Tutorial 3

Exercise 1 (compulsory)

Consider the following claim:

> **Claim:** If L is a decidable language and $L' \subseteq L$, then L' is a decidable language too.

Is this claim true? If yes, prove it. If not, give a counter-example.

Exercise 2 (compulsory)

Consider the following problem:

> "Does a given regular expression R over $\Sigma = \{0, 1\}$ describe a language, which contains at least one string starting with 11 and ending with 0?"

1. Express the problem as a language RU_{110}.
2. Prove that RU_{110} is decidable.

Exercise 3 (compulsory)

Prove that the class of decidable languages is closed under union, concatenation and Kleene star.

Exercise 4 (compulsory)

Prove that the class of recognizable languages is closed under intersection, concatenation and Kleene star.

Exercise 5 (compulsory)

Consider the following two proofs that try to show that recognizable languages are closed under complement.

Proof 1:

> Let L be a recognizable language. By definition there is a Turing machine M such that $L(M) = L$. Consider the following machine M':
>
> 1. On input x:
> 2. Simulate M on x.
> 3. If M accepted then M' *rejects*. If M rejected then M' *accepts.*
>
> Now we can see that $L(M') = \overline{L}$. Hence $\overline{L}$ is recognizable.

Proof 2:

Let L be a recognizable language. By definition there is a Turing machine M such that $L(M) = L$. Consider the following machine M':

1. On input x:
2. Simulate M on x.
3. If M accepted then M' *rejects*. If M rejected or looped on x then M' *accepts.*

Now we can see that $L(M') = \overline{L}$. Hence $\overline{L}$ is recognizable.

As we already know, recognizable languages are not closed under complement and hence there must be an error in both proofs. Locate the errors by underlining them and explain (e.g. by giving a counter example) where is the problem.

Exercise 6 (optional and challenging)

Problem 4.28 on page 212 (in international edition) or Problem 4.14 on page 211 (in standard edition).