Exercise Session 03

Solve the following exercises. The exercises marked with a star are more challenging than the others.

Exercise 1.
Run the merge sort algorithm on the following array of numbers: $[3, 41, 52, 26, 38, 57, 49, 9]$. Give the state of the array after five calls of the algorithm MERGE are performed during the execution of MERGE-SORT.

Exercise 2.
Solve exercise CLRS 2.3–3, 2.3–4, and 2.3–6.

Exercise 3.
Consider the problem of finding the smallest element in a nonempty array of numbers $A[1\,..\,n]$.

(a) Write an *incremental* algorithm that solves the above problem and determine its asymptotic worst-case running time.

(b) Write a *divide-and-conquer* algorithm that solves the above problem and determine its asymptotic worst-case running time.

(c) Assume that the length of A is a power of 2. Write a recurrence describing how many comparison operations (among elements of A) your divide-and-conquer algorithm performs, and solve the recurrence using the recursion-tree method.

Remark: count ONLY the comparisons performed among elements in A. E.g., a comparison like $i \leq A.length$ shall not be counted, whereas $A[i] \leq k$ where k is a variable storing some element of A shall be counted. Moreover, if you use expressions like $\min(A[i], A[j])$ for some indices i, j, that also counts as 1 comparison.

Hint: A full binary tree with n leaves has $n - 1$ internal nodes (see CLRS B.5.3 pp.1177–1179).

★ **Exercise 4.**
Solve exercise CLRS 2.3–7.