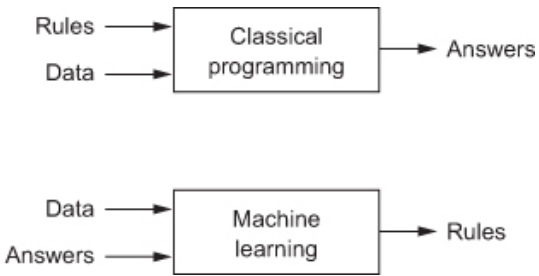
# Text Data in Business and Economics

Basel University – Autumn 2023

6. Machine Learning with Text

# Outline

# What is machine learning?




- In classical computer programming, humans input the rules and the data, and the computer provides answers.
- In machine learning, humans input the data and the answers, and the computer learns the rules.

# What do ML Algorithms do? Fit a function to data points

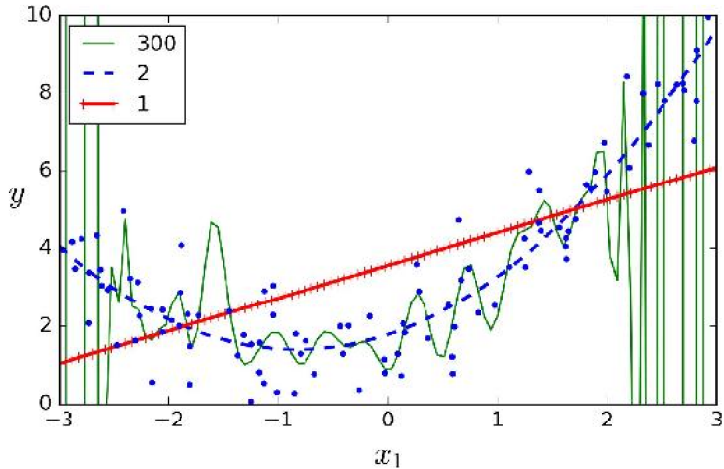



*Figure 4-14. High-degree Polynomial Regression*

# What do ML Algorithms do? Minimize a cost function

- A typical cost function (or loss function) for regression problems is Mean Squared Error (MSE):

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(x_i; \theta) - y_i)^2$$

  - $n_D$, the number of rows/observations
  - $x$, the matrix of predictors, with row $x_i$
  - $y$, the vector of outcomes, with item $y_i$
  - $h(x_i; \theta) = \hat{y}$ the model prediction (hypothesis)

The **data** $(x, y)$ are taken as given, and the ML algorithm searches for **parameters** $\theta$ to minimize the cost function.

# Linear Regression is Machine Learning

- Ordinary Least Squares Regression (OLS) assumes the functional form $f(x;\theta) = x_i'\theta$ and minimizes the mean squared error (MSE)

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- This minimand has a closed form solution

$$\hat{\theta} = (\boldsymbol{x}'\boldsymbol{x})^{-1}\boldsymbol{x}'\boldsymbol{y}$$

  - most machine learning models do **not** have a closed form solution $\rightarrow$ use numerical optimization instead (gradient descent).

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(\theta; \boldsymbol{x}_i) - y_i)^2$$

- The partial derivative for feature $j$ is

$$\frac{\partial \text{MSE}}{\partial \theta_j} = \frac{2}{n_D} \sum_{i=1}^{n_D} (\underbrace{h(\theta; \boldsymbol{x}_i) - y_i}_{\text{error for this obs}}) \underbrace{\frac{\partial h(\theta; \boldsymbol{x}_i)}{\partial \theta_j}}_{\text{how } \theta_j \text{ shifts } h(\cdot)}$$

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- The ***gradient*** $\nabla$ gives the vector of these partial derivatives for all features:

$$\nabla_\theta \text{MSE} = \begin{bmatrix} \frac{\partial \text{MSE}}{\partial \theta_1} \\ \frac{\partial \text{MSE}}{\partial \theta_2} \\ \vdots \\ \frac{\partial \text{MSE}}{\partial \theta_{n_x}} \end{bmatrix}$$

- ***Gradient descent*** nudges $\theta$ against the gradient (the direction that reduces MSE):

$$\theta_{t+1} = \theta_t - \eta \nabla_\theta \text{MSE}$$

- $\eta$ = learning rate

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- Each document $i$ has an associated outcome or label $\boldsymbol{y}_i$ with dimensions $n_y \geq 1$
- Some documents are labeled and some are unlabeled $\rightarrow$
  - we would like to learn a function $\hat{\boldsymbol{y}}(d_i)$ based on the labeled data ...
  - ... to machine-classify the unlabeled data.

