# Exercise Session 04

**Exercise 1.**
Consider the following recurrence $T(n) = T(2n/3) + \Theta(1)$. Prove that $T(n) = O(\lg n)$.

**Exercise 2.**
Consider the following recurrence:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n = 1 \\ T(n-1) + \Theta(n) & \text{if } n > 1 \end{cases}$$

Prove that $T(n) = O(n^2)$ using the substitution method.

**Exercise 3.**
Consider the following recurrence:

$$T(n) = \begin{cases} \Theta(1) & \text{if } n \leq 1 \\ T(\lfloor n/2 \rfloor) + T(\lceil n/2 \rceil - 1) + \Theta(n) & \text{if } n > 1 \end{cases}$$

Use the substitution method to prove that $T(n) = O(n \lg n)$.
*Hint: be careful when you choose the base case because $n = 0$ and $n = 1$ may not work*

**Exercise 4.**
The factorial of $n$, is usually recursively defined as

$$n! = \begin{cases} 1 & \text{if } n = 0, \\ n \cdot (n-1)! & \text{if } n > 0 \end{cases}$$

(a) Prove that $n! = \Omega(2^n)$.

(b) Prove that $n! = O(n^n)$.

(c) Prove that $\lg n! = O(n \lg n)$.

★ **Exercise 5.**
Consider the recurrence $T(n) = T(9n/10) + T(n/10) + cn$ where $c$ is a constant such that $c > 0$. Prove that $T(n) = O(n \lg n)$.