Tutorial 6

Exercise 1 (compulsory)

We know that the problem ALL_{CFG} (does a given context-free grammar recognize all strings from Σ^*?) is undecidable.

Consider the problem $EQ_{CFG} \stackrel{\text{def}}{=} \{\langle G_1, G_2\rangle \mid \ G_1 \text{ and } G_2 \text{ are CFGs such that } L(G_1) = L(G_2)\ \}$.

- By reduction from ALL_{CFG} prove that EQ_{CFG} is undecidable.
- Prove that EQ_{CFG} is co-recognizable.
- Can EQ_{CFG} be also recognizable?

Exercise 2 (compulsory)

Consider the following instance P of Post correspondence problem.

$$P = \{[\frac{ab}{abab}], [\frac{b}{a}], [\frac{aba}{b}], [\frac{aa}{a}]\}$$

Find a match of P (a nonempty sequence of indices between 1 and 4).

Exercise 3 (compulsory)

Consider the problem *silly Post correspondence problem* (SPCP). An instance P of SPCP is

$$P = \{[\frac{t_1}{b_1}], [\frac{t_2}{b_2}], \ldots, [\frac{t_k}{b_k}]\}$$

where $t_i, b_i \in \Sigma^+$ and moreover $|t_i| = |b_i|$ for all i, $1 \leq i \leq k$. In other words in each pair the top string has the same length as the bottom string. The question is whether P contains a match (like in standard PCP).

- Is the problem SPCP decidable or undecidable? Give a precise proof of your claim.

Exercise 4 (compulsory)

Consider the problem *binary Post correspondence problem* (BPCP). An instance P of BPCP is

$$P = \{[\frac{t_1}{b_1}], [\frac{t_2}{b_2}], \ldots, [\frac{t_k}{b_k}]\}$$

where $t_i, b_i \in \{0,1\}^+$ for all i, $1 \leq i \leq k$.

In other words BPCP instance is like PCP instance, we only restrict the alphabet for the top and bottom strings to consist only of two symbols.

- Is the problem BPCP decidable or undecidable? Give a precise proof of your claim.

Hint: Consider an instance of standard PCP over some general alphabet $\Sigma = \{a_1, a_2, \ldots, a_n\}$. Is it possible to construct an instance of BPCP which has a match if and only if the original instance has a match?

Exercise 5 (optional)

Consider the problem whether two given context-free grammars have a nonempty intersection of the languages they generate.

- Describe the decision problem as a language $INTERSECTION_{CFG}$.
- Prove that $INTERSECTION_{CFG}$ is undecidable by reduction from PCP.

Hint: For a given PCP instance

$$P = \{ [\frac{t_1}{b_1}], [\frac{t_2}{b_2}], \ldots, [\frac{t_k}{b_k}] \}$$

you might find useful to consider the following grammars G_1 and G_2 (where $i_1, i_2, \ldots, i_k$ are new terminal symbols).

$$G_1 : \quad S_1 \to t_1 S_1 i_1 \mid \ldots \mid t_k S_1 i_k \mid t_1 i_1 \mid \ldots \mid t_k i_k$$
$$G_2 : \quad S_2 \to b_1 S_2 i_1 \mid \ldots \mid b_k S_2 i_k \mid b_1 i_1 \mid \ldots \mid b_k i_k$$

Exercise 6 (optional)

Show that any Turing machine that is allowed to use only the first $2|w|$ tape cells when run on an input w is equivalent to LBA (which can use only $|w|$ of the first tape cells).

Exercise 7 (optional)

Exercise 5.8 on page 239.