

# Database Systems Part-2

Relation, FD, and Keys

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# Schedule for first half

Intro & ER	Sep 5 Monday (12:30-14:15), Lecture - FRB 7H	No exercise
Relation, FD, Keys	Sep 12 Monday (12:30-14:15), Lecture - FRB 7H	Exercise-1 on Intro & ER (14:30-16:30)
Normalization	Sep 19 Monday (12:30-14:15), Lecture - FRB 7H	Exercise-2 on Relation, FD, Keys (14:30-16:30)
Relational Algebra	Exercise-3 on Normalization (8:15-10:00)	Sep 23 Friday (10:15-12:00), Lecture - NOVI8
Release of Self- Study-1	Sep 26 Monday, No lecture, No exercise	
SQL – Part I	Oct 3 Monday (12:30-14:15), Lecture - FRB 7H	Exercise-4 on Relational Algebra (14:30-16:30)
Feedback of Self- Study-1	Oct 10 Monday (12:30-14:15), During lecture - FRB 7H	Exercise-5 on SQL – Part I (14:30-16:30)

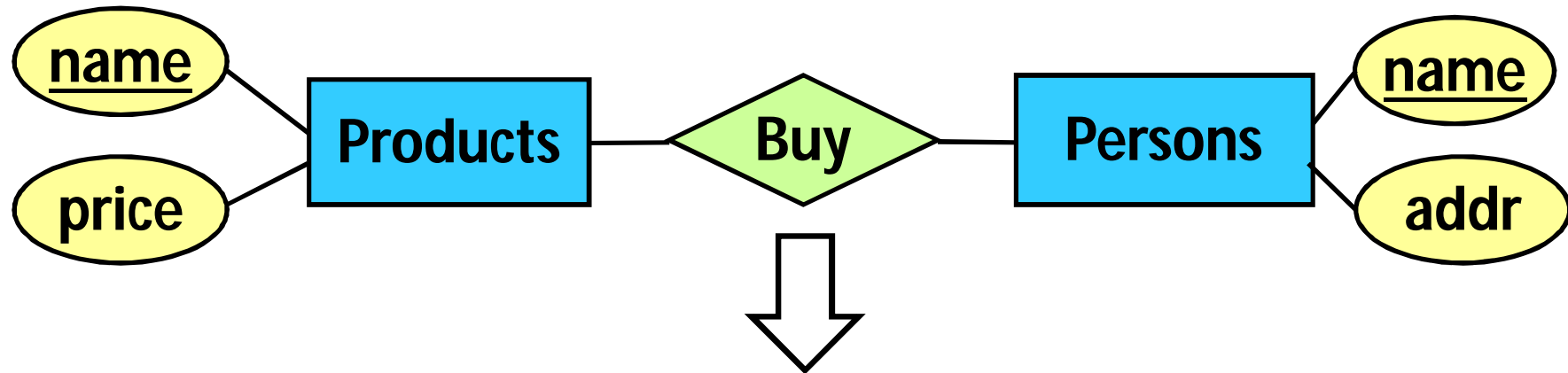
# Ask Questions!



The important thing is not to  
stop questioning.

