Tutorial 7 (extra)

Exercise 1 (highly recommended)

Consider the following claim that some of you used during the second test.

Claim: If a language is co-recognizable then it cannot be decidable.

Does this claim hold or not? Give arguments for your answer.

Exercise 2 (for further practice on the simplest reductions)

1. Prove that $HALT_{TM} \leq_m A_{TM}$. (Note that your task is to find a mapping reduction in the opposite direction than the one provided in the book in Example 5.24 on page 236).
2. Prove that the language

$$EPSILON_{TM} \stackrel{\text{def}}{=} \{\langle M\rangle \mid \ M \text{ is a TM and } \epsilon \in L(M)\ \}$$

is undecidable. Is $EPSILON_{TM}$ recognizable? Is $EPSILON_{TM}$ co-recognizable?

3. Prove that E_{TM} is undecidable. First, define the problem and then provide either the standard reduction or mapping reduction from a suitable undecidable problem.

Exercise 3 (for even further practice)

Prove that the problem

$$INFINITE_{TM} \stackrel{\text{def}}{=} \{\langle M\rangle \mid \ M \text{ is a TM and } L(M) \text{ contains infinitely many strings }\}$$

is undecidable.

Exercise 4 (only for ”feinschmeckers”)

The credit for this exercise goes to Morten Dahl and Morten Kühnrich.

Assume that $L \subseteq \{0,1\}^*$ is an undecidable language. Prove that $L' \stackrel{\text{def}}{=} L \cup F$ remains undecidable for any finite language $F \subseteq \{0,1\}^*$. Is this the case also if we allow F to be an infinite language?

Hint: Prove the undecidability claim by reduction from L to L'. The details of the proof are rather delicate and in some sense a part of the proof is non-constructive.