D001 Economic Analysis of Non-Standard Data

Benjamin W. Arold

5. Unsupervised and Supervised Learning from Text

Different Goals, Different Methods

- Unsupervised Learning
  - algorithm discovers themes/patterns in text (or other high-dimensional data)
  - human interprets the results (e.g. inspect content of topics or clusters)

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- Both strategies amplify human effort, each in different ways
- Distinctions are not clear-cut:
  - unsupervised learning models can be used in service of prediction or known goals
  - supervised learning models can be used to discover themes/patterns

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4. Empirical analysis
   - Produce statistics or predictions with the trained model
   - **Answer the research question**

Outline

The Document-Term Matrix is high-dimensional

The **document-term matrix $\boldsymbol{X}$**:

- each row d represents a **document,** while each column w represents a word (or term more generally, e.g. n-grams).
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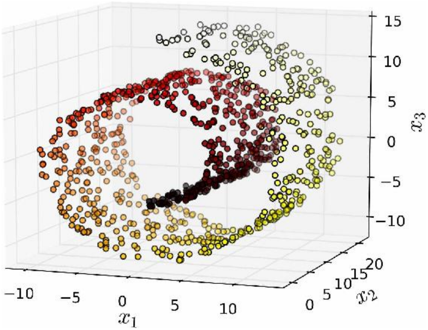
$\rightarrow$ $\boldsymbol{X}$ often has billions of cells.

Distribution of Datasets

- Datasets are not distributed uniformly across the feature space
- They have a lower-dimensional latent structure – a **manifold** – that can be learned

"The Swiss Roll"



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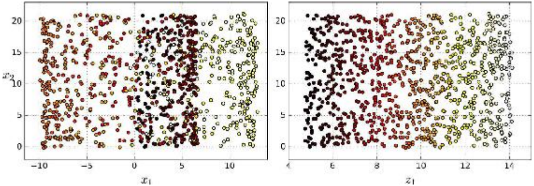

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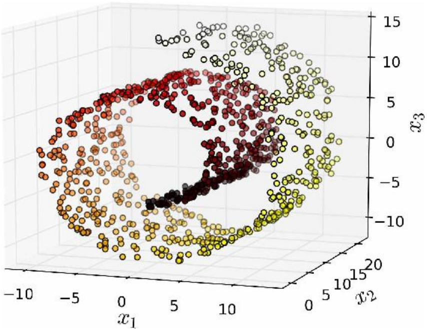
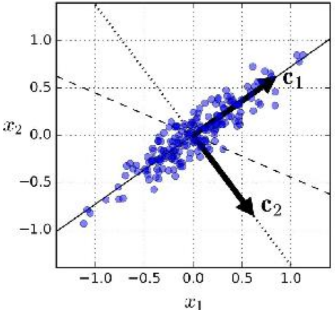


- **Dimensionality reduction** makes data more interpretable – for example by projecting down to two dimensions for visualization
- improves computational tractability
- can improve model performance

What dimension reductions have you already tried in this class?

PCA (principal component analysis)



- PCA computes the dimension in data explaining most variance.

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from sklearn.decomposition import PCA
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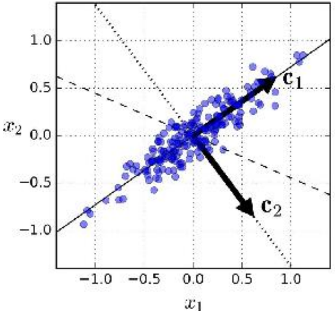

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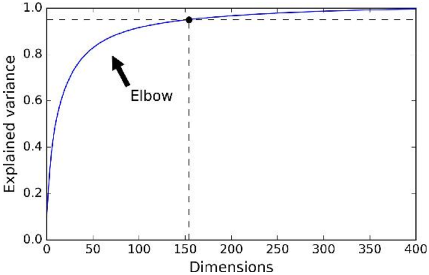

