Tutorial 7

Exercise 1 (compulsory)

Show that $\leq_m$ is a transitive relation.

Solution:
Solution to Problem 5.6 on page 242.

Exercise 2 (compulsory)

Show that A_{TM} is not mapping reducible to E_{TM}. In other words, show that there is no computable function that reduces A_{TM} to E_{TM}.

Hint: Use proof by contradiction and the known facts about these two problems. Also the fact that $A \leq_m B$ iff $\overline{A} \leq_m \overline{B}$ might be handy.

Solution:
Solution to Problem 5.5 on page 242.

Exercise 3 (compulsory)

Show that if A is recognizable and $A \leq_m \overline{A}$, then A is decidable.

Solution:
Solution to Problem 5.7 on page 242.

Exercise 4 (compulsory)

Show that $REGULAR_{TM}$ is neither recognizable nor co-recognizable.

Hint: Modify a similar proof for EQ_{TM} from Lecture 7.

Solution:
In order to prove the claim, it is enough to show that $\overline{A_{TM}} \leq_m REGULAR_{TM}$, and $\overline{A_{TM}} \leq_m \overline{REGULAR_{TM}}$.

- We prove that $\overline{A_{TM}} \leq_m REGULAR_{TM}$ by showing that

$$A_{TM} \leq_m \overline{REGULAR_{TM}}.$$

M_f = "On input $\langle M, w\rangle$ where M is a TM:
1. Construct the following machine M_1:
   M_1 = "On input x:
   If x is of the form $a^n b^n$ then run M on w and <u>M_1 accepts</u> iff M accepted.
   If x is not of the form $a^n b^n$ then <u>M_1 rejects</u>.
2. Output $\langle M_1\rangle$."

Note: M_f halts for any given input and:

- If M accepts w then $L(M_1) = \{a^n b^n \mid n \geq 0\}$ is not regular.
- If M does not accept w then $L(M_1) = \emptyset$ is regular.

Hence f is a mapping reduction from A_{TM} to $\overline{REGULAR_{TM}}$.

- We prove that $\overline{A_{TM}} \leq_m \overline{REGULAR_{TM}}$ by showing that

$$A_{TM} \leq_m REGULAR_{TM}.$$

M_f = "On input $\langle M, w\rangle$ where M is a TM:
1. Construct the following machine M_1:
   M_1 = "On input x:
   If x is of the form $a^n b^n$ then run M on w and $\underline{M_1 \text{ accepts}}$ iff M accepted.
   If x is not of the form $a^n b^n$ then $\underline{M_1 \text{ accepts}}$.
2. Output $\langle M_1\rangle$."

Note: M_f halts for any given input and:

– If M accepts w then $L(M_1) = \Sigma^*$ is regular.

– If M does not accept w then $L(M_1) = \Sigma^* \setminus \{a^n b^n \mid n \geq 0\}$ is not regular (can you see why?).

Hence f is a mapping reduction from A_{TM} to $REGULAR_{TM}$.

Exercise 5 (optional)

Exercise 5.4 on page 239.

Exercise 6 (optional)

Problem 5.9 on page 239 (in international edition), or Problem 5.21 (in standard edition).