Albert Einstein

# ER Diagram → Relational Schema



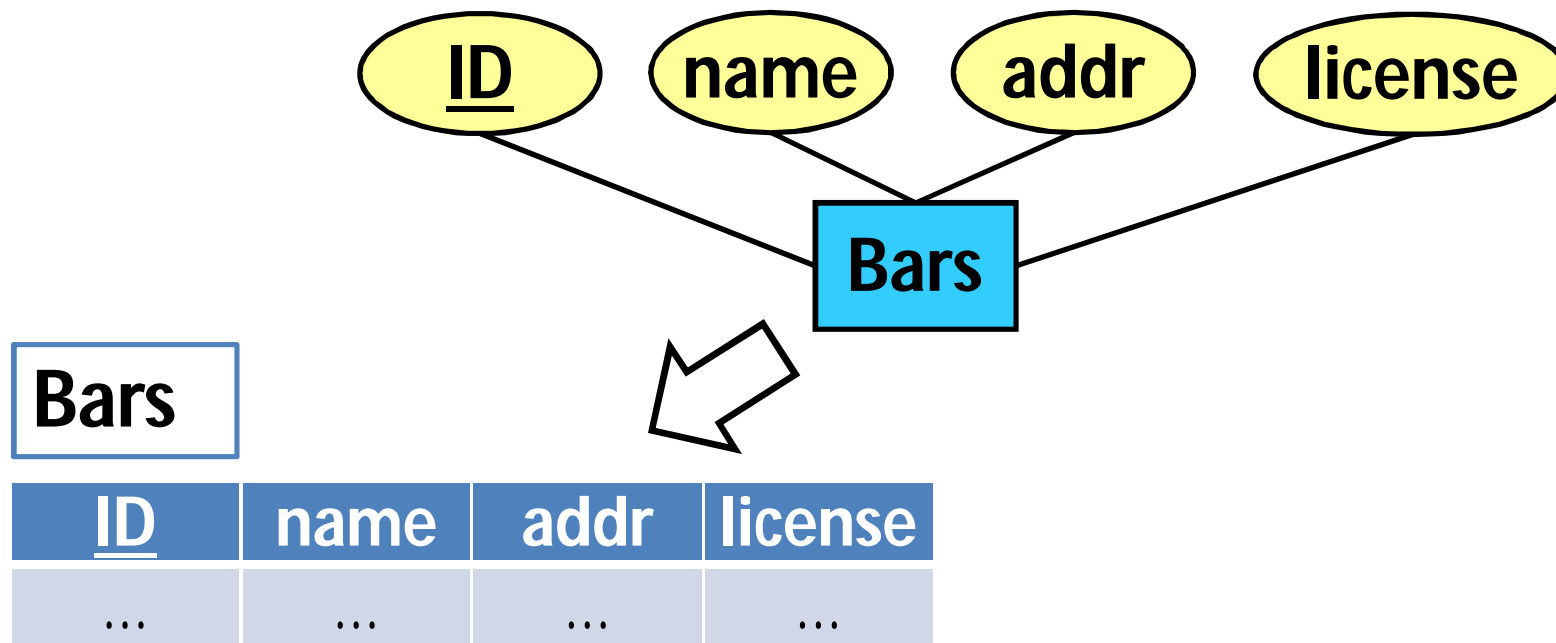
- Products (name, price)
- Persons (name, addr)
- Buy (product\_name, person\_name)
- Terminology
  - A **relation schema** = the name of a table + names of its attributes
  - A **database schema** = a set of relation schemas

# ER Diagram → Relational Schema



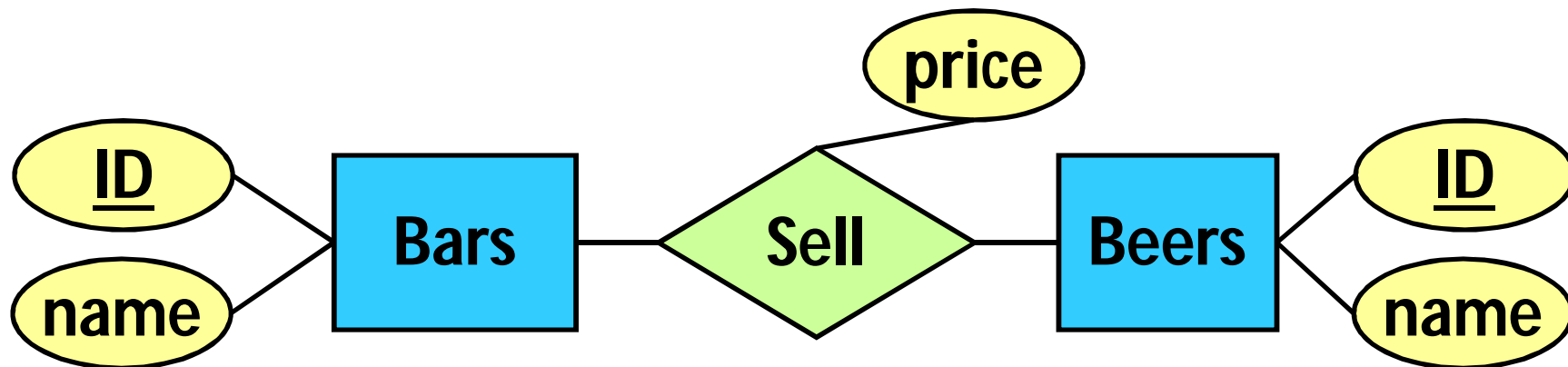
- General rules:
  - Each entity set becomes a relation
  - Each many-to-many relationship becomes a relation
- Special treatment needed for:
  - Weak entity sets
  - Subclasses
  - Many-to-one and one-to-one relationships