- after the first component, subsequent components learn the (orthogonal) dimensions explaining most variance in dataset after projecting out first component

PCA and LSA

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- This is known as “latent semantic analysis”
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- PCA factors are not interpretable
  - For non-negative data (e.g. counts or frequencies), **Non-negative Matrix Factorization (NMF)** provides more interpretable factors than PCA

Outline

Topic Models in Economics/Social Science

- Core methods for topic models were developed in computer science and statistics
  - summarize unstructured text
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- Economists use topics as a form of measurement
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  - tell a story not just about what, but how and why
  - **topic models are more interpretable** than other dimension reduction methods, such as PCA

Latent Dirichlet Allocation

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Latent Dirichlet Allocation

- Each topic is a distribution over words
- Each document is a distribution over topics
- Input: $N \times M$ document-term count matrix X
- Like NMF, LDA works by factorizing X into:
  - an $N \times K$ document-topic matrix
  - an $K \times M$ topic-term matrix
- Assume: there are K topics (tunable hyperparameter, use coherence)
- Unlike NMF, LDA is probabilistic:
  - Goal: Estimate the latent (hidden) topic structure that best explains the word co-occurrences in documents
  - Algorithm maximizes the likelihood of observed documents given the latent topic structure
  - Output: Estimates of probabilities that each word and each document are composed of each topic
  - LDA is a generative probabilistic model, meaning it assumes documents were generated by an underlying topic structure, and it tries to reverse-engineer that structure
  - Dirichlet priors: Hyperparameters for topics per document (alpha), and words per topic (beta)

Using an LDA Model

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Can then use the topic proportions as variables in a economics analysis.

- e.g., Catalinac (2016) shows that after a Japanese political reform that reduced intraparty competition, candidate platforms reduced pork-barrel policies and increased national ones

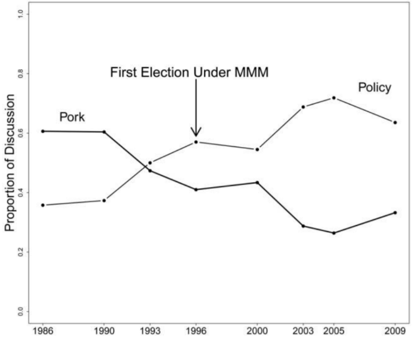

TABLE 1 A Summary of Common Assumptions and Relative Costs Across Different Methods of Discrete Text Categorization

	Method				
A. Assumptions	*Reading*	*Human Coding*	*Dictionaries*	*Supervised Learning*	*Topic Model*
Categories are known	No	Yes	Yes	Yes	No
Category nesting, if any, is known	No	Yes	Yes	Yes	No
Relevant text features are known	No	No	Yes	Yes	Yes
Mapping is known	No	No	Yes	No	No
Coding can be automated	No	No	Yes	Yes	Yes
B. Costs					
Preanalysis Costs					
Person-hours spent conceptualizing	Low	High	High	High	Low
Level of substantive knowledge	Moderate/High	High	High	High	Low
Analysis Costs					
Person hours spent per text	High	High	Low	Low	Low
Level of substantive knowledge	Moderate/High	Moderate	Low	Low	Low
Postanalysis Costs					
Person-hours spent interpreting	High	Low	Low	Low	Moderate
Level of substantive knowledge	High	High	High	High	High

Recommended: read this part of Quinn, Monroe, Colaresi, Crespin, and Radev (2010).

