Exercise Session 07

Exercise 1.
(CLRS 11.1-1) Suppose that a dynamic set S is represented by a direct-address table T of length m. Describe a procedure that finds the maximum element of S . What is the worst-case performance of your procedure?

Exercise 2.
(CLRS 11.2-2) Demonstrate what happens when we insert the keys $5; 28; 19; 15; 20; 33; 12; 17; 10$ into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \mod 9$.

Exercise 3.
(CLRS 11.2-5) Suppose that we are storing a set of n keys into a hash table of size m. Show that if the keys are drawn from a universe U with $|U| > nm$, then U has a subset of size n consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is $\Theta(n)$.

Hint: The Dirichlet's box principle —a.k.a. pigeon hole principle— states that for $n, m \in \mathbb{N}$, if $nm+1$ objects are distributed among m sets, then at least one of the sets will contain at least $n+1$ objects.

Exercise 4.
(CLRS 11.4-1) Consider inserting the keys $10; 22; 31; 4; 15; 28; 17; 88; 59$ into a hash table of length $m = 11$ using *open addressing* with the auxiliary function $h'(k) = k$. Illustrate the result of inserting these keys using linear probing, using quadratic probing with $c_1 = 1$ and $c_2 = 3$, and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \mod (m-1))$.

Exercise 5.
(CLRS 11.4-3) Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is $3/4$ and when it is $7/8$.