Solutions to Re-Exam 2019

Exercise 1

1.1.1 (c)
$\lg n + \mathbf{3^n} + n \cdot \lg n + n^2$

1.1.2. (d)
$\lg n + \lg n + \mathbf{n^3} + 19990$

2 (b)
a: $\lg(f(n)) = \lg n^3 \approx \Theta(\lg n)$, wrong.
b: $\lg(f(n)) = \lg(3^{2n} \cdot n^4) = \lg(9^n \cdot n^4) = n \cdot \lg 9 + 4 \cdot \lg n \approx \Theta(n)$, correct.
c: $\lg(f(n)) = \lg e^{n^2} = n^2 \lg e \approx \Theta(n^2)$, wrong.
d: $\lg(f(n)) = \lg(n+1) \approx \Theta(\lg n)$, wrong

3 (b)
Master method, 1st case.
$T(n) = a \cdot T(n/b) = 8 \cdot T(n/2)$, $a = 8, b = 2$.
$f(n) = n^2 \approx O(n^{\log 2^8 - 1})$, $\epsilon = 1 > 0$.
$T(n) \approx \Theta(n^{\log 2^8}) = \Theta(n^3)$.

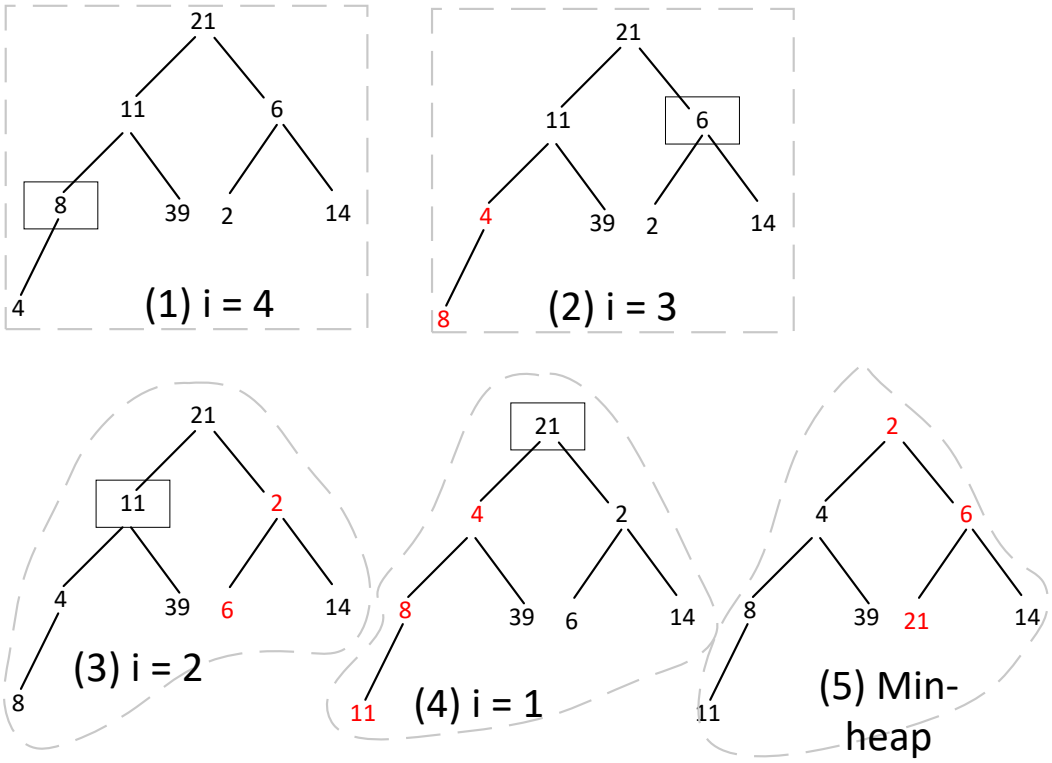
4 (a)

After $j = 2 : [71, 91, 29, 43, 97, 59, 17, 93, 61, 13]$
After $j = 3 : [29, 71, 91, 43, 97, 59, 17, 93, 61, 13]$
After $j = 4 : [29, 43, 71, 91, 97, 59, 17, 93, 61, 13]$

5 (d)
The pre-order traversal of the tree is: 9, 5, 3, 1, 4, 8, 6, 20, 1, 12, 10, 11, 30, 21, 31.
The in-order traversal of the tree is: 1, 3, 4, 5, 6, 8, 9, 10, 11, 12, 20, 21, 30, 31.
The post-order traversal of the tree is: 1, 4, 3, 6, 8, 5, 11, 10, 12, 21, 31, 30, 20, 9.

