Tutorial 9

Exercise 1 (compulsory)

Consider the following context-free grammar G in Chomsky normal form:

$$\begin{aligned} S &\rightarrow AA \mid \epsilon \\ A &\rightarrow BB \mid AB \mid a \\ B &\rightarrow BA \mid b \end{aligned}$$

Following the algorithm in the proof of Theorem 7.16 (see also Lecture 9) compute the entries $table(i, j)$ for all $1 \leq i \leq j \leq 4$ in order to show that $abba \in L(G)$.

Exercise 2 (compulsory)

Prove that the class P is closed under intersection, complement and concatenation.

Exercise 3 (compulsory)

Prove the following theorem.

Theorem: Let $t(n) \geq n$ be a function from natural numbers to positive reals. Then for every language $L \in$ NTIME$(t(n))$ there is a constant c such that $L \in$ TIME$(2^{c \cdot t(n)})$.

Exercise 4 (compulsory)

Every week someone manages to "prove" that P $=$ NP or that P $\neq$ NP. Last week a very famous professor published the following proof that P $\neq$ NP:

Proof: Consider the following decider for $HAMPATH$:

"On input $\langle G, s, t \rangle$:

1. Generate all possible permutations of nodes from G.
2. If one of these permuations (sequences of nodes) forms a Hamiltonian path, then accept.
3. Otherwise reject."

Because there are $n!$ different permuations of nodes to examine, the algorithm clearly does not run in polynomial time. Therefore we have proved that $HAMPATH$ has exponential time complexity and this means $HAMPATH \notin$ P. Because we know that $HAMPATH \in$ NP, we conclude that P $\neq$ NP.

Describe the error in the above proof.

Exercise 5 (optional, but highly recommended)

Prove that the class P is closed under Kleene star. (**Hint:** use dynamic programming.)