## Home Task

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# Task 2 – Vacuum Tube

# Why is vacuum important in the tube? List at least two major reasons.

The use of vacuum inside the acceleration tube is essential for both the performance of the capsule and the structural integrity of the tube. Several key reasons are outlined below:

- 1. Reduction of aerodynamic drag. In continuum flow (atmospheric conditions), drag follows  $F_d = \frac{1}{2}\rho v^2 C_d A$  where  $F_d \propto v^2$ . At hypersonic speeds ( $v \sim 7000 \text{m/s}$ ), even trace atmospheric densities produce enormous opposing forces. However, in the free molecular flow regime achieved by vacuum (Kn  $\gg 1$ ), drag becomes  $F_d = \frac{2}{3} n m_{\text{mol}} v A \alpha$  where  $F_d \propto v$ . This linear velocity dependence, combined with drastically reduced molecular density, suppresses drag by orders of magnitude and allows efficient acceleration. Even trace atmospheric densities lead to prohibitive drag and heating (see Appendix A.1), which highlights why ultra-high vacuum is essential.
- 2. Reduction of aerodynamic heating. In continuum flow, heating scales as  $P \propto \rho v^3$  due to stagnation temperature effects. For  $M \gg 1$ , stagnation temperatures reach tens of thousands of kelvins, far above material limits. In free molecular flow achieved by vacuum, heating scales as  $P \propto nv$  where individual molecules transfer kinetic energy upon collision. This fundamental change in heating mechanism, combined with reduced molecular density, makes thermal management feasible.
- 3. **Prevention of shockwave formation and choking.** In a confined tube with residual gas, shockwaves would form, reflect, and interfere, creating unstable oscillations and potentially choking the flow. Vacuum prevents these destructive phenomena by eliminating the continuous medium required for shock propagation.
- 4. **Structural safety of the tube.** Residual gas allows shockwaves to form and reflect inside the tube. These shocks impose oscillating loads on the walls, which can couple into the tube's natural vibration modes. Over repeated launches this resonance may cause fatigue or even structural failure. Maintaining vacuum prevents shockwave formation and thus protects the tube from these oscillatory stresses.
- 5. **Energy efficiency.** Without vacuum, a large portion of input energy would be wasted compressing and heating the residual air instead of accelerating the capsule. Vacuum ensures that nearly all electromagnetic energy is converted into kinetic energy.
- 6. **Stable capsule dynamics.** Residual atmosphere at high accelerations would introduce asymmetric forces on the capsule, risking destabilization or tumbling. Vacuum ensures predictable, symmetric acceleration governed solely by the coil fields.

# What would be the approximate vacuum level required? What are the main considerations for this required level?

We require operation in the free molecular regime with Knudsen number Kn > 10, i.e. mean free path  $\lambda > 12$  m for a 1.2 m capsule. This corresponds to a required pressure of

$$p < 6.1 \times 10^{-6}$$
 mbar.

At this pressure, drag and heating are manageable:

$$F_d \approx 0.16 \text{ N}, \qquad P_{\text{heat}} \approx 1.36 \text{ kW}, \qquad \Delta T_{\text{bulk}} \approx 0.0065^{\circ} \text{C}, \qquad \Delta T_{\text{surface}} \approx 0.037^{\circ} \text{C}.$$

Both temperature increases are essentially irrelevant compared to aluminum's melting point (660 °C), so heating is not a design driver at this vacuum level. Derivations are provided in Appendix A.2.

List at least two methods for the capsule to exit the vacuum tube, with the pro's and con's of each method. Think of at least one very simple method, and another that's much more technological. What would you choose?

Several approaches exist for allowing the capsule to exit the vacuum tube into the atmosphere. Table 1 summarizes the main options.

Method	Pros	Cons	
Thin burstable membrane (simple)	Simple, cheap, no moving parts, immediate availability	Single-use, debris risk, requires cleanup. Asymmetric tearing gives $\theta \approx 4 \times 10^{-6}$ rad for 100 kg capsule (< 40 cm miss at 100 km) (see Appendix A.3).	
Fast mechanical gate (technological)	Reusable, clean opening, no debris	Complex, expensive, requires $\mu$ s timing precision at $v=7000$ m/s, single-point failure risk	
Supersonic gas curtain (intermediate)	No physical exit hardware, self-healing, moderate cost	Continuous $N_2$ consumption ( $\sim 10$ kg/s), degrades upstream vacuum to $\sim 0.1$ Pa	
Plasma window (advanced)	Continuous operation, no mechanical parts	Extreme power requirement (~90 GW), stability issues, electrode erosion	

Table 1: Comparison of capsule exit methods from the vacuum tube.

