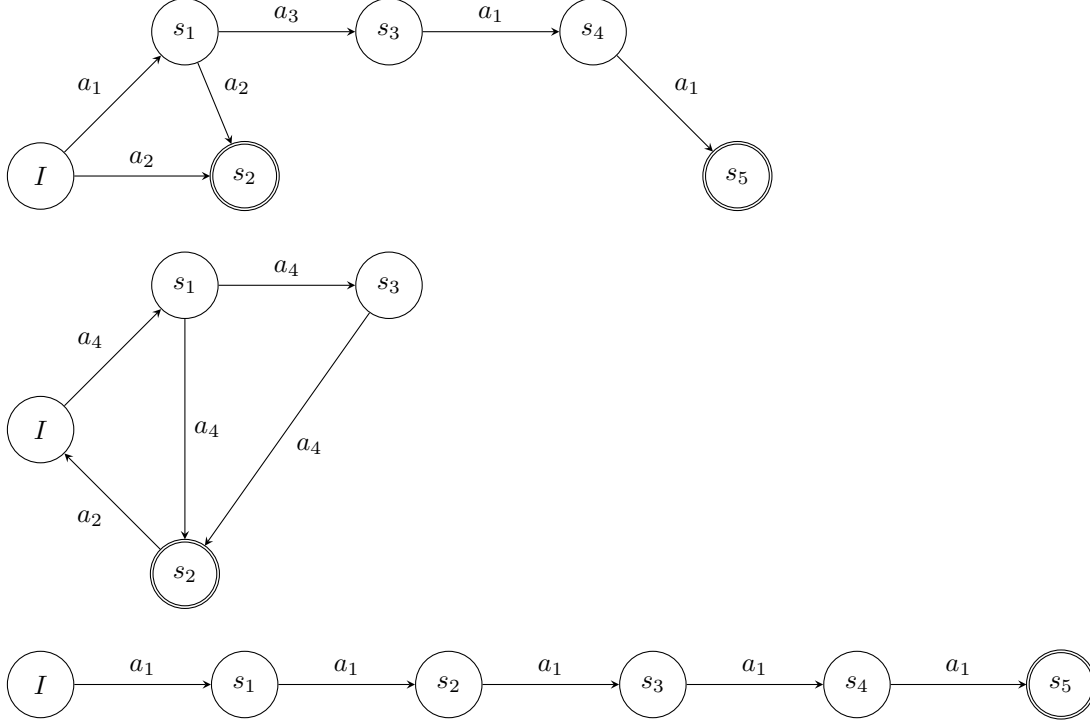## Exercise 1

We have a planning problem with four actions, a_1, a_2, a_3, a_4, and have computed several abstract transition systems as follows (self-loop transitions are ommited):

i) Which abstractions are orthogonal?

ii) What is the best heuristic value that we can obtain for the initial state using the notion of orthogonal abstractions?

iii) Consider the abstractions in the given order, compute a saturated cost partitioning. For each abstraction, mark what is the cost function, as well as the heuristic value for each abstract state. What value do we obtain in total for the initial state?

iv) Consider the abstractions in the reverse order (from bottom to top), compute a saturated cost partitioning. For each abstraction, mark what is the cost function, as well as the heuristic value for each abstract state. What value do we obtain in total for the initial state? Is it the same as in the other order?

v) Repeat the saturated cost partitioning with order from top to bottom, but now, instead of assuming that all actions cost 1, our original problem had the following costs: $c(a_1) = 10$, $c(a_2) = 2$, $c(a_3) = 1$, $c(a_4) = 5$.

vi) Is the obtained heuristic with the method of orthogonallity and with the method of saturated cost partitioning admissible? Justify your answer.

Exercise 2

Consider a Blocksworld task with three blocks A, B, C with initial state and goal as shown in Figure 1.

The encoding has three types of variables, $on(x)$ to describe which block is on top of block x (so on "points upwards"); $ontable(x)$, to say if a block is currently on the table, and $holding$, describing which block is currently held in the hand. The $\perp$ symbol represents the fact that *nothing* is on top of a block, respectively *nothing* is held in the hand.

There are four types of actions, $(un)stack(x, y)$, to $(un)stack$ block x onto/away from block y; $pickup(x)$ and $drop(x)$ to $pickup$ block x from the table into the hand, or $drop$ it to the table if it is held in
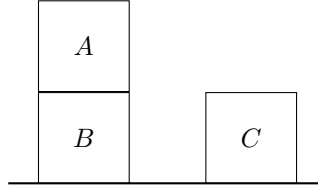
Figure 1: Initial state of the running example.

the hand. In the initial state depicted in Figure 1, the only two applicable actions are $unstack(A, B)$ and $pickup(C)$.

Formally, the FDR encoding of the task, $\Pi = (V, A, I, G)$, is defined as follows:

- Variables: $V = \{on(x), holding, ontable(x)\}$ for $x \in \{A, B, C\}$ with domains

  - $D(ontable(x)) = \{\top, \bot\}$,
  - $D(on(x)) = \{A, B, C, \bot\} \setminus \{x\}$, and
  - $D(holding) = \{A, B, C, \bot\}$.

- Actions: $A = \{stack(x, y), unstack(x, y), pickup(x), drop(x)\}$ (uniform action costs)

  - $stack(x, y)$, with $x \neq y \in \{A, B, C\}$
    pre: $\{holding = x, on(y) = \bot\}$,
    eff: $\{holding = \bot, on(y) = x\}$
  - $unstack(x, y)$, with $x \neq y \in \{A, B, C\}$
    pre: $\{holding = \bot, on(y) = x, on(x) = \bot\}$,
    eff: $\{holding = x, on(y) = \bot\}$
  - $pickup(x)$, with $x \in \{A, B, C\}$
    pre: $\{holding = \bot, ontable(x) = \top, on(x) = \bot\}$,
    eff: $\{holding = x, ontable(x) = \bot\}$
  - $drop(x)$, with $x \in \{A, B, C\}$
    pre: $\{holding = x\}$,
    eff: $\{holding = \bot, ontable(x) = \top\}$

- Initial state:

$$I = \{on(A) = \bot, on(B) = A, on(C) = \bot, holding = \bot,$$
$$ontable(A) = \bot, ontable(B) = \top, ontable(C) = \top\}$$

- Goal: $G = \{on(B) = A, on(C) = B\}$

(i) Indicate which of the following pairs of patterns are orthogonal. For those that are not orthogonal, explain why in a short sentence:

a) $\{\{holding, on(A)\}, \{ontable(C)\}\}$.
b) $\{\{on(B)\}, \{on(C)\}\}$.
c) $\{\{holding, on(A)\}, \{holding, ontable(C)\}\}$.
d) $\{\{ontable(A), on(B)\}, \{ontable(B), on(A)\}\}$.

(ii) For one of the pairs of patterns that are not orthogonal, is there a cost-partitioning that will make them additive? If yes, indicate what cost partitioning you would use to obtain a heuristic that is as good as possible, and explain why you choose it. Only consider 0-1 cost-partitionings where actions can only have a cost of 0 or 1, so you do not need to consider assigning fractional costs.

Exercise 3

- Goal: A and B both true.
- Initial state: A and B both false.
- Actions: cA effect A cost 1; cB effect B cost 1; fC effect A and B cost 1.5.
- Patterns $P_1 = \{A\}$ and $P_2 = \{B\}$.

Provide an LP encoding that computes the optimal cost-partitioning. Follow the following steps:

1. Draw the abstract state space of both PDBs.

2. What are the variables of the LP?

3. What is the objective function?

4. What are the constraints?

5. What would be a solution for the linear program?