Algorithms and Data Structures (DAT3/SW3)

Re-Exam Assignments

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This exam consists of four exercises and there are **three** hours (13.00 p.m. to 16.00 p.m.) to solve them. When answering the questions in exercise 1, mark the check-boxes on this paper. Remember also to put your name and your student number, if you use any additional sheets of paper for exercises 2, 3 and 4.

During the exam you are allowed to consult books and written notes. However, the use of any kind of electronic devices with communication functionalities, e.g., laptops, tablets, and mobile phones, is **NOT** permitted. You can bring old fashion calculators.

- *Read* carefully *the text of each exercise before solving it! Pay particular attentions to the terms in* **bold**.
- **CLRS** *refers to the textbook—T.H. Cormen, Ch. E. Leiserson, R. L. Rivest, C. Stein,* Introduction to Algorithms *(3rd edition).*
- *For exercises 2, 3 and 4, it is important that your solutions are presented in a readable form. In particular, you should provide precise descriptions of your algorithms using pseudo-code or reference existing pseudo-code from the textbook CLRS. To get partial points for not completely correct answers, it is also worth to write two or three lines in English to describe informally what the algorithm is supposed to do.*
- *Make an effort to use a readable handwriting and to present your solutions neatly.*

Exercise 1 [60 points in total]

Note that each of the following questions only has **a single correct solution**.

1. Identifying asymptotic notation. (Note: lg means logarithm base 2).

1.1. (*3 points*) $3 \cdot \lg n + 3^n + 7n \cdot \lg n + 100 \cdot n^2$ is:

☐ **a)** $\Theta(\lg n)$ ☐ **b)** $\Theta(n \cdot \lg n)$ ☐ **c)** $\Theta(3^n)$ ☐ **d)** $\Theta(n^2)$

1.2. (*3 points*) $0.6 \cdot \lg n^2 + 35 \cdot \lg \sqrt{n} + \sqrt{0.3 \cdot n^6} + 19990$ is:

☐ **a)** $\Theta(n \lg n)$ ☐ **b)** $\Theta(n^{\frac{1}{2}})$ ☐ **c)** $\Theta(lgn)$ ☐ **d)** $\Theta(n^3)$

2. (*7 points*) Please choose the function which makes the following statement be true.

$$\lg(f(n)) = \Theta(n)$$

☐ **a)** $f(n) = n^3$
☐ **b)** $f(n) = 3^{2n} \cdot n^4$
☐ **c)** $f(n) = e^{n^2}$
☐ **d)** $f(n) = n + 1$

3 (*7 points*) Consider the following recurrence:

$$\begin{aligned} T(1) &= \Theta(1) \\ T(n) &= 8 \cdot T(n/2) + n^2 \qquad (n > 1), \end{aligned}$$

Which of the following is correct? T(n) is:

☐ **a)** $\Theta(n)$ ☐ **b)** $\Theta(n^3)$ ☐ **c)** $\Theta(n^2)$ ☐ **d)** $\Theta(n \cdot lgn)$

4. (*8 points*) Given an insertion sort algorithm INSERTION-SORT(A).

```
INSERTION-SORT(A)
for j = 2 to A.length
    key = A[j]
    i = j − 1
    while i > 0 and A[i] > key
        A[i + 1] = A[i]
        i = i − 1
    A[i + 1] = key
```

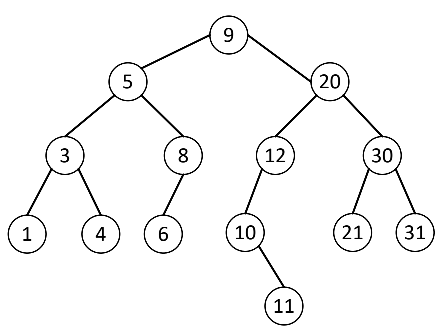
Given an array A, specified in the following table, and its index, specified on top of the table,

1	2	3	4	5	6	7	8	9	10
91	71	29	43	97	59	17	93	61	13

which of the following result comes **after** applying $j = 4$ in algorithm INSERTION-SORT(A)?

☐ **a)** $[29, 43, 71, 91, 97, 59, 17, 93, 61, 13]$
☐ **b)** $[13, 17, 29, 43, 59, 61, 71, 91, 97, 93]$
☐ **c)** $[71, 29, 43, 59, 17, 61, 13, 91, 97, 93]$
☐ **d)** $[29, 43, 91, 71, 97, 59, 17, 93, 61, 13]$

5. (*8 points*) Given the following tree, which statement is true.



☐ **a)** The in-order traversal is: 9, 5, 3, 1, 4, 8, 6, 20, 12, 10, 11, 30, 21, 31.
☐ **b)** The post-order traversal is: 1, 3, 4, 5, 6, 8, 9, 10, 11, 12, 20, 21, 30, 31.
☐ **c)** The pre-order traversal is: 1, 4, 3, 6, 8, 5, 11, 10, 12, 21, 31, 30, 20, 9.
☐ **d)** The pre-order traversal is: 9, 5, 3, 1, 4, 8, 6, 20, 12, 10, 11, 30, 21, 31.

6. (*8 points*) Suppose you have the following hash table, implemented using linear probing to resolve conflicts. We assume that when a conflict happens, you check cells to the right but not to the left. The hash function you use is $h(x) = x\%9$.

0	1	2	3	4	5	6	7	8
9	18		12	3	14	4	21	

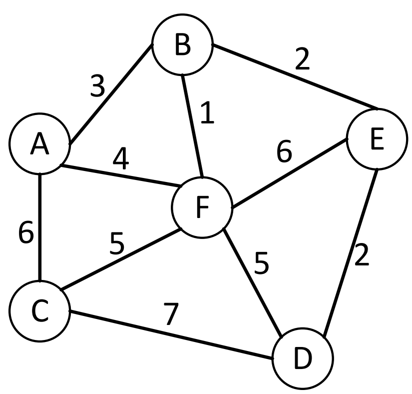
Assume that the hash table has never been resized, and no elements have been deleted yet. In which order could the elements be added to the hash table?