**Choice.** For proof-of-concept: **Membrane** for simplicity, cheapness, and adequate precision. For an operational system: **Mechanical gate**. The gas curtain offers an interesting middle ground but degrades vacuum performance. Plasma windows remain impractical due to prohibitive power requirements.

What are the main elements of the vacuum system (that creates and holds the vacuum)? Find an actual vacuum pump that will do the job and link to its specifications sheet. Note: It can be one "big" pump, or many smaller pumps.

We segment the 3 km tube into **30 cells of 100 m**, each isolated by a gate interface with **parallel (coaxial) redundancy**, two independent actuators on the same closure so that either one can open the bore. Each cell is locally pumped with a turbomolecular pump (e.g., Edwards nEXT730D [2]) backed by a dry pump (e.g., Edwards iH1000 [3]). UHV practices are mandatory: electropolished 316L, full bakeout ( $\sim 300^{\circ}$ C), TiZrV NEG activation, and distributed cryopanels for water/hydrogen capture.

At 300 K, a 100 m cell has high molecular-flow conductance ( $\sim 9.6 \times 10^3$  L/s), so the effective speed is pump-limited. With  $S_{\rm pump} \approx 730$  L/s and  $V_{\rm cell} \approx 1.43 \times 10^5$  L, the time constant is  $\tau = V/S \approx 1.96 \times 10^2$  s. After roughing to  $10^{-2}$  mbar, a few  $\tau$  bring each cell to the  $10^{-5}$  mbar regime in  $\sim 40$  minutes. All 30 cells pump in parallel. Reliability drives the interface design: a single 99.9% gate per interface gives system launch reliability  $(0.999)^{29} \approx 97.1\%$ . Using parallel redundancy per interface raises station reliability to  $1 - (1 - 0.999)^2 = 0.999999$ , giving system reliability  $\approx 99.99\%$  across 29 interfaces.

Details and derivations (conductance, pump-down, outgassing, reliability) are in Appendix A.4.

# A Appendix

## A.1 Continuum Flow Analysis (Baseline for Comparison)

The drag force in continuum (viscous) flow is:

$$F_d = \frac{1}{2} \rho v^2 C_d A \tag{1}$$

For streamlined hypersonic bodies,  $C_d \approx 0.55$  [1]. The tube's inner diameter = 1.35 m, capsule diameter d = 1.20 m (for clearance), giving frontal area:

$$A = \frac{\pi d^2}{4} \approx 1.131 \,\mathrm{m}^2 \tag{2}$$

The instantaneous power dissipated into the flow is

$$P = F_d v = \frac{1}{2} \rho v^3 C_d A. \tag{3}$$

At sea level  $\rho_0 = 1.225 \text{ kg/m}^3$ , with v = 7000 m/s:

$$\rho = 0.01 \rho_0 = 0.01225 \,\text{kg/m}^3 \implies F_d = 1.87 \times 10^5 \,\text{N}, \quad P = 1.32 \times 10^9 \,\text{W}$$
 (4)

$$\rho = 0.0001 \rho_0 = 1.225 \times 10^{-4} \,\mathrm{kg/m}^3 \implies F_d = 1.87 \times 10^3 \,\mathrm{N}, \quad P = 1.32 \times 10^7 \,\mathrm{W}$$
 (5)

Even at just  $10^{-4}$  of sea-level density, drag forces remain kilonewtons and heating remains at the megawatt level.

#### A.2 Free Molecular Flow Analysis

The flow regime is characterized by the Knudsen number:

$$Kn = \frac{\lambda}{d} \tag{6}$$

where  $\lambda$  is the mean free path and d is the capsule diameter.

The mean free path for air molecules is:

$$\lambda = \frac{kT}{\sqrt{2}\sigma p} = \frac{1.38 \times 10^{-23} \times 300}{\sqrt{2} \times 4 \times 10^{-19} \times p} = \frac{7.324 \times 10^{-3}}{p} \text{ m}$$
 (7)

For free molecular flow (Kn > 10) with d = 1.2 m:

$$\lambda > 12 \text{ m} \Rightarrow p < 6.1 \times 10^{-4} \text{ Pa} = 6.1 \times 10^{-6} \text{ mbar}$$
 (8)

In free molecular flow, drag becomes:

$$F_d = \frac{2}{3} n m_{\text{mol}} v A \alpha \tag{9}$$

where n=p/(kT) is molecular number density,  $m_{\rm mol}=4.8\times 10^{-26}$  kg, and  $\alpha\approx 1$  is the accommodation coefficient.