Structural Topic Model = LDA + Metadata

Roberts, Stewart, and Tingley

STM provides two ways to include contextual information:

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- Topic prevalence can vary by metadata
  - e.g. Republicans talk about military issues more then Democrats
- Topic content can vary by metadata
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- Structural topic model is not a prediction model:
  - it will tell you which topics or features correlate with an outcome, but it will not provide an in-sample or out-of-sample prediction for an outcome
- It actually uses another distribution of the priors (*not* Dirichlet) such that without covariates it replicates the correlated topic model (Blei and Lafferty, 2005)

Recent Advances in Topic Model

- Keyword-Assisted topic model (Eshima, Imai, and Sasaki, 2024)
    - allows *semi-supervised* creation of topics
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- Keyword-Assisted topic model (Eshima, Imai, and Sasaki, 2024)
  - allows *semi-supervised* creation of topics
  - input seed dictionaries and labeled topics
- Problems with unstructured data and casual inference (Battaglia et al., 2024)
  - shows that two-steps strategy leads to invalid inference
  - propose solutions with bias correction and a one-step strategy

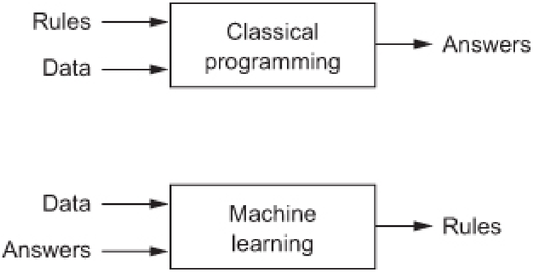
Outline

Dimensionality Reduction

Topic Models

Supervised Learning
- Overview
- Regression / Regularization
- Binary Classification
- Multi-Class Models

What is supervised machine learning?



- In classical computer programming, humans input the rules and the data, and the computer provides answers.
- In machine learning, humans input the data and the answers, and the computer learns the rules.

What do ML Algorithms do? Fit a function to data points

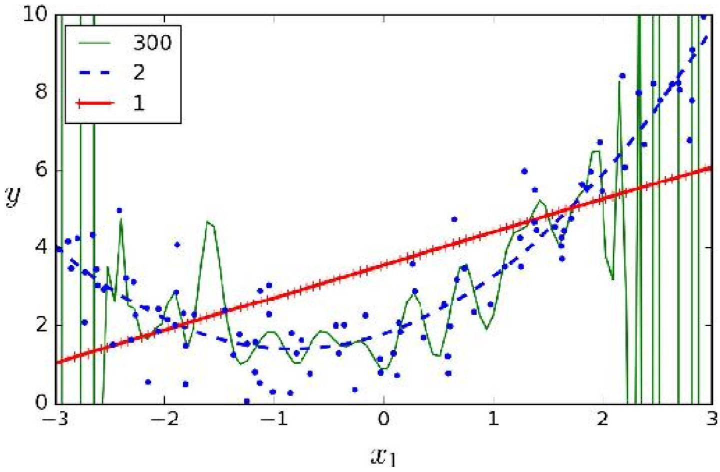


Figure 4-14. High-degree Polynomial Regression

What do ML Algorithms do? Minimize a cost function

- A typical cost function (or loss function) for regression problems is Mean Squared Error (MSE):

$$\text{MSE}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} (h(x_i;\theta) - y_i)^2$$

  - n_D, the number of rows/observations
  - x, the matrix of predictors, with row x_i
  - y, the vector of outcomes, with item y_i
  - $h(x_i;\theta) = \hat{y}$ the model prediction (hypothesis)

The **data** (x, y) are taken as given, and the ML algorithm searches for **parameters** θ to minimize the cost function

Loss functions, more generally

- The loss function $L(\hat{\boldsymbol{y}}, \boldsymbol{y})$ assigns a score based on prediction and truth:
  - Should be bounded from below, with the minimum attained only for cases where the prediction is correct
- The average loss for the test set is

$$\mathcal{L}(\theta) = \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i)$$

- The estimated parameter matrix θ solves

$$\hat{\theta} = \arg\min_{\theta} \mathcal{L}(\theta)$$

Linear Regression is Machine Learning

- Ordinary Least Squares Regression (OLS) assumes the functional form $f(x;\theta) = x_i'\theta$ and minimizes the mean squared error (MSE)

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- This minimand has a closed form solution

$$\hat{\theta} = (\boldsymbol{x}'\boldsymbol{x})^{-1}\boldsymbol{x}'\boldsymbol{y}$$

  - most machine learning models do **not** have a closed form solution $\rightarrow$ use numerical optimization instead (gradient descent).

