Exercise 1

Perform a trace of DPLL until you find the first conflict. Start by using the splitting rule and assign the value T to the proposition S and then assign T to Q.

$$\Delta_b = \{ \{\neg P, Q\}, \{\neg Q, E\}, \{\neg E, S\}, \{\neg Q, F\}, \{\neg E, \neg F\}, \{P, F\}, \{\neg E, \neg S, G\}, \{\neg P, \neg Q, \neg F\} \}$$

Whenever you encounter a conflict, draw the corresponding implication graph as well as its conflict graph, and mention which clause can be learned with the clause learning method. Highlight vertices of choice and implied literals with different colors. What is the learned clause?

Note: You do not need to continue with the DPLL with clause learning procedure after learning the clause.

Exercise 2

Perform DPLL with clause learning as explained in the slides of Chapter 8 on the following set of clauses (give the full DPLL trace). Start by using the splitting rule and assign the value T to the proposition S and then assign T to Q. Whenever you encounter a conflict, draw the corresponding implication graph as well as its conflict graph, and mention which clause can be learned with the clause learning method. Highlight vertices of choice and implied literals with different colors. Then use this information and continue with the DPLL procedure, backtracking the last choice as specified in slide 33 of Chapter 8. Do this until the clause set is proven to be satisfiable or unsatisfiable.

$$\Delta_a = \{\{R, P, \neg Q\}, \{\neg P, \neg Q\}, \{\neg R, Q\}, \{\neg R, \neg Q\}, \{R, \neg P, Q\}, \{\neg S, Q, R\}, \{S, \neg P\}\}$$

Exercise 3

Consider the following statements. For each statement say whether it is correct or not and briefly explain why in a couple of sentences.

- 1. Every implication graph results in exactly one conflict graph.
- 2. If a branch of DPLL ends in the empty clause, we can always learn a new clause.
- 3. If a branch of DPLL ends in the empty clause without having applied the splitting rule, we cannot learn a new clause.
- 4. If a branch of DPLL ends in the empty clause, can we learn more than one clause such that none of them is a subset of the other? If yes, provide an example where this happens. If no, explain why not.

Exercise 4

Considering the procedure to generate random k-SAT formulas by with n variables and m clauses by Mitchell et al. (1992), indicate for each statement below whether it is correct or not and briefly explain why in one or two sentences.

- 1. Increasing the number of clauses makes the problem more constrained.
- 2. Problems more constrained are harder to solve.
- 3. Increasing the number of clauses makes the problem harder to solve.
- 4. Problems more constrained are easier to solve.
- 5. Increasing the number of clauses makes the problem easier to solve.