At  $p = 6.1 \cdot 10^{-4}$  Pa and v = 7000 m/s:

$$n = \frac{6.1 \times 10^{-4}}{1.38 \times 10^{-23} \times 300} = 1.47 \times 10^{17} \text{ molecules/m}^3$$
 (10)

$$F_d = \frac{2}{3} \times 1.47 \times 10^{17} \times 4.8 \times 10^{-26} \times 7000 \times 1.131 \times 1 = 0.16 \text{ N}$$
 (11)

Heating becomes dominated by molecular kinetic energy transfer:

$$q'' = nv E_{\text{mol}} \alpha = 1.47 \times 10^{17} \times 7000 \times 1.18 \times 10^{-18} \times 1 = 1200 \text{ W/m}^2$$
(12)

Total heating power:  $P = q'' \times A = 1200 \times 1.131 = 1.36 \text{ kW}$ 

For transit time t = 3000/7000 = 0.43 s through the tube:

Bulk temperature rise (lumped capacity):

$$\Delta T_{\text{bulk}} = \frac{P \cdot t}{mc_p} = \frac{1.36 \cdot 10^3 \times 0.43}{100 \times 900} = 0.0065^{\circ} \text{C}$$
 (13)

Surface temperature rise (semi-infinite solid):

$$\Delta T_{\text{surface}} = \frac{2q''\sqrt{\alpha t}}{k\sqrt{\pi}} \tag{14}$$

For aluminum ( $\alpha = 97 \times 10^{-6} \text{ m}^2/\text{s}, k = 237 \text{ W/m·K}$ ):

$$\Delta T_{\text{surface}} = \frac{2 \times 1200 \times \sqrt{97 \times 10^{-6} \times 0.43}}{237 \times \sqrt{\pi}} = 0.037^{\circ} \text{C}$$
 (15)

Both temperature rises are well within aluminum's operating range (melting point 660 °C).

#### A.3 Membrane Deflection Analysis

Asymmetric membrane tearing can impart lateral impulse. For a membrane with areal density  $\sigma$  and asymmetry fraction  $\varepsilon$ :

$$\theta \approx \varepsilon \frac{m_{\text{membrane}}}{m_{\text{capsule}}} = \varepsilon \frac{\sigma A}{m_{\text{capsule}}}$$
 (16)

For 25  $\mu$ m PET film ( $\sigma = 0.035 \text{ kg/m}^2$ ) and  $A = 1.131 \text{ m}^2$ :

$$m_{\text{membrane}} = 0.035 \times 1.131 = 0.040 \text{ kg}$$
 (17)

With  $\varepsilon = 0.01$ :

100 kg capsule: 
$$\theta = 0.01 \times \frac{0.040}{100} = 4 \times 10^{-6} \text{ rad}$$
 (18)

Miss distance at 100 km: 
$$\Delta y = \theta \times 10^5 = 0.4 \text{ m}$$
 (19)

This deflection improves with larger capsule mass.

#### A.4 Vacuum System Engineering Analysis

Creating and maintaining vacuum in a 3-kilometer tube presents extraordinary engineering challenges. The tube's volume is

$$V_{\text{total}} = \pi r^2 L = \pi (0.675)^2 \times 3000 \approx 4.3 \times 10^6 \text{ L}$$
 (20)

At ultra-low pressures, molecules travel in molecular flow. The conductance of a tube in this regime can be derived from kinetic theory.

The mean molecular speed is

$$\bar{v} = \sqrt{\frac{8RT}{\pi M}} \tag{21}$$

The molecular flux through an aperture of area A is

$$\Phi = \frac{1}{4}n\bar{v}A\tag{22}$$

For a long cylindrical tube, only a fraction of molecules transmit. The transmission (Clausing) probability is approximately

$$\tau \approx \frac{4}{3} \frac{D}{L}, \qquad (L \gg D)$$
 (23)

The conductance of the tube then follows as aperture conductance times transmission probability:

$$C = \frac{1}{4}\bar{v}A\tau = \frac{\pi}{12}\frac{D^3}{L}\bar{v}$$
 (24)

Substituting constants and converting to liters per second gives the handbook form:

$$C = 12.1 \frac{D^3}{L} \sqrt{\frac{T}{M}} \quad [L/s] \tag{25}$$

with D, L in cm, T in K, and M in g/mol. For air at 300 K,  $\sqrt{T/M} \approx 3.22$ . For a 3 km tube of diameter 1.35 m, the conductance is

$$C_{3000\,\mathrm{m}} \approx 3.2 \times 10^2 \,\mathrm{L/s}$$
 (26)