# Entity Set $\rightarrow$ Relation



- Each entity set is converted into a relation that contains all its attributes
- The key of the relation = the key of the entity set

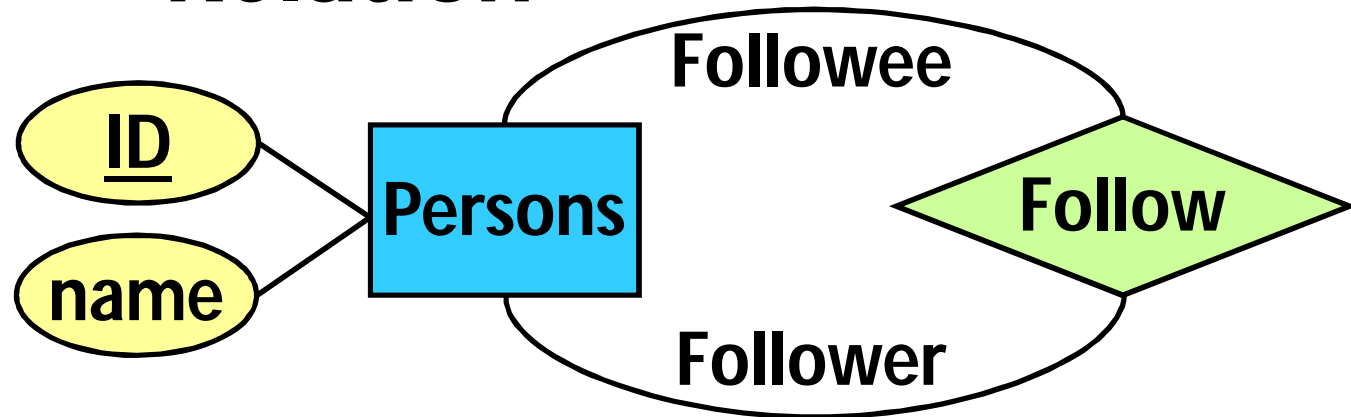
# Many-to-Many Relationship → Relation



- Converted into a relation that contains
  - all keys of the participating entity sets, and
  - the attributes of the relationship (if any)
- Key of relation = Keys of the participating entity sets

Sell	<u>Bars-ID</u>	<u>Beers-ID</u>	price
	...	...	...

