Rank-based Approach on Graphs with Structured Neighborhood

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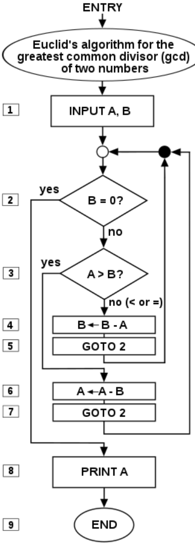
An algorithm



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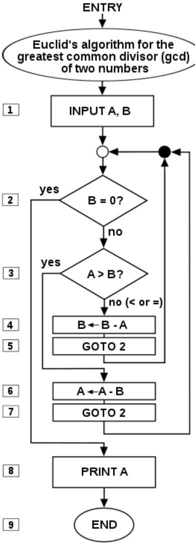

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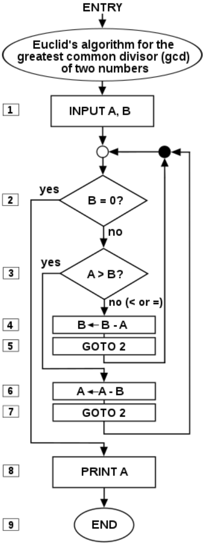

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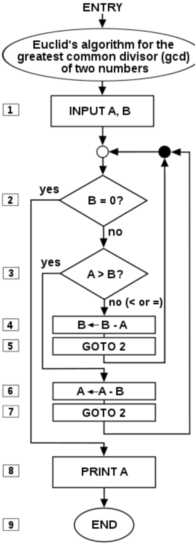

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- solves a problem

Introduction to Complexity

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- We measure running times in function of input size in the worst case
- In theory, constant factor does not matter.
  $\Longrightarrow$ We use big O notation

Running time: example

Seeking an element in a sequence: $O(n)$

Example: looking for 8

11	6	78	1	0	54	7	25	42	42	33	9

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If the sequence is sorted: $O(\log(n))$

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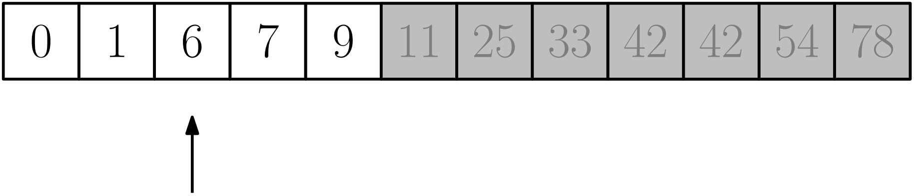

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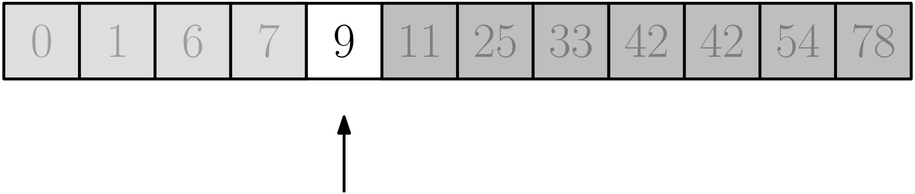

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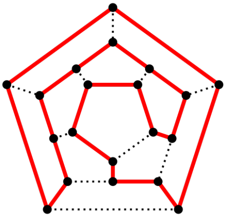
- Does a graph admit a Hamiltonian Cycle ?
  $\Longrightarrow O(2^n \cdot n^2)$ exponential in the input size.



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  - Finding the divisors of an integer harder than computing the GCD.

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Theorem (Karp 1972)

Hamiltonian Cycle *is NP-hard !*

A dead end ?

Theorem (In particular: Garey and Johnson 1979)

Thousands of problems are NP-hard !

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Conjecture

$P \neq NP$: No polynomial algorithms for NP-hard problems.

P vs NP problem: one of the biggest open questions in discrete math.

NP-hard problems everywhere !

We cannot ignore them !

$\Longrightarrow$ They have tons of applications in the real-world in many fields:

- Computer Science,
- Industries, enterprises
  $\Longrightarrow$ Optimization, logistic, planning,...
- Biology
  $\Longrightarrow$ DNA sequencing,...
- Chemistry
- Social choice

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Question

Can the theory explain these good performances ?
Can we characterize these hidden structures ?

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- Good in practice !

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  $\Longrightarrow$ Unless ETH fails, there is no $2^{o(k)} \cdot n^{O(1)}$ algorithm...