# First Problem

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- The solution: all the methods from previous lectures for extracting informative numerical information from documents:
  - style features
  - counts over dictionary patterns
  - tokens
  - n-grams
  - principal components
  - topic shares
  - etc.
- documents can thus be **featurized** – represented as a matrix of vectors $\boldsymbol{x}$ with $n_x \geq 1$ features.

# Three Types of (Standard) Machine Learning Problems

Determined by the data type of the outcome variable (or label):

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  - e.g., number of days of prison assigned
- **Multinomial Classification**: Three or more discrete, un-ordered outcomes.
  - e.g., predict what judge is assigned to a case: Alito, Breyer, or Cardozo

# Loss functions, more generally

- The loss function $L(\hat{\boldsymbol{y}}, \boldsymbol{y})$ assigns a score based on prediction and truth:
  - Should be bounded from below, with the minimum attained only for cases where the prediction is correct.
- The average loss for the test set is

$$\mathcal{L}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i)$$

- The estimated parameter matrix $\theta$ solves

$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta)$$

$\hookrightarrow$ optimizes over parameter space; treats the data as constants.

# Gradient Descent

- even when cost function is not convex (eg neural nets), gradient descent often gets decent results.
- ***Stochastic* gradient descent (SGD)** computes the gradient for a single randomly sampled instance (at each iteration).
  - Much faster, still works well.

# Data Prep for Machine Learning

- Data Pre-Processing: See Geron Chapter 2 for pandas and sklearn syntax:
  - imputing missing values.
  - feature scaling (often helpful/necessary for ML models to work well)
    - if predictors are sparse (e.g. bag-of-words), use `StandardScaler(with_mean=False)`.
  - encoding categorical variables.
  - **Best practice: reproducible data pipeline**.

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- Train/Test Split:
  - ML models can achieve arbitrarily high accuracy in-sample, so performance should be evaluated out-of-sample.
  - standard approach: randomly sample 80% training dataset to learn parameters, form predictions in 20% testing dataset for evaluating performance.

# Use Cross-Validation During Model Training

- Within the training set:
  - Use cross-validation with grid search to get model performance metrics across subsets of data using different hyperparameter specs.
  - Find the best hyperparameters for out-of-fold prediction in the training set.
- Then evaluate model performance in the test set using these hyperparameters.

# Model Evaluation in Test Set

Evaluating a "good" model is context-dependent. Here are some basics.

Regression:

- mean squared error (MSE)
- R-squared (same ranking as MSE, but units are more interpretable)
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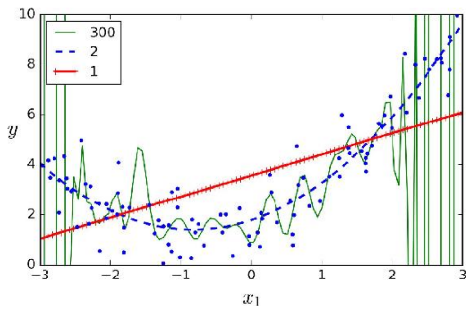
- What if one of the outcomes is over-represented – e.g., 19 out of 20? Then I can guess the modal class and get 95% accuracy.
  - Some alternative classifier metrics designed to address class imbalance (more below).

# Regression models $\leftrightarrow$ Continuous outcome

- If the outcome is continuous (e.g., $Y =$ tax revenues collected, or criminal sentence imposed in months of prison):
  - Need a regression model.
- Problems with OLS:
  - tends to over-fit training data.
  - cannot handle multicollinearity.

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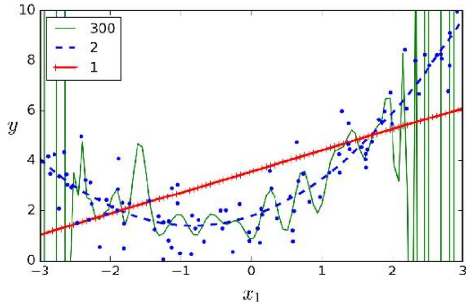


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- ***Regularization***: model training methods designed to reduce/prevent over-fitting.