# Many-to-Many Relationship → Relation

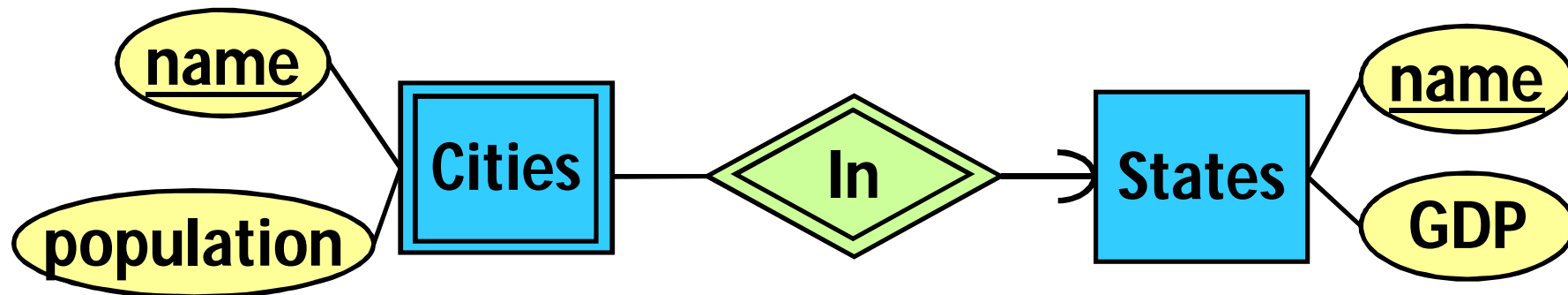


- If an entity is involved multiple times in a relationship
  - Its key will appear in the corresponding relation multiple times
  - The key is re-named according to the corresponding role

<b>Follow</b>	<u>Follower-ID</u>	<u>Followee-ID</u>
	...	...



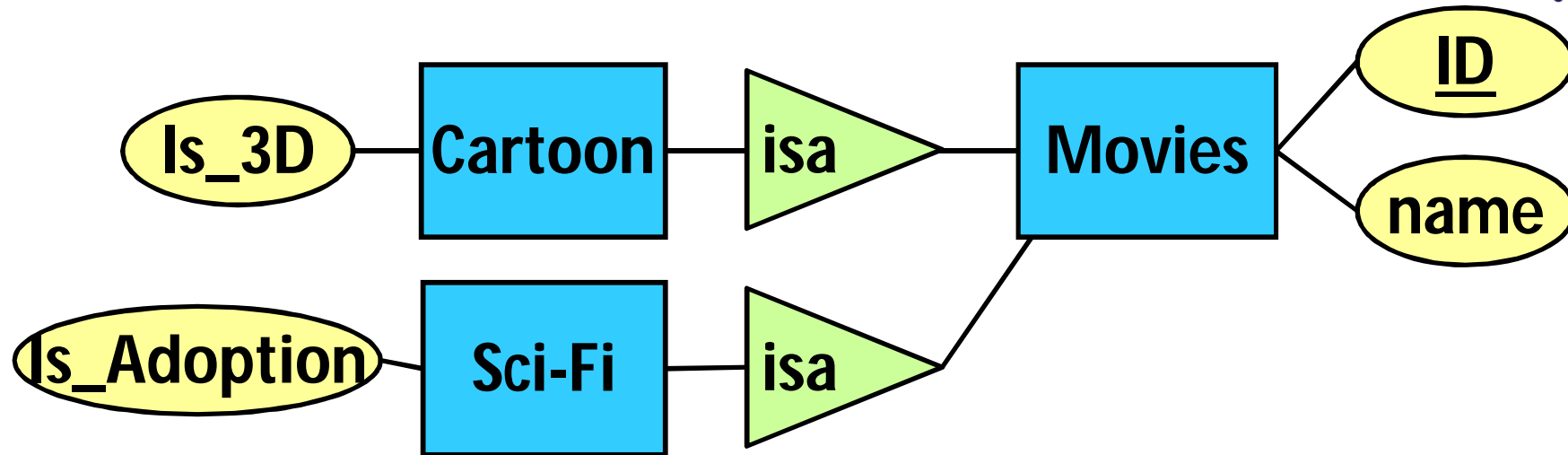
# Weak Entity Set $\rightarrow$ Relation



- Each weak entity set is converted to a relation that contains
  - all of its attributes, and
  - the key attributes of the supporting entity set
- The supporting relationship is ignored