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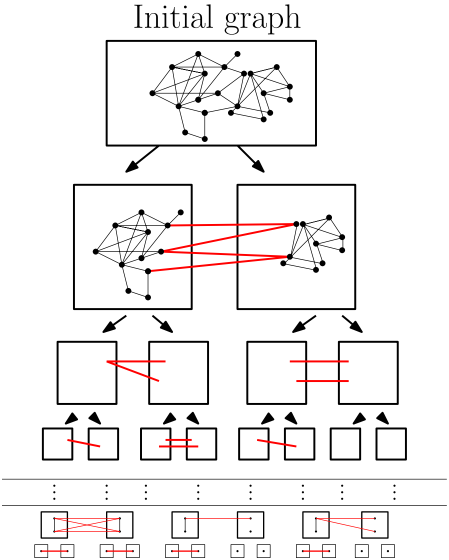
There exists some special kind of parameters: width measures !

Width measures: intuitions



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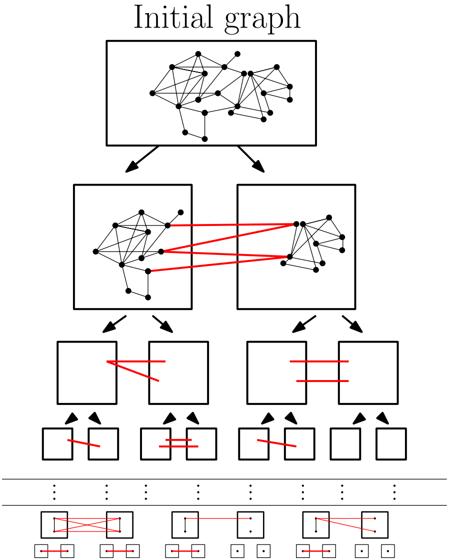

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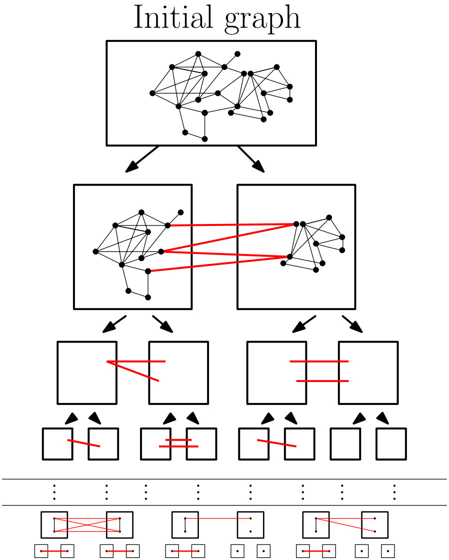

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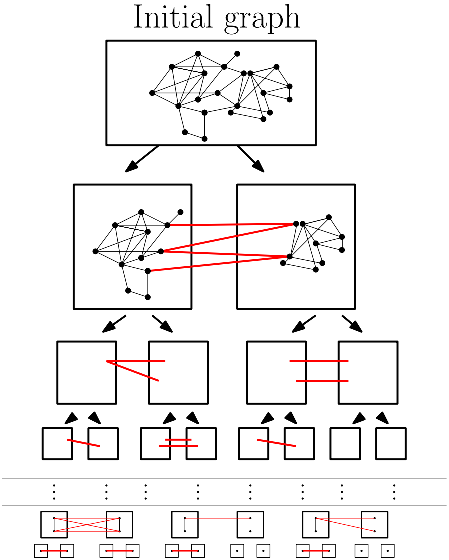

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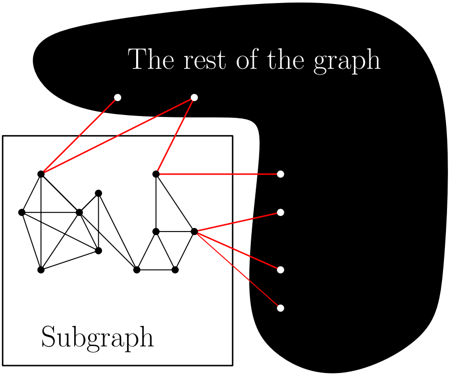

Example: Hamiltonian Cycle

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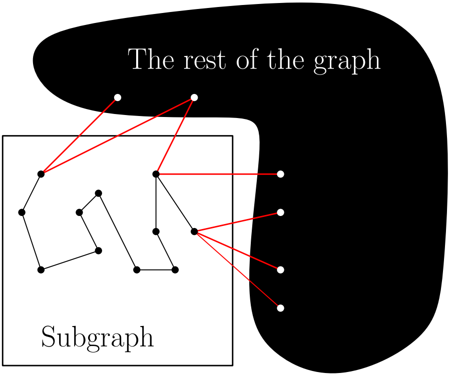

