Solutions to Re-Exam 2020

Exercise 1

1.1.1 (b)
$5(n^3)^2 = n^6 \sim \Theta(n^6)$
$\sqrt{n^7} = n^{3.5} \sim \Theta(n^{3.5})$

1.1.2. (d)
$n^2\sqrt{\lg n}$ is $\Theta(n^2\sqrt{\lg n})$, and thus $\Omega(n)$

2.1. (c) This can be solved by the master method, the second case.
2.2. (a) $a = 4$, $b = 4$, and $f(n) = n$, so $f(n) = \Theta(n^{\log 4^4})$.
2.3. (a) $T(n) = \Theta(n^{\log 4^4} \lg n) = \Theta(n \lg n)$.

3.1. (c)
(a) The worst case time complexity of merge sort is $\Theta(n \lg n)$.
(b) Merge sort is not an in place sorting algorithm..
(c) The average case time complexity of Merge sort is $\Theta(n \lg n)$

3.2. (d)
First call on Merge(,) A[1], A[2] are merged and sorted. Second call on Merge(,) A[3], A[4] are merged and sorted. Third call, A[1..2] and A[3..4] are merged and sorted.

4. (c)

5. (d)

6.1 (c)
6.2 (a)

Exercise 2

2.1 (*10 points*) ENQUEUE(*List* L, *int* a): when inserting a new element with key being a into a queue, we need to insert the element to the end of the linked list L.

The pseudo code assumes that the list is not empty and has already N elements in the list.

```
ENQUEUE(List $L$, int a)
pointer $x = L.head$
while $x.next \neq NILL$ do
    $x = x.next$
Create a new element $y$ such that $y.next = null$ and $y.key = a$.
$x.next = y$;
```

Asymptotic complexity: $\Theta(N)$

2.2 (*10 points*) Return the element pointed by the list's head pointer. And move head pointer to its next element.

```
DEQUEUE(List $L$)
integer m = L.head.key;
L.head = L.head.next;
return m;
```

Asymptotic complexity: $\Theta(1)$

Exercise 3

3.1 (*7 point*)

```
MinimumInteger(Array $A$, Integer $p$, Integer $r$)
if $p < r$
    q=$\lfloor \frac{(p+r)}{2} \rfloor$
    Integer minleft=MinimumInteger$(A, p, q)$
    Integer minright=MinimumInteger$(A, q+1, r)$
    return min(minleft, minright);
else return $A[p]$;
```

Call MinimumInteger$(A, 1, n)$ in the beginning.

3.2 (*6 point*) $T(n) = 2T(n/2) + \Theta(1)$; $T(1) = \Theta(1)$; two subproblems, each subproblem is half size of the original problem. merging is constant time.

3.3 (*4 point*) Master method, first case. $\Theta(n)$.

3.4 (*3 point*) Constant space overhead.