# Regularized Loss Function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i) + \lambda R(\theta)$$

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- Elastic Net: $R_{\text{enet}} = \lambda_1 R_1 + \lambda_2 R_2$

# Binary Outcome $\leftrightarrow$ Binary Classification

- Binary classifiers try to match a boolean outcome $y \in \{0, 1\}$.
  - The standard approach is to apply a transformation (e.g. sigmoid/logit) to normalize $\hat{y} \in [0, 1]$.
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- The binary cross-entropy (or log loss) is:

$$L(\theta) = \underbrace{-\frac{1}{n_D}}_{\text{negative}} \sum_{i=1}^{n_D} [\underbrace{y_i}_{y_i=1} \underbrace{\log(\hat{y}_i)}_{\text{log prob} y_i=1} + \underbrace{(1-y_i)}_{y_i=0} \underbrace{\log(1-\hat{y}_i)}_{\text{log prob} y_i=0}]$$

- In **logistic regression** we use a sigmoid transformation:

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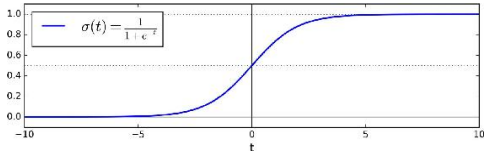

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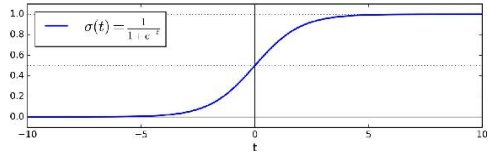


- Plugging into the binary-cross entropy loss gives the logistic regression cost objective:

$$\min_{\theta} \sum_{i=1}^{n_D} -y_i \log(\text{sigmoid}(\boldsymbol{x}_i \cdot \theta)) - [1 - y_i] \log(1 - \text{sigmoid}(\boldsymbol{x}_i \cdot \theta))$$

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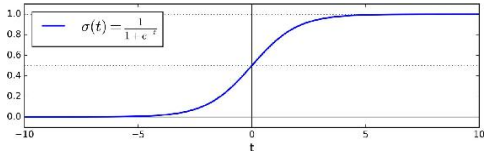


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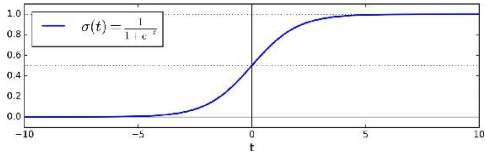


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- Like linear regression, logistic regression can be regularized with L1 or L2 penalties.

A **Confusion Matrix** is a nice way to visualize classifier performance:

| | | **Predicted Class** | |
|---|---|---|---|
| | | Negative | Positive |
| 2***True Class** | Negative | **# True Negatives** | # False Positives |
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- Recall decreases with false negatives. "When this outcome occurs, I don't miss it."

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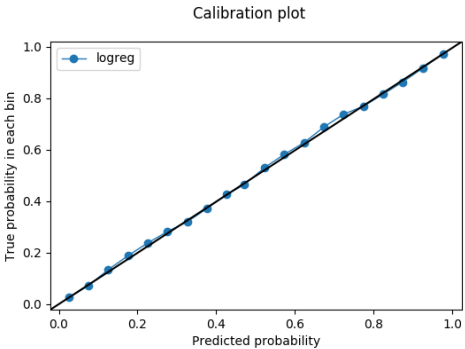
**AUC-ROC = Area Under the Receiver Operating Characteristic Curve**

- provides an aggregate measure of performance across all possible classification thresholds.
- Interpretation: randomly sample one positive and one negative example. AUC = probability that the model correctly guesses which is which.

# Evaluating Classification Models: Calibration Curves




- Plotting the binned fraction in a category (Y axis) against the predicted probability in a category (X axis):
- Provides evidence of whether the classifer is replicating the conditional distribution of the outcome.