Shorter segments improve conductance substantially:

$$C_{100\,\mathrm{m}} \approx 9.6 \times 10^3 \,\mathrm{L/s}$$
 (27)

$$C_{10 \,\mathrm{m}} \approx 9.6 \times 10^4 \,\mathrm{L/s}$$
 (28)

We therefore divide the tube into 300 segments of 10 m each. Each segment has

$$V_{\text{seg}} = \pi (0.675)^2 \times 10 \approx 1.43 \times 10^4 \text{ L}$$
 (29)

$$A_{\text{seg}} = \pi DL \approx 4.24 \times 10^5 \text{ cm}^2 \tag{30}$$

For baked 316L stainless steel  $(q \sim 10^{-11} \text{ mbar} \cdot \text{L/s} \cdot \text{cm}^2)$ , the outgassing load per segment is

$$Q_{\text{seg}} = qA_{\text{seg}} \approx 4.2 \times 10^{-6} \text{ mbar} \cdot \text{L/s}$$
 (31)

To hold  $p = 6.1 \cdot 10^{-6}$  mbar requires only

$$S_{\text{req}} = \frac{Q}{p} \approx 0.69 \text{ L/s} \tag{32}$$

far below typical pump capacities. In practice, NEG coatings and cryopanels are used to handle water and hydrogen and to maintain UHV margins.

Since conductance is high for short cells, the effective pumping speed is pump-limited:

$$S_{\text{eff}} \approx S_{\text{pump}}$$
 (33)

For a 730 L/s turbomolecular pump, the characteristic time constant is

$$\tau = \frac{V}{S_{\text{eff}}} \approx \frac{V}{S_{\text{pump}}} \tag{34}$$

and the pressure evolution is

$$p(t) = p_0 \exp\left(-\frac{S_{\text{pump}}}{V}t\right) \iff t = \frac{V}{S_{\text{pump}}} \ln\left(\frac{p_0}{p}\right).$$
 (35)

Using  $p_0=1$  mbar (after roughing) and  $p=6\times 10^{-6}$  mbar,  $\ln(p_0/p)=12.024$ , the pump-down characteristics are:

Segment length	Volume [L]	$S_{\text{eff}} [\text{L/s}]$	$\tau = V/S$ [s]	$t_{1 \text{ mbar} \rightarrow 6 \times 10^{-6} \text{ mbar}}$
3000 m (single tube)	$4.3 \times 10^{6}$	$2.2 \times 10^{2}$	$1.95 \times 10^4 \text{ s (5.43 h)}$	$\sim 65~\mathrm{h}$
100 m segment	$1.43 \times 10^{5}$	$7.3 \times 10^{2}$	$1.96 \times 10^2 \text{ s } (3.3 \text{ min})$	$\sim 39 \mathrm{\ min}$
10 m segment	$1.43 \times 10^{4}$	$7.3 \times 10^{2}$	$1.96 \times 10^{1} \text{ s (20 s)}$	$\sim 3.9 \mathrm{\ min}$

Table 2: Pump-down characteristics for different segmentation choices (air, 300 K, 730 L/s TMP per segment).

System operation is limited by valve reliability. With 29 valves at 99.9% reliability each, the overall launch reliability is

$$R_{\text{system}} = (0.999)^{29} \approx 0.971$$
 (36)

meaning roughly 1 in 35 launches would fail. With parallel redundancy (two independent actuators per valve), the effective per-station reliability is

$$R_{\text{station}} = 1 - (1 - 0.999)^2 = 0.999999$$
 (37)

which improves system reliability to

$$R_{\text{system}} = (R_{\text{station}})^{29} \approx 0.99997 \tag{38}$$

Thus, the 100 m segmented system is pump-speed limited rather than conductance-limited, can reach  $10^{-6}$  mbar within 39 minutes per segment after roughing, and is dominated by the engineering challenge of valve redundancy and sequencing.

#### References

- [1] Kirk, D. B., and Miller, R. J. (1962). Free-Flight Tests of Fifth-Stage Scout Entry Vehicle at Mach Numbers of 5 and 17. NASA Technical Note D-1425. Available at: https://ntrs.nasa.gov/citations/19620006855
- [2] Edwards Vacuum. nEXT730 Turbomolecular Pump Datasheet. Available at: https://ptbsales.com/edwards-next-730d-new.html
- [3] Edwards Vacuum. iH1000 Dry Pump Datasheet. Available at: https://ptbsales.com/edwards-ih1000-new.html