Exercise Session 02

Solve the following exercises. The exercises that are more involved are marked with a star. If you need some guidance for solving the exercise, place your trash bin in front of your group room's door and the first available teaching assistant will come to help you out.

Exercise 1.

Solve exercises CLRS 3.1–1, and 3.1–4

Remark: A function $f(n)$ is asymptotically nonnegative if there exists n_0 such that $f(n) \geq 0$ for all $n \geq n_0$.

Exercise 2.

Consider the algorithm SumUpTo that takes as input a natural number $n \in \mathbb{N}$.

```
SumUpTo(n)
1  s = 0
2  for i = 1 to n
3        s = s + i
4  return s
```

Prove that given $n \in \mathbb{N}$, SumUpTo terminates and returns $\frac{n(n+1)}{2}$.

Exercise 3.

By getting rid of the asymptotically insignificant parts on the expressions, give a simplified asymptotic tight bounds (big-theta notation) for the following functions in n. Here, $k \geq 1$, $e > 0$ and $c > 1$ are constants.

(a) $0.001n^2 + 70000n$

(b) $2^n + n^{10000}$

(c) $n^k + c^n$

(d) $\log^k n + n^e$

(e) $2^n + 2^{n/2}$

(f) $n^{\log c} + c^{\log n}$ (hint: look at some properties of the logarithm at page 56 in CLRS)

★ **Exercise 4.**

Consider the following algorithm that takes an array $A[1 \ldots n]$ and rearrange its elements in nondecreasing order.

```
Sort(A)
1  for i = 1 to A.length
2        for j = i + 1 to A.length
3              if A[i] > A[j]
4                    key = A[i]
5                    A[i] = A[j]
6                    A[j] = key
```

(a) Try Sort(A) on the the instance $A = [4, 2, 8, 7, 1]$. Explain in your words how the algorithms works in general;

(b) Prove that Sort solves the sorting problem (hint: determine suitable invariants for both loops);

(c) Determine the asymptotic worst-case running time using the Θ notation.