Gradient Descent

$$\text{MSE}(\theta) = \frac{1}{n_D}\sum_{i=1}^{n_D}(h(\theta;\boldsymbol{x}_i) - y_i)^2$$

- The partial derivative for feature j is

$$\frac{\partial \text{MSE}}{\partial \theta_j} = \frac{2}{n_D}\sum_{i=1}^{n_D}(\underbrace{h(\theta;\boldsymbol{x}_i) - y_i}_{\text{error for this obs}}) \underbrace{\frac{\partial h(\theta;\boldsymbol{x}_i)}{\partial \theta_j}}_{\text{how } \theta_j \text{ shifts } h(\cdot)}$$

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- The ***gradient*** ∇ gives the vector of these partial derivatives for all features:

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- If the cost function is convex, gradient descent is guaranteed to find the global minimum

Gradient Descent

- even when cost function is not convex (eg neural nets), gradient descent often gets decent results
- ***Stochastic* gradient descent (SGD)** computes the gradient for a single randomly sampled data point (at each iteration)
  - Much faster, still works well

Evaluation: Use Cross-Validation During Model Training

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  - ML models can achieve arbitrarily high accuracy in-sample, so performance should be evaluated out-of-sample

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- Train/Test Split:
  - ML models can achieve arbitrarily high accuracy in-sample, so performance should be evaluated out-of-sample
  - standard approach: randomly sample 80% training dataset to learn parameters, form predictions in 20% testing dataset for evaluating performance
- Within the training set:
  - Use cross-validation with grid search to get model performance metrics across subsets of data using different hyperparameter specs.
  - Find the best hyperparameters for out-of-fold prediction in the training set
- Then evaluate model performance in the test set using these hyperparameters

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- Each document i has an associated outcome or label $\boldsymbol{y}_i$ with dimensions $n_y \geq 1$
- Some documents are labeled and some are unlabeled $\rightarrow$
  - we would like to learn a function $\hat{\boldsymbol{y}}(d_i)$ based on the labeled data ...
  - ... to machine-classify the unlabeled data.

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- The solution: all the methods from previous lectures for extracting informative numerical information from documents:
  - style features
  - counts over dictionary patterns
  - tokens
  - n-grams
  - principal components
  - topic shares
  - etc.
- documents can thus be **featurized** – represented as a matrix of vectors $\boldsymbol{x}$ with $n_x \geq 1$ features.

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Determined by the data type of the outcome variable (or label):

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- **Binary classification**: two choices, normalized to zero and one.
  - e.g., guilty or innocent
- **Multinomial Classification**: Three or more discrete, un-ordered outcomes.
  - e.g., predict what judge is assigned to a case: Alito, Breyer, or Cardozo

Model Evaluation in Test Set

Evaluating a "good" model is context-dependent. Here are some basics

Regression:

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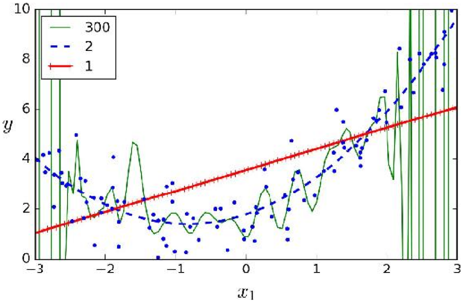
  accuracy $=$ (# correct test-set predictions) / (# of test-set observations)
- What if one of the outcomes is over-represented – e.g., 19 out of 20? Then I can guess the modal class and get 95% accuracy
  - Some alternative classifier metrics designed to address class imbalance (more below)

