Tutorial 11

Exercise 1 (compulsory)

Is the following definition of NP-completeness correct? If no, give arguments why not.

Definition: A Turing machine M is NP-complete if $M \in$ NP and for every $L \in$ NP we have $L \leq_P M$.

Exercise 2 (compulsory)

Consider the following claims and answer which of them are correct and which of them not. Give precise arguments.

1. If L_1 and L_2 are NP-complete, then $L_1 \leq_P L_2$ and $L_2 \leq_P L_1$.
2. If L_1 and L_2 are NP-complete, then $L_1 \leq_m L_2$ and $L_2 \leq_m L_1$.
3. If $L_1 \leq_P L_2$, $L_2 \leq_P L_1$, and $L_1, L_2 \in$ NP, then L_1 and L_2 are both NP-complete.
4. If L_1 is NP-complete and $L_1 \leq_P L_2$, then L_2 is NP-complete.

Exercise 3 (compulsory)

Consider a nondeterministic Turing machine M over the input alphabet $\Sigma = \{a\}$, tape alphabet $\Gamma = \{a, \sqcup\}$, control states $Q = \{q, q_{accept}, q_{reject}\}$ and the following δ function:

- $\delta(q, a) = \{(q, a, R), (q_{reject}, \sqcup, L)\}$
- $\delta(q, \sqcup) = \{(q_{accept}, a, R)\}$

Fill in all legal windows (recall the proof of Cook-Levin Theorem) with the following chosen first rows. Note that this covers only some of the legal windows. The number of windows in every row indicates how many possible legal windows you should be able to find. You might use the abbreviations q_a for q_{accept} and q_r for q_{reject}.

a	a	a

a	a	a

a	a	a

a	a	a

#	a	a

#	a	a

a	q	a

a	q	a

a	q	$\sqcup$

q	a	a

q	a	a

q	a	a

q	a	a

a	a	q

a	a	q

Exercise 4 (compulsory)

Show that if P=NP then every language $B \in$ P, except for $\emptyset$ and Σ^*, is NP-complete.

Exercise 5 (optional)

Define the language

$$A_{BTM} = \{\langle M, x, 1^t \rangle \mid M \text{ is a NTM which accepts the input } x \text{ after no more than } t \text{ steps}\}$$

where 1^t here denotes the number t written in unary, i.e., the string $\underbrace{1 \ldots 1}_{t \text{ occurrences}}$. Prove that A_{BTM} is NP-complete. *Hint:* Make a direct proof, similarly as Cook-Levin Theorem.