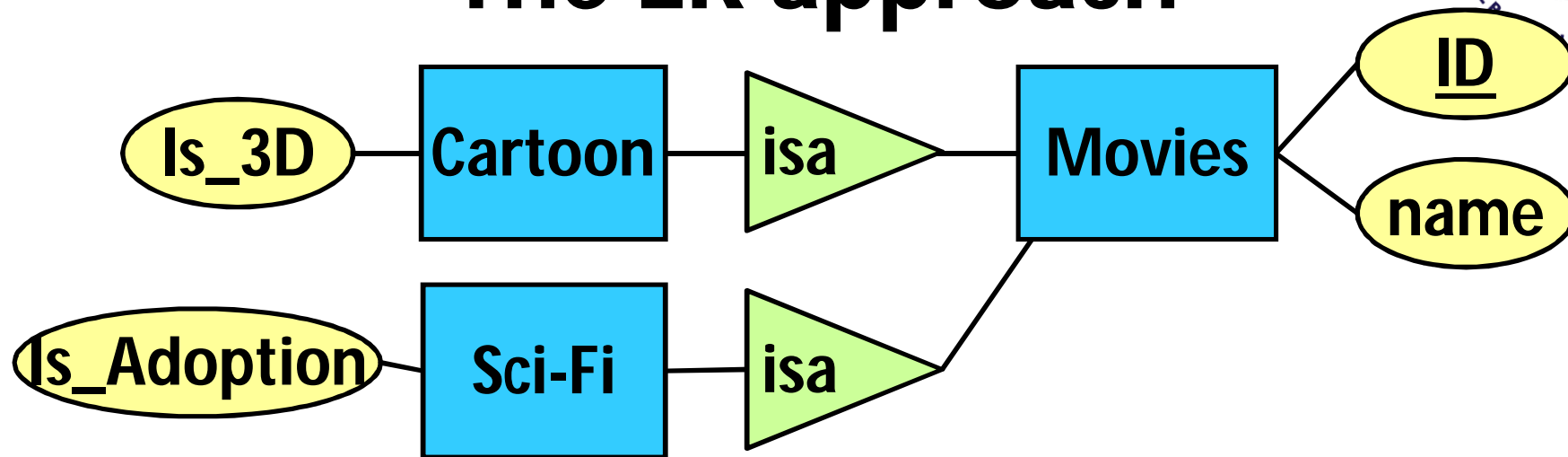
Cities	<u>state-name</u>	<u>city-name</u>	population
	...	...	...

# Subclass → Relation



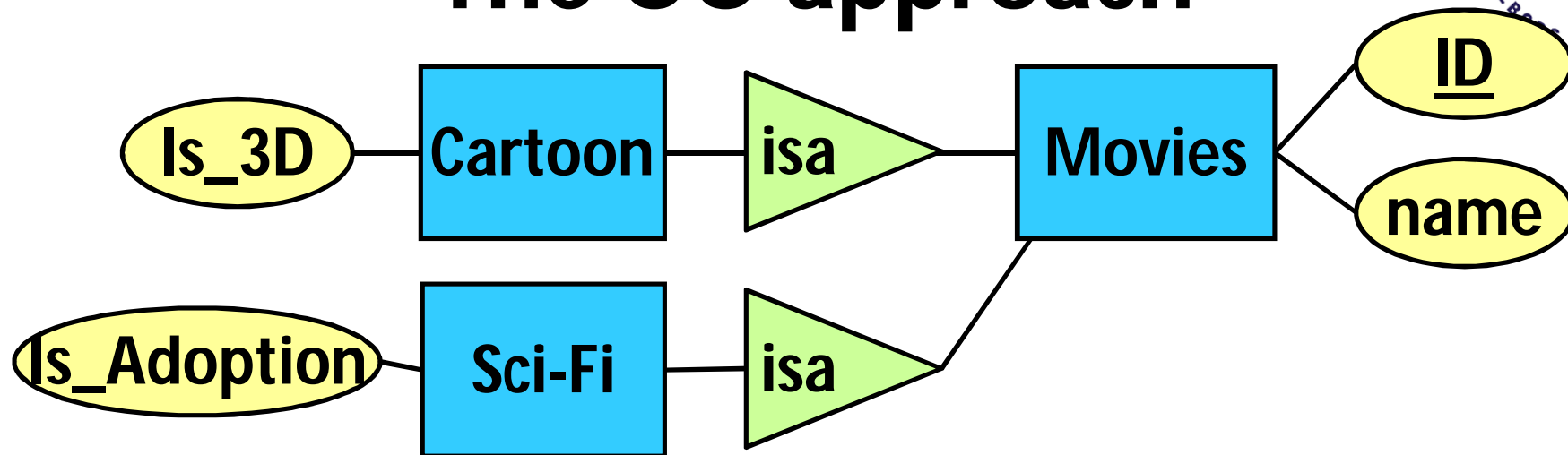
- There are three different ways
  - The ER approach
  - The OO approach
  - The NULL approach