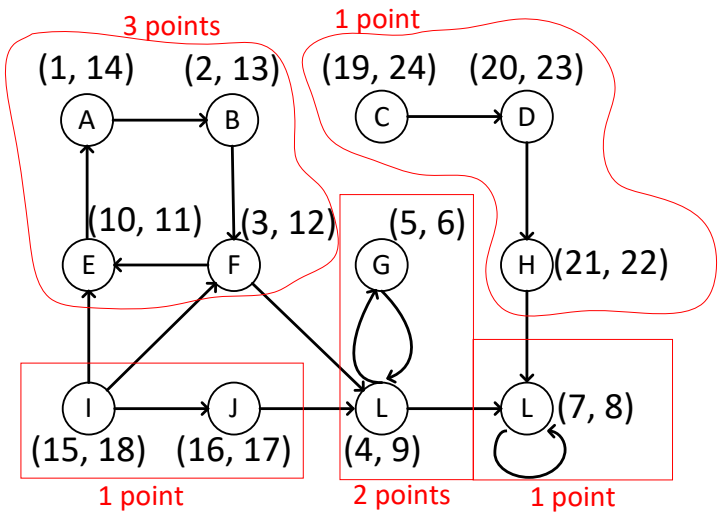
6 (c)
a) 4 should be at index 4, not at 6.
b) 18 should be at index 0, not at 1.
c) correct.
d) 21 should be at index 6, not at 7.

7 (a)



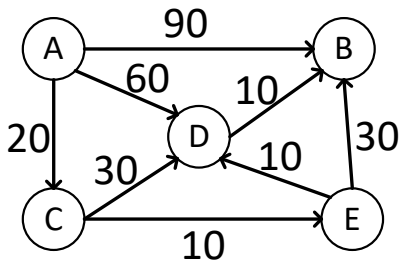
8.1 d)
8.2 a)
extracted vertex, weight, parent vertex;
A, 0, NULL;
B, 3, A;
F, 1, B;
E, 2, B;
D, 2, E;
C, 5, F;

Ex. 2



Exercise 3

3.1 (*2 points*)



3.2 (*2 points*)

The adjacency matrix is:

$$\begin{matrix} 0 & 90 & 20 & 60 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 30 & 10 \\ 0 & 10 & 0 & 0 & 0 \\ 0 & 30 & 0 & 10 & 0 \end{matrix}$$

3.3.a (*2 points*) The input is a directed weighted graph $G = (V, E)$, where

1. $V = \{A, B, C, D, E\}$ is a vertex set,
2. $E = \{(A, B, 90), (A, C, 20), (A, D, 60), (C, D, 30), (C, E, 10), (D, B, 20), (E, B, 30), (E, D, 10)\}$, where the numeric value of each triple indicates the weight on an edge;

3. source node A;

4. destination node B.

3.3.b (*2 points*) The output includes

1. a sequence of edges, whose nodes start from A and end at B, and

2. a numeric value that minimizes the weight of the edge sequence.

3.4.a (*4 points*)

(*2 points*) The path is $(A, C), (C, E), (E, D), (D, B)$
(*2 points*) The minimum travel cost is 50 minutes.

3.2.2 (*8 points*)

A	B	C	D	E
$0/-$	∞	∞	∞	∞
C	D	B	E	
$20/A$	$60/A$	$90/A$	∞	
E	D	B		
$30/C$	$50/C$	$90/A$		
D	B			
$40/E$	$60/E$			
B				
$50/D$				

Exercise 4

4.1 (*4 points*)

n	0	1	2	3	4	5	6	7	8	9	10
F_n	0	1	3	6	10	15	21	28	36	45	55

4.2 (*8 points*)
(*6 points*)

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BOTTOM-UP(n)
 1  F[0] ← 0
 2  for i = 1 to n
 3      F[i] ← F[i − 1] + i
 4  Return F[n]
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(*2 points*) Time complexity: $\Theta(n)$.