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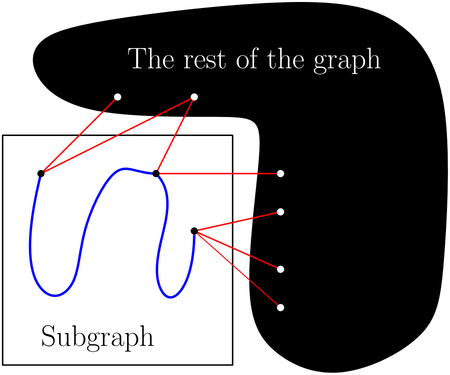
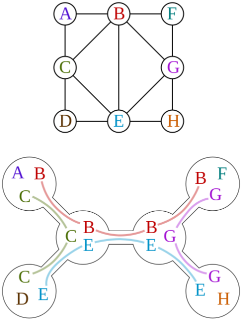


Example: Hamiltonian Cycle

- How to use the simple boundary:
  1. Observe how partial solutions interact with it.
  2. Bound the amount of information we need to store.

Tree-width

Certainly the most studied and famous graph parameter !



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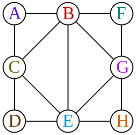

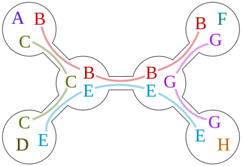

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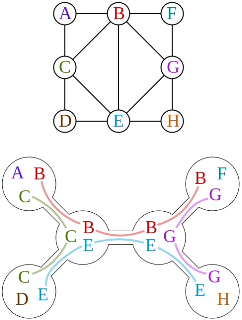

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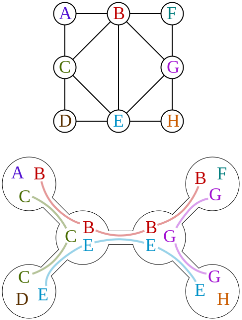

- Measure the tree-likeness.
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- Computable efficiently: $2^{O(\text{tw})} \cdot n$ constant factor approximation.

Tree-width against NP-hard problems

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- Being naive is not that bad for these problems !

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- Naive lower bound: $2^{o(\mathsf{tw})} \cdot n^{O(1)}$ unless ETH fails.
- Being naive is not enough.
  $\Longrightarrow$ Can we have $2^{O(\mathsf{tw})} \cdot n^{O(1)}$ algorithm or $\mathsf{tw}^{O(\mathsf{tw})} \cdot n^{O(1)}$ is optimal ?

Tree-width against NP-hard problems

Surprisingly:

Theorem [Cygan et al. FOCS 2011]

Monte Carlo $2^{O(\mathsf{tw}(G))} \cdot n^{O(1)}$ time algorithms for many connectivity problems.

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Theorem [Bodlaender et al. 2015]

Deterministic $2^{O(\mathsf{tw}(G))} \cdot n$ time algorithms for a wider range of connectivity problems.

Other width measures

Main drawback of tree-width : can be bounded only on sparse graph
$\Longrightarrow$ cliques have tree-width $n-1$.

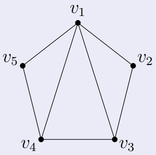
- Dense graphs can be simple too: cliques, distance hereditary graphs,...
- Many NP-hard problems are tractable on these dense graph classes.
- This can be explained through other width measures !
  $\Longrightarrow$ clique-width, rank-width, maximum induced matching width,...
- Most of these width measures are defined through the notion of rooted layout.

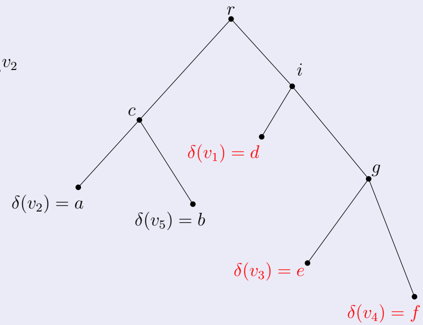
Rooted Layout

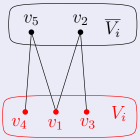
A rooted layout of a graph G is a pair (T, δ) with

- T a rooted tree,
- δ a bijection between the leaves of T and the vertices of G.

