Tutorial 9

Exercise 1 (compulsory)

Which of these definitions of the O-notation are correct?

1. $f = O(g)$ iff
   there exist positive integers c and n_0 such that for all $n \geq n_0$ we have that $f(n) \geq c \cdot g(n)$
2. $f = O(g)$ iff
   for all positive integers c and n_0 it is the case that for all $n \geq n_0$ we have that $f(n) \leq c \cdot g(n)$
3. $f = O(g)$ iff
   there exist positive integers c and n_0 such that for all $n \geq n_0$ we have that $f(n) \leq c \cdot g(n)$
4. $f = O(g)$ iff
   for all positive integers c and n_0 it is the case that there exists an $n \geq n_0$ such that $f(n) \leq c \cdot g(n)$

Exercise 2 (compulsory)

Which of the following claims are true? Give precise arguments (proofs) for your answers.

1. $3n^2 + 2n + 7 = O(n^2)$
2. $n^2 = O(n \log n)$
3. $3^n = 2^{O(n)}$ (Hint: $3 = 2^{\log_2 3}$)
4. $n^2 = o(n^3)$
5. $n^2 = o(n^2)$
6. $n = o(2n)$

Exercise 3 (compulsory)

Assume a 5-tape Turing machine M running in time $O(n^3)$. What is the time complexity of the corresponding single-tape Turing machine simulating M?

Exercise 4 (compulsory)

Which of the following statements about the class P are correct?

1. P is the class of all languages that are decidable by deterministic single-tape Turing machines running in polynomial time.
2. P is the class of all languages such that if $w \in P$ then there is a deterministic single-tape Turing machine which accepts the string w in polynomial time.
3. P is the class of all languages that are decidable by deterministic multi-tape Turing machines running in polynomial time.

4. A language L belongs to P iff there is a constant k and a decider M running in time $O(n^k)$ such that $L = L(M)$.

5. A language L belongs to P iff $L \in$ TIME(2^n).

Give 5 languages that are in the class P. Does A_{TM} belong to P?

Exercise 5 (compulsory)

Define the language ALL_{DFA} and show that $ALL_{DFA} \in P$.

Exercise 6 (optional but easy)

Show that if L is a regular language then $L \in$ TIME(n).