- ☐ **a)** 9, 14, 4, 18, 12, 3, 21
- ☐ **b)** 12, 3, 14, 18, 4, 9, 21
- ☐ **c)** 12, 14, 3, 9, 4, 18, 21
- ☐ **d)** 12, 9, 18, 3, 14, 21, 4

7. (*8 points*) Let's consider a scenario that Jakob would like to build a **min-heap** on the following numbers (21, 11, 6, 8, 39, 2, 14, 4). After building the min-heap, which of the following should Jakob get?

- ☐ **a)** $2, 4, 6, 8, 39, 21, 14, 11$
- ☐ **b)** $2, 4, 8, 11, 39, 6, 21, 14$
- ☐ **c)** $2, 4, 21, 11, 39, 6, 14, 8$
- ☐ **d)** None of the above is correct.

8 (*8 points*) Consider the following undirected, weighted graph. Assume that we use the Prim's algorithm from vertex A to identify the minimum spanning tree of the graph.



8.1 (*4 points*) What is the correct order of the vertices that are extracted from the min-priority queue used by the Prim's algorithm.

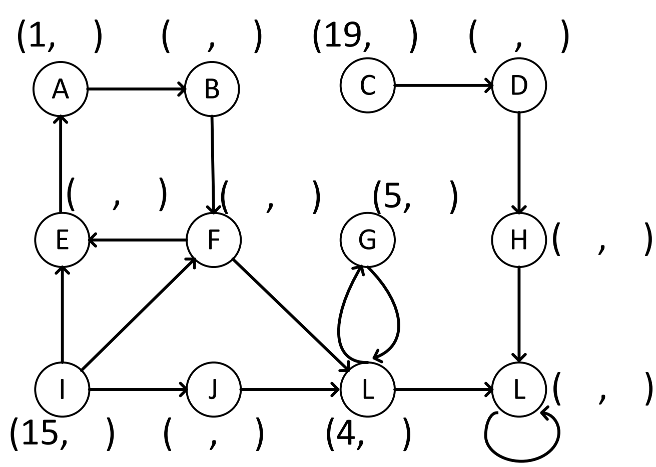
☐ **a)** A, B, F, C, D, E ☐ **b)** A, C, F, D, B, E
☐ **c)** D, A, F, C, B, E ☐ **d)** A, B, F, E, D, C

8.2 (*4 points*) What are the corresponding parent nodes for the vertices that you have just chosen.

☐ **a)** null, A, B, B, E, F ☐ **b)** null, A, D, F, C, B
☐ **c)** null, A, F, C, B, E ☐ **d)** null, A, D, F, C, B

Exercise 2 [8 points]

Given the directed graph below, fill in one possible depth-first-search (DFS) traversal starting from node A, in the form of: (discovered timestamp, finishing timestamp) near each node.
When there are multiple choices to visit a node, a possible discovered timestamp has already been filled in. It means that you should follow this order to conduct the DFS traversal.



Exercise 3 [20 points]

A Kommune has built 5 new villages A, B, C, D, E and 8 roads to connect them as follows:

$(A, B, 90)$, $(A, C, 20)$, $(A, D, 60)$, $(C, D, 30)$,
$(C, E, 10)$, $(D, B, 10)$, $(E, B, 30)$, $(E, D, 10)$.

For example, the triple $(A, B, 90)$ specifies that traveling from village A to village B takes 90 minutes. (*Hint: since there is no (B, A, x), where x is a possible numeric value, it means that people cannot travel from B to A.*)

1. (*2 points*) Draw a **directed** graph to model the 5 villages and 8 roads. You need to include travel time between two villages in the graph.

2. (*2 points*) Write down the adjacency matrix for the graph.

3. (*4 points*) We would like to compute a path with the minimum travel time from village A to village B. (*Hint: Slightly different from the single source problem, i.e., one source node to all reachable destination nodes. Here, we*

only consider one source node to one destination node.) Show how to model this problem as a graph problem. Specifically, write down:

(a) (*2 points*) Input.

(b) (*2 points*) Output.

4. (*12 points*) Using Dijkstras' algorithm to compute the minimum travel time from A to B, and write down:

   (a) (*4 points*) A path that has the minimum travel time from A to B, and the minimum travel time.

   (b) (*8 points*) Steps on how to obtain the path and the value. Please show the contents of the priority queue in each iteration, including nodes left in the priority queue, the distance from A to each node, and each node's parent. Filling cells in the tables according to your need - some empty cells/tables are fine.

Iteration 1:

Nodes	A	B	C	D	E
Distance/Parent	$0/-$				

Iteration 2:

Nodes	C				
Distance/Parent	$20/A$				

Iteration 3:

Nodes					
Distance/Parent					

Iteration 4:

Nodes					
Distance/Parent					

Iteration 5:

Nodes					
Distance/Parent					

Exercise 4 [12 points]

Consider the sequence $F_0, F_1, F_2, \ldots$ defined recursively as follows.

$$
\begin{aligned}
F_0 &= 0 \\
F_n &= F_{n-1} + n \qquad (n > 1).
\end{aligned}
$$

1. (*4 points*) Compute and tabulate F_n for n values from 0 to 10.
2. (*8 points*) Design a dynamic programming algorithm for computing F_n. Please be aware that you should consider n but not just n up to 10.
   (a) (*6 points*) Write down the pseudo-code of your algorithm.
   (b) (*2 points*) Write down the time complexity of your algorithm.