# The ER approach



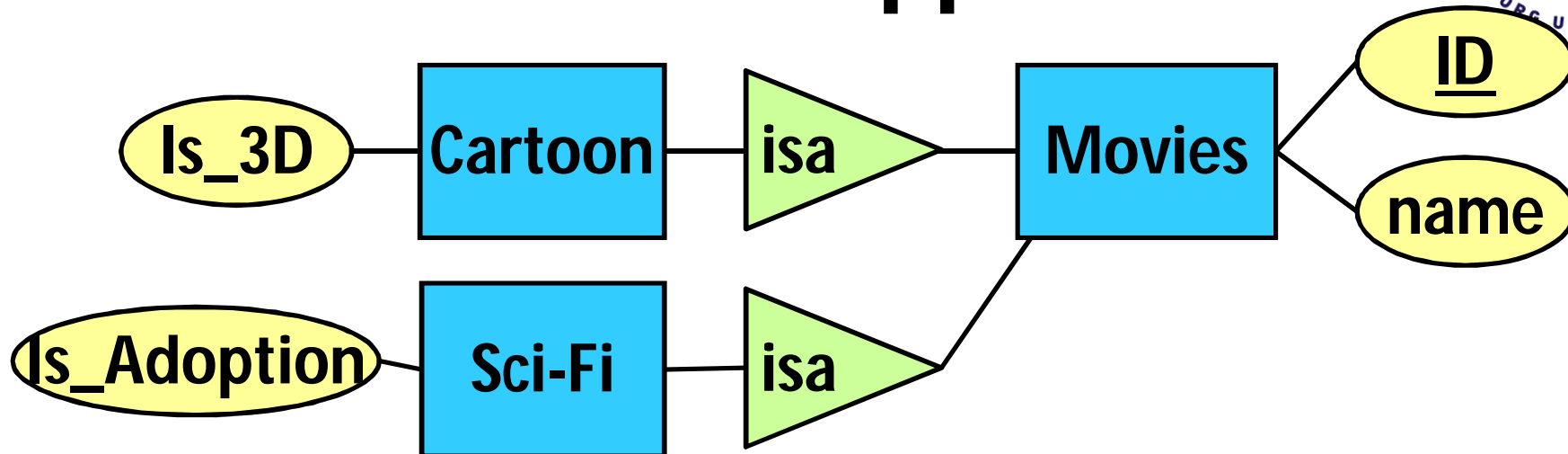
- One relation for each entity set
  - Movies( ID, name )
  - Cartoon( ID, Is\_3D )
  - Sci-Fi( ID, Is\_Adoption )
- A record may appear in multiple relations

# The OO approach



- One relation for each entity set and each **possible subclass combination**
  - Movies( ID, name )
  - Cartoon( ID, name, Is\_3D )
  - Sci-Fi( ID, name, Is\_Adoption )
  - Sci-Fi-Cartoon( ID, name, Is\_3D, Is\_Adoption )
- Each record appears in only one relation

# The NