Tutorial 1

Exercise 1 (compulsory)

Answer the following questions (and justify your answers):

1. Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
2. Can the tape alphabet Γ be equal to the input alphabet Σ?
3. Can the head of a Turing machine ever stay on the same cell for two subsequent steps of a computation?
4. Can the state set of a Turing machine consist of only a single state?

Exercise 2 (compulsory)

Some of the following definitions are correct and some wrong (and some are even pure nonsense). Mark the correct definitions and for the incorrect ones underline the part of the definition which is wrong and explain why.

1. A language L is recognizable if the language halts in the state q_{accept} whenever $x \in L$.
2. A language L is recognizable if there exists a Turing machine M such that M, given input x, halts in the state q_{accept} if and only if $x \in L$.
3. A language L is recognizable if it halts in an accepting state q_{accept} whenever $x \in L$.
4. A language L is recognizable if there exists a Turing machine M which has a state q_{accept} and is a member of L.
5. A language L is recognizable if there exists a Turing machine M such that for any given input x the machine M run on x halts in the state q_{accept} if $x \in L$, and it either loops or halts in q_{reject} if $x \notin L$.
6. A language L is recognizable if every Turing machine M when run on a string x halts in the state q_{accept} if and only if $x \in L$.

Exercise 3 (compulsory)

Complete the definition below and be formally precise.

A language $L \subseteq \Sigma^*$ is decidable iff ...

Exercise 4 (compulsory)

Answer the following questions and give precise arguments.

1. Suppose that a Turing machine has its head at a symbol s and is in a state p which is different from q_{accept} and q_{reject}. How many distinct states may the machine be in after a transition?
2. Is it always the case that if a language is decidable then it is also recognizable?

Exercise 5 (compulsory)

As a preparation for this exercise solve Exercise 3.2 part a) on page 187. Then you can proceed with the following tasks:

1. Draw a state diagram of a Turing machine M recognizing the language $\{a^n b^n a^n \mid n \geq 0\}$ over the alphabet $\Sigma = \{a, b\}$.
2. Consider the input string $w = aabbaa$. Write the whole sequence of configurations that M will enter when run on w.
3. Does M accept w?

Exercise 6 (optional, if you need more practice on TM basics)

If you don't feel comfortable with the computation of a TM and its design, try Exercise 3.2 on page 187 and Exercise 3.8 on page 188 (give the implementation-level descriptions but you can also draw the full state diagrams).

Exercise 7 (optional and mind-challenging)

Problem 3.9 on page 188 (in international edition), or Problem 3.22 on page 190 (in standard edition).