# Multiple Classes: Setup

- The outcome is $y_i \in \{1, ..., k, ..., n_y\}$ output classes, which can also be represented as a one-hot vector

$$\boldsymbol{y}_i = \{\mathbf{1}[y_i = 1], ..., \mathbf{1}[y_i = n_y]\}$$

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- We want to learn a vector function

$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \theta)$$

taking text features $\boldsymbol{x}$ as inputs and outputing a vector of probabilities across outcome classes:

$$\hat{\boldsymbol{y}} = \{\hat{y}^1, ..., \hat{y}^{n_y}\}, \sum_{k=1}^{n_y} \hat{y}^k = 1, \hat{y}^k \geq 0 \ \forall k$$

- for prediction step, can select the highest-probability class:

$$\tilde{y} = \arg\max_k \hat{y}_{[k]}$$

# Categorical Cross Entropy

- The standard loss function in multinomial classification is **categorical cross entropy**:

$$L(\theta) = -\sum_{k=1}^{n_y} \boldsymbol{y}^k \log(\hat{y}^k(\boldsymbol{x}, \theta))$$

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- Since there is just one true class ($y = 1$ for one class $k^*$, and zero for others), simplifies to

$$L(\theta) = -\log(\hat{y}^{k^*}(\boldsymbol{x}, \theta))$$

  - Rewards putting higher probability on the true class, ignores distribution of probabilities on other classes.
  - function is convex $\rightarrow$ gradient descent will find the optimum.

# Multinomial Logistic Regression

Multinomial logistic regression computes probabilities for each class $k$ using the softmax transformation

$$\hat{y}_k(\boldsymbol{x}_i) = \Pr(y_i = k) = \frac{\exp(\theta_k' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)}$$

- softmax is the multiclass generalization of sigmoid $\rightarrow$ can then interpret $\hat{y}$ as probabilities.
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The **L2-penalized logistic regression** has loss function

$$\mathcal{L}(\theta) = -\frac{1}{n_D} \sum_{i=1}^{n_D} \log \frac{\exp(\theta_{k^*}' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)} + \lambda \sum_{j=1}^{n_x} \sum_{k=1}^{n_y} (\theta_{[j,k]})^2$$

- $\lambda$ = strength of L2 penalty (could also add lasso penalty)
  - as before, predictors should be scaled to the same variance.

| | | Predicted Class | | |
|---|---|---|---|---|
| | | Class A | Class B | Class C |
| 3***True Class** | Class A | Correct A | A, classed as B | A, classed as C |
| | Class B | B, classed as A | Correct B | B, classed as C |
| | Class C | C, classed as A | C, classed as B | Correct C |

More generally, with **multi-class confusion matrix** $M$ with items $M_{ij}$ (row $i$, column $j$):

$$\text{Precision for } k = \frac{\text{True Positives for } k}{\text{True Positives for k} + \text{False Positives for } k} = \frac{M_{kk}}{\sum_l M_{lk}}$$

$$\text{Recall for } k = \frac{\text{True Positives for } k}{\text{True Positives for } k + \text{False Negatives for } k} = \frac{M_{kk}}{\sum_l M_{kl}}$$

$$F_1(k) = 2 \times \frac{\text{precision}(k) \times \text{recall}(k)}{\text{precision}(k) + \text{recall}(k)}$$

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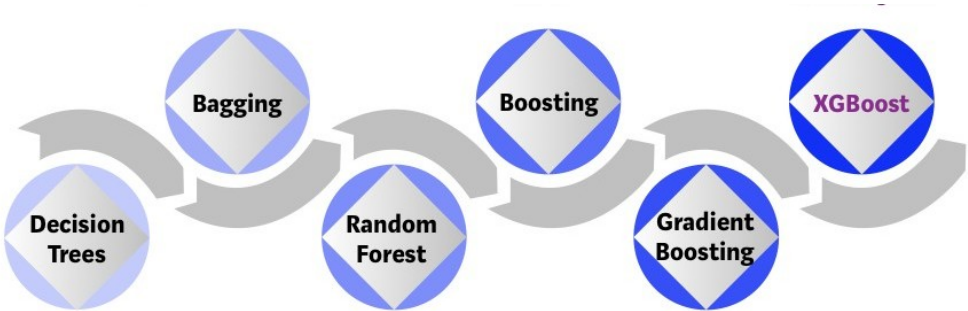
Can average these metrics across classes to get aggregate metrics.

