Tutorial 13

Exercise 1 (compulsory)

Show that the class PSPACE is closed under union, intersection, concatenation, Kleene star and complement. **Hint:** You can conveniently use the fact that PSPACE=NPSPACE.

Exercise 2 (compulsory)

Prove that co-NP $\subseteq$ PSPACE.

Exercise 3 (compulsory)

Let f be a function such that $f(n) \geq n$. Which of the following claims are true?

1. *CLIQUE* $\in$ PSPACE
2. *VERTEX-COVER* $\notin$ PSPACE
3. *CLIQUE* $\in$ SPACE(n)
4. *CLIQUE* $\notin$ NSPACE(n)
5. SPACE($f(n)$) $\subseteq$ NSPACE($f(n)$)
6. NSPACE($f(n)$) $\subseteq$ SPACE($f^2(n)$)
7. NSPACE($f(n)$) $\subseteq$ SPACE($f^3(n)$)
8. SPACE($f(n)$) $\subseteq$ NSPACE($f^2(n)$)

Exercise 4 (compulsory)

Assume a deterministic decider M with space complexity n^3. How many steps does the machine M at most perform on an input w of length n?

Exercise 5 (compulsory)

Argue that given any TM M, we can without loss of generality assume that M satisfies the unique-accept-configuration condition:

> whenever M enters an accepting configuration, then the configuration is exactly this one: $q_{accept}\sqcup$.

In other words, provide a polynomial time reduction from A_{TM} to $A_{TM,unique}$, where

$A_{TM,unique} \stackrel{\text{def}}{=}$
$\{\langle M, w\rangle \mid$ M is a TM which accepts w and M satisfies the unique-accept-configuration condition $\}$.

Exercise 6 (optional)

Let L be a language that can be decided by a deterministic Turing machine in space $f(n)$ where $f(n) \geq n$ for all n. Prove that for any real number c, $0 < c < 1$, L can be decided by a deterministic Turing machine M_c with space complexity $cf(n)$. **Note:** Here we consider the exact space complexity, not an approximation using O-notation, so you cannot simply write that O-notation allows us to disregard constant factors.