Tutorial 12

Exercise 1 (compulsory)

Prove that the class NP is closed under union, intersection, concatenation and Kleene star. Is the class NP closed also under complement?

Exercise 2 (compulsory)

Consider the language $A = \{a^n b^n \mid n \geq 0\}$, which is a language in P. We will now try to show that *VERTEX-COVER* $\leq_P A$. The reduction is

$$f(\langle G, k\rangle) = \begin{cases} aabb & \text{if } G \text{ has a vertex cover of size } k \\ aab & \text{otherwise} \end{cases}$$

Since *VERTEX-COVER* is NP-complete, *VERTEX-COVER* $\leq_P A$ and $A \in$ P, we get that P=NP.

Explain carefully what is the flaw is in this ”proof”.

Exercise 3 (compulsory)

A Boolean formula ϕ is a *tautology* if every truth assignment will cause ϕ to evaluate to true. Consider the problem

> ”Given a formula ϕ, is it the case that ϕ is *not* a tautology?”

1. Express this problem as a language called *NOTA*.
2. Show that *NOTA* is NP-complete.

Exercise 4 (compulsory)

Consider the following formula ϕ in cnf.

$$(x_1 \vee \overline{x_2} \vee x_3 \vee \overline{x_4}) \wedge (\overline{x_1} \vee x_4)$$

Using the reduction described in the proof of *CNF-SAT* $\leq_P$ *3SAT* construct a formula ϕ' in 3-cnf such that ϕ is satisfiable if and only if ϕ' is satisfiable.

Exercise 5 (compulsory)

Consider the following formula ϕ in 3-cnf.

$$(x \vee x \vee \overline{y}) \wedge (\overline{x} \vee y \vee y) \wedge (x \vee y \vee y)$$

Using the reduction described in the proof of *3SAT* $\leq_P$ *VERTEX-COVER* construct an undirected graph G and a number k such that G has k-vertex cover if and only if ϕ is satisfiable. List at least one k-vertex cover of the graph and find a corresponding satisfying truth assignment of the formula ϕ.

Exercise 6 (optional)

Consider the language *SUBSET-SUM*. In its variant discussed in the book, we are given a multiset S of numbers (that means that some of the numbers in S can repeat several times) and we try to select some of the numbers from S that add up to a given number t. We know that this problem is NP-complete. Show that a slight variant of *SUBSET-SUM* where S is given as a set of numbers (which means that numbers cannot repeat) is also NP-complete.