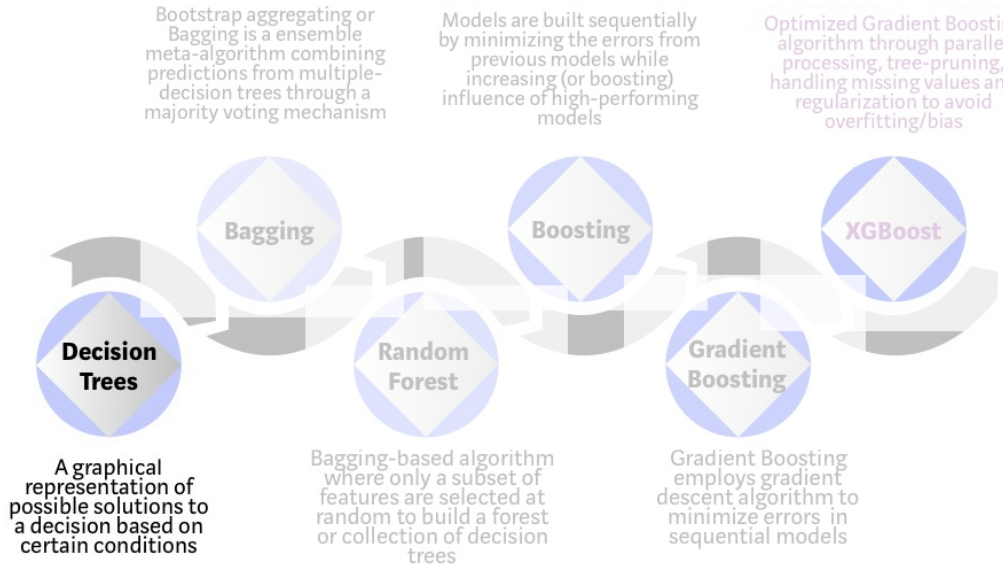
- e.g., balanced accuracy = unweighted average of recalls across classes.
- can weight classes by their frequency in dataset

# Outline

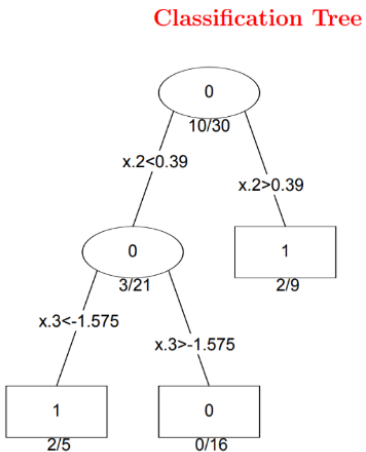
# XGBoost: Overview



# XGBoost Ingredients: Decision Trees

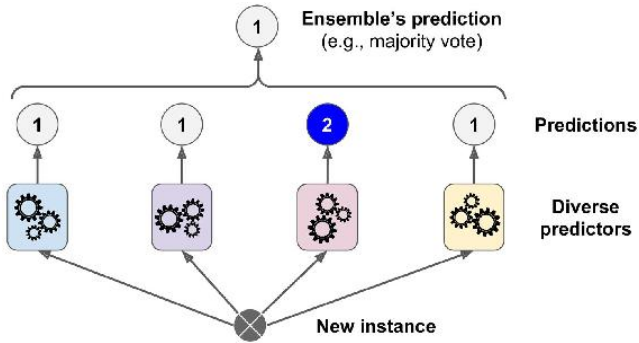


# Decision Trees



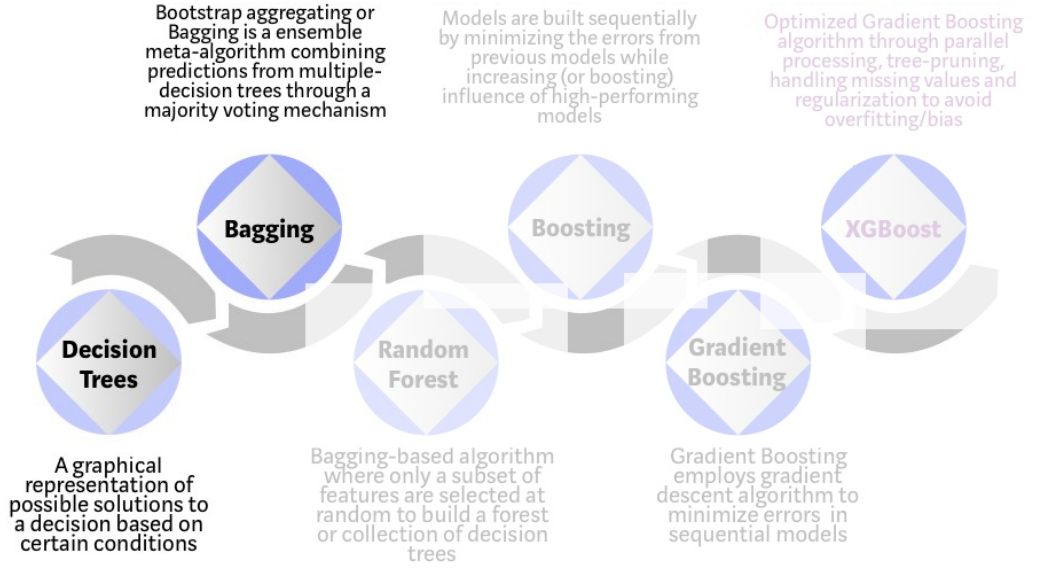

- Decision trees learn a series of binary splits in the data based on hard thresholds.
  - if yes, go right; if no, go left.
- Can have additional splits as you move through the tree.
- fast and interpretable, but performance is often poor.

# Voting Classifiers



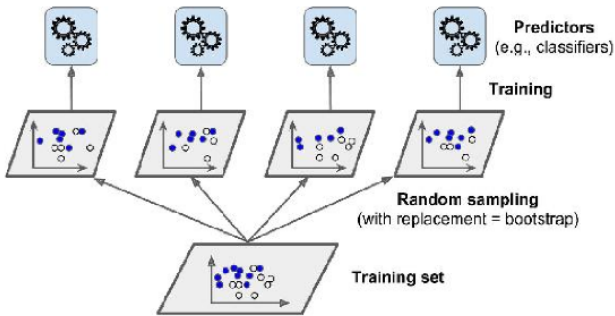

- voting classifiers (ensembles of different models that vote on the prediction) generally out-perform the best classifier in the ensemble.
  - more diverse algorithms will make different types of errors, and improve your ensemble's robustness.

# XGBoost Ingredients: Bootstrapping



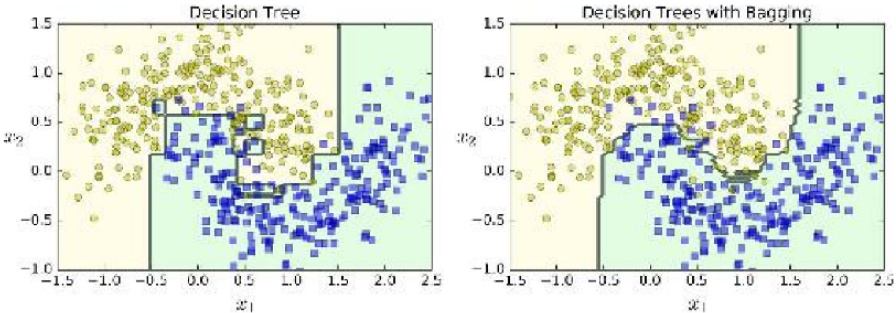
# Bootstrapping

- Rather than use the same data on different classifiers, one can use different subsets of the data on the same classifier:



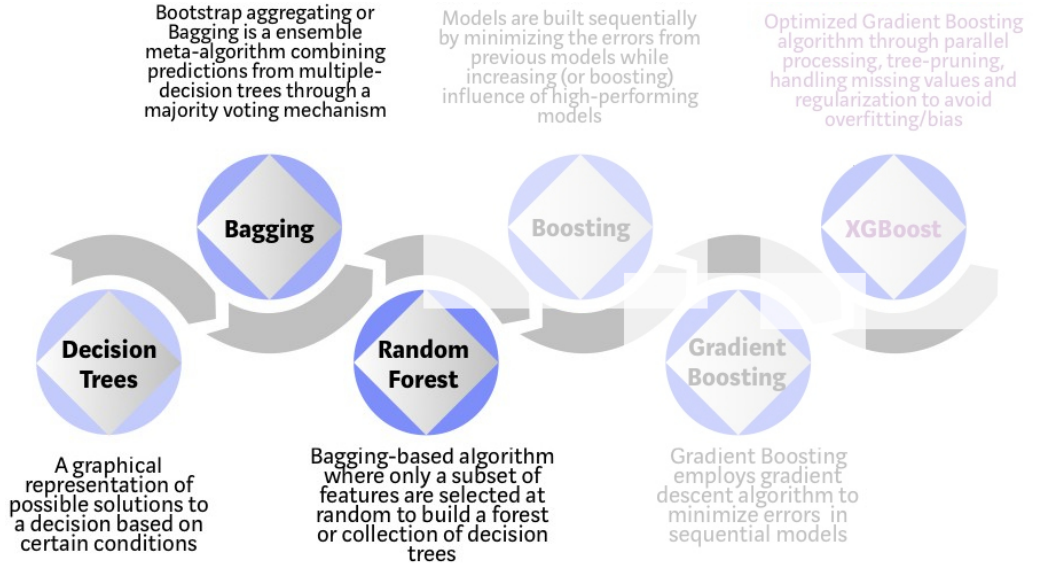

  - can also use different subsets of features across subclassifiers.

# Bootstrapping Benefits




- A bootstraped ensemble generally has a similar bias but lower variance than a single predictor trained on all the data.
- Predictors can be trained in parallel using separate CPU cores.

# XGBoost Ingredients: Random Forests



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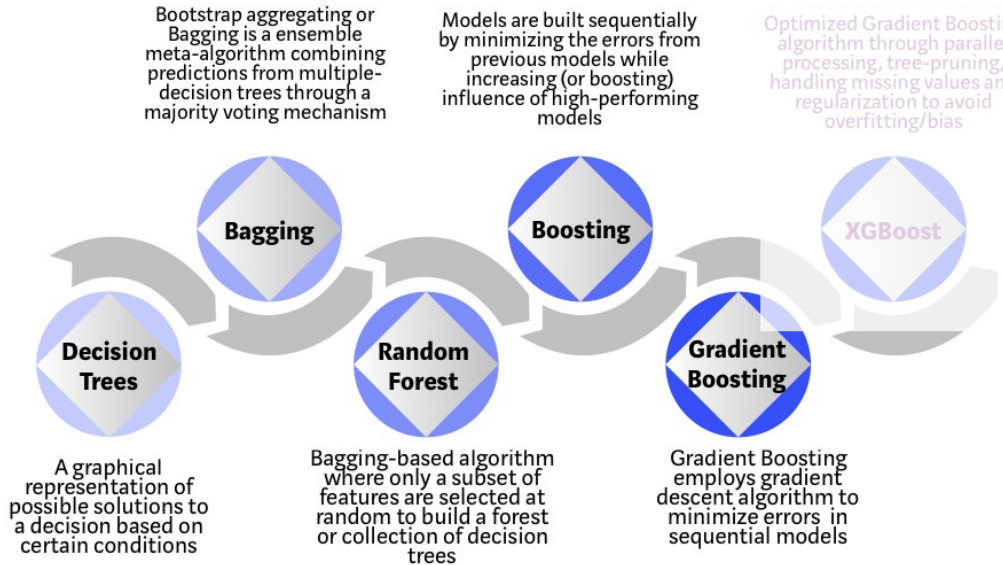
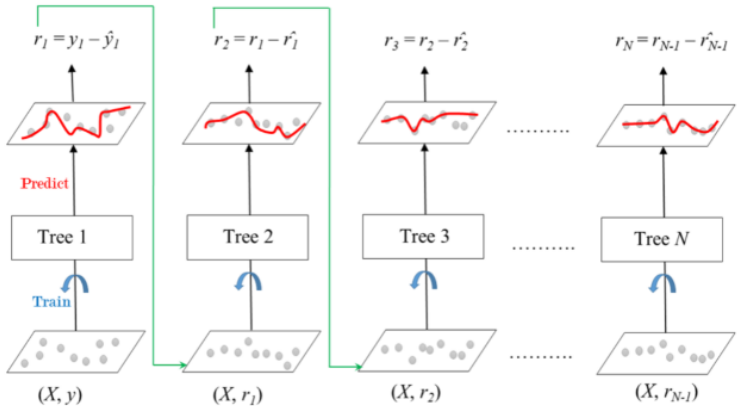
3. For each tree, error rate is computed using data outside its bootstrap sample.