Each node x of T is associated with a vertex set V_x of G:





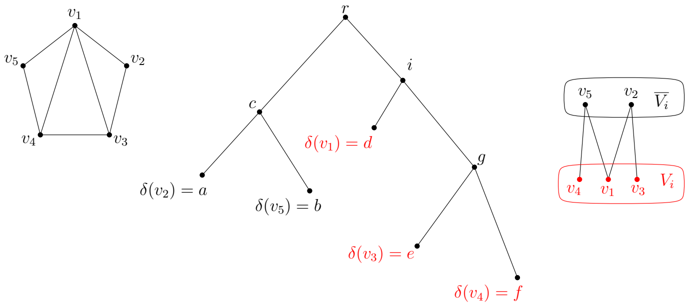


f-width

Given a set function $\mathsf{f} : 2^{V(G)} \to \mathbb{N}$, we define the f-width of

- a rooted layout (T, δ) as $\max_{x \in V(T)} \mathsf{f}(V_x)$,
- $\mathsf{f}(G) := \min \mathsf{f}(\mathcal{L})$ over the rooted layout $\mathcal{L}$ of G.

- f is intend to measure the simplicity of a boundary



Rank-width

The rank-width of G is the rw-width of G where rw(A) is the rank of the adjacency matrix between A and $\overline{A}$.

Mim-width

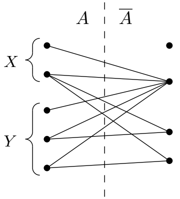
The mim-width of G is the mim-width of G where mim(A) is the size of a maximum induced matching of the bipartite graph between A and $\overline{A}$.

Let G be a graph and $A \subseteq V(G)$.

d-neighbor equivalence

For $d \in \mathbb{N}$, $X, Y \subseteq A$ are d-neighbor equivalent over A if for all $v \in \overline{A}$:

- $\min(d, |N(v) \cap X|) = \min(d, |N(v) \cap Y|)$.
- (If $d = 1$, then $N(X) \cap \overline{A} = N(Y) \cap \overline{A}$.



Number of neighbor in

X	Y
0	0
2	3
2	2
2	2

X and Y are 2-neighbor equivalent but not 3-neighbor equivalent.

d-neighbor width

For every $A \subseteq V(G)$, s-nec$_d(A)$ is the maximum between the number of equivalence classes of

- the d-neighbor equivalence relation over A,
- the d-neighbor equivalence relation over $\overline{A}$.

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Given a rooted layout $\mathcal{L}$, we have the following upper bounds on $\text{s-nec}(\mathcal{L})$

Clique-width	Rank-width	Mim-width
$(d+1)^{\mathsf{cw}(\mathcal{L})}$	$2^{d \cdot \mathsf{rw}(\mathcal{L})^2}$	$n^{d \cdot \mathsf{mim}(\mathcal{L})}$

A useful tool for problem with locally checkable property

Thanks to d-neighbor width, we obtained:

- the best (up to a constant in the exponent) algorithm

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We can design $\text{s-nec}_d(\mathcal{L}) \cdot n^c$ time algorithm for some constants d and c.

What we do

Theorem [Bodlaender et al. 2015]

Deterministic $2^{O(\mathsf{tw}(G))} \cdot n$ time algorithms for a wider range of connectivity problems.

Bodlaender et al. introduced a technique call rank-based approach.

B. and Kanté 2018

We can use the rank-based approach with d-neighbor-width to design:

- $\mathsf{s\text{-}nec}_d(\mathcal{L}) \cdot n^c$ time algorithm
- for the connected variant of problem with locally checkable property $\Longrightarrow$ Connected Dominating Set, Connected Vertex Cover,...

Consequences

Corollary

We obtained algorithms with the following running times

Clique-width	Rank-width	Mim-width
$(d+1)^{\mathsf{cw}(\mathcal{L})} \cdot n$	$2^{d \cdot \mathsf{rw}(\mathcal{L})^2} \cdot n^{O(1)}$	$n^{d \cdot \mathsf{mim}(\mathcal{L})}$

These running times match (up to a constant in the exponent) the best known running times for Vertex Cover and Dominating Set.

Extension to Acyclicity

B. and Kanté

We obtained algorithms with the following running times

Clique-width	Rank-width	Mim-width
$(d+1)^{\mathsf{cw}(\mathcal{L})} \cdot n$	$2^{d \cdot \mathsf{rw}(\mathcal{L})^2} \cdot n^{O(1)}$	$n^{d \cdot \mathsf{mim}(\mathcal{L})}$

for the acyclic variant of problems with locally checkable property
$\Longrightarrow$ Maximum Induced Tree, Feedback Vertex Set, Longest Induced Path,...

- We did not obtained $\mathsf{s\text{-}nec}_d(\mathcal{L}) \cdot n^c$ time algorithm...
- But we heavily rely on the d-neighbor equivalence relation !

Conclusion

- d-neighbor equivalence is incredibly useful.
- Not only for local problems.
- Works also for tree-width.
  $\Longrightarrow$ Maximum matching width !