Regression models $\leftrightarrow$ Continuous outcome

- If the outcome is continuous (e.g., $Y =$ tax revenues collected, or criminal sentence imposed in months of prison):
    - Need a regression model
- Problems with OLS:
    - tends to over-fit training data
    - cannot handle multicollinearity



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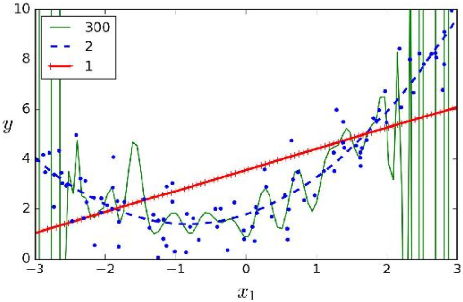

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- ***Regularization***: model training methods designed to reduce/prevent over-fitting

Regularized Loss Function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i; \theta), \boldsymbol{y}_i) + \lambda R(\theta)$$

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Regularized Loss Function

$$\hat{\boldsymbol{\theta}} = \arg\min_{\theta} \frac{1}{n_D} \sum_{i=1}^{n_D} L(h(\boldsymbol{x}_i;\theta), \boldsymbol{y}_i) + \lambda R(\theta)$$

- $R(\theta)$ is a "regularization function" or "regularizer", designed to reduce over-fitting
- λ is a hyperparameter where higher values increase regularization

In particular:

- "Lasso" (or L1) penalty:

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- Elastic Net: $R_{\text{enet}} = \lambda_1 R_1 + \lambda_2 R_2$

Binary Outcome $\leftrightarrow$ Binary Classification

- Binary classifiers try to match a boolean outcome $y \in \{0, 1\}$.
  - The standard approach is to apply a transformation (e.g. sigmoid/logit) to normalize $\hat{y} \in [0, 1]$.
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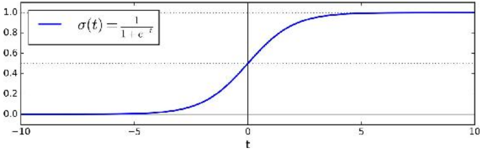
- The binary cross-entropy (or log loss) is:

$$L(\theta) = \underbrace{-\frac{1}{n_D}}_{\text{negative}} \sum_{i=1}^{n_D} [\underbrace{y_i}_{y_i=1} \underbrace{\log(\hat{y}_i)}_{\text{log prob} y_i=1} + \underbrace{(1 - y_i)}_{y_i=0} \underbrace{\log(1 - \hat{y}_i)}_{\text{log prob} y_i=0}]$$

- In **logistic regression** we use a sigmoid transformation:

$$\hat{y} = \text{sigmoid}(\boldsymbol{x} \cdot \theta) = \frac{1}{1 + \exp(-\boldsymbol{x} \cdot \theta)}$$



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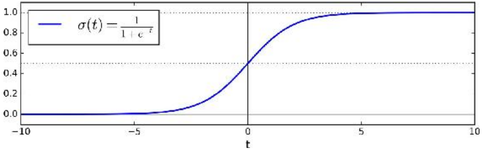

- Plugging into the binary-cross entropy loss gives the logistic regression cost objective:

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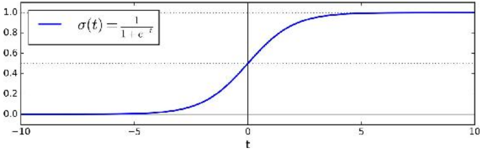

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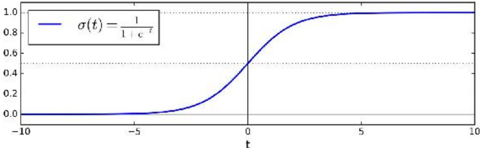

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- Like linear regression, logistic regression can be regularized with L1 or L2 penalties.

Evaluating Binary Classification Models

A **Confusion Matrix** is a nice way to visualize classifier performance:

		Predicted Class	
		Negative	Positive
2***True Class**	Negative	**# True Negatives**	# False Positives
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$$\text{Recall (for positive class)} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

- Recall decreases with false negatives. “When this outcome occurs, I don’t miss it.”

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AUC-ROC = Area Under the Receiver Operating Characteristic Curve

- Provides an aggregate measure of performance across all possible classification thresholds
- False Positive Rate (FPR) vs. True Positive Rate (TPR)
- Interpretation: randomly sample one positive and one negative example AUC = probability that the model correctly guesses which is which

Multiple Classes: Setup

- The outcome is $y_i \in \{1, ..., k, ..., n_y\}$ output classes, which can also be represented as a one-hot vector

$$\boldsymbol{y}_i = \{\mathbf{1}[y_i = 1], ..., \mathbf{1}[y_i = n_y]\}$$

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$$\boldsymbol{y}_i = \{\mathbf{1}[y_i = 1], ..., \mathbf{1}[y_i = n_y]\}$$
- We want to learn a vector function
$$\boldsymbol{y} = \boldsymbol{h}(\boldsymbol{x}, \theta)$$
taking text features $\boldsymbol{x}$ as inputs and outputting a vector of probabilities across outcome classes:
$$\hat{\boldsymbol{y}} = \{\hat{y}^1, ..., \hat{y}^{n_y}\}, \sum_{k=1}^{n_y} \hat{y}^k = 1, \hat{y}^k \geq 0 \ \forall k$$
- for prediction step, can select the highest-probability class:
$$\tilde{y} = \arg\max_k \hat{y}_{[k]}$$
- The standard loss function in multinomial classification is **categorical cross entropy**:
$$L(\theta) = -\sum_{k=1}^{n_y} \boldsymbol{y}^k \log(\hat{y}^k(\boldsymbol{x}, \theta))$$

Multinomial Logistic Regression

Multinomial logistic regression computes probabilities for each class k using the softmax transformation

$$\hat{y}_k(\boldsymbol{x}_i) = \Pr(y_i = k) = \frac{\exp(\theta_k' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)}$$

- softmax is the multiclass generalization of sigmoid $\rightarrow$ can then interpret $\hat{y}$ as probabilities.
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The **L2-penalized logistic regression** has loss function

$$\mathcal{L}(\theta) = -\frac{1}{n_D} \sum_{i=1}^{n_D} \log \frac{\exp(\theta_k' \boldsymbol{x}_i)}{\sum_{l=1}^{n_y} \exp(\theta_l' \boldsymbol{x}_i)} + \lambda \sum_{j=1}^{n_x} \sum_{k=1}^{n_y} (\theta_{[j,k]})^2$$

- $\lambda =$ strength of L2 penalty (could also add lasso penalty)
  - as before, predictors should be scaled to the same variance.

		Predicted Class		
		Class A	Class B	Class C
3* **True Class**	Class A	Correct A	A, classed as B	A, classed as C
	Class B	B, classed as A	Correct B	B, classed as C
	Class C	C, classed as A	C, classed as B	Correct C

More generally, with **multi-class confusion matrix** M with items M_{ij} (row i, column j):

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Can average these metrics across classes to get aggregate metrics

- e.g., balanced accuracy = unweighted average of recalls across classes
- can weight classes by their frequency in dataset

Ensemble Methods

Key Idea: Combine multiple models to improve accuracy and reduce errors

- Voting classifiers (ensembles of different models that vote on the prediction) generally out-perform the best classifier in the ensemble
- More diverse algorithms will make different types of errors, and improve your ensemble's robustness

Types of Ensemble Methods:

- Bagging – Combine independent models (Random Forest)
- Boosting – Sequentially improve weak models (Gradient Boosting: XGBoost)

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6. Answer the research question!