# XGBoost Ingredients: Gradient Boosting



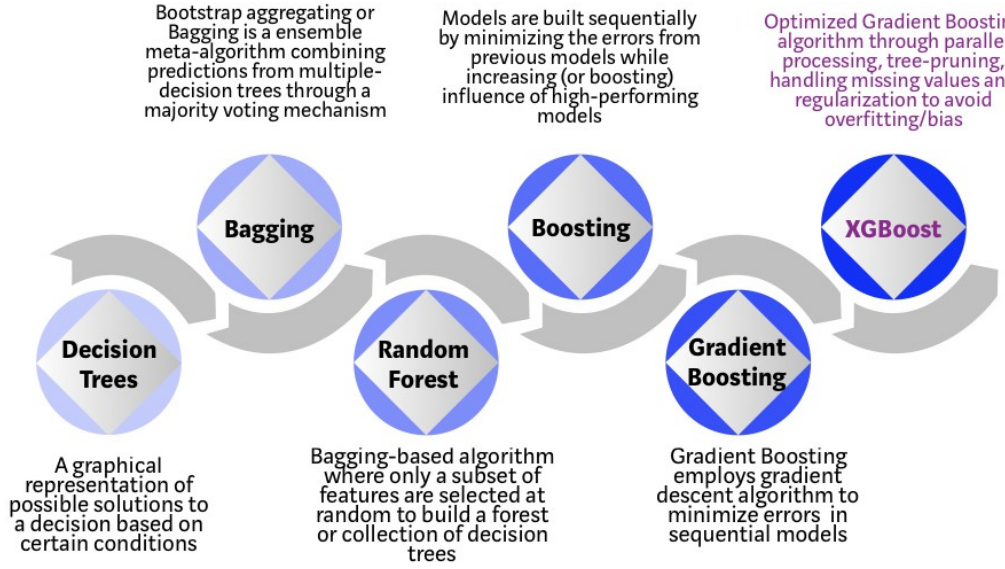
# Gradient Boosting Machines

- Gradient boosting refers to an additive ensemble of trees:




- Adds additional layers of trees to fit the residuals of the first layers

# XGBoost Ingredients

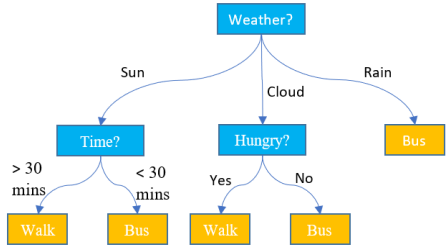


# XGBoost

- Feurer et al (2018) find that XGBoost beats a sophisticated AutoML procedure with grid search over 15 classifiers and 18 data preprocessors.
- A good starting point for any machine learning task.
- easy to use
- actively developed
- efficient / parallelizable
- provides model explanations
- takes sparse matrices as input

# Tree Ensembles are Black Boxes

- Small decision trees have the advantage of being highly interpretable.



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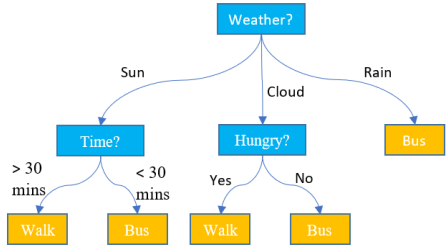


- Larger trees and ensembles (e.g. XGBoost) lose this nice feature.
- Best-performing ML models are hard to interpret because they use lots of features and exploit non-linearities and interactions.

# Interpreting Tree Ensembles

XGBoost's Feature Importance Metric:

- At each decision node, compute **information gain** for feature $j$ **(change in predicted probability**).
- Average across all nodes for each $j$.

Ranks predictors by their relative contributions.

```
from xgboost import plot_importance
plot_importance(xgb_reg, max_num_features=10)
```

# Feature Importance

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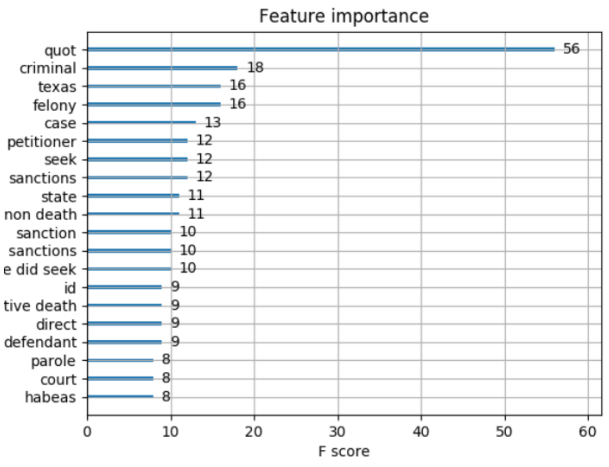
```
<IPython.core.display.Javascript object>
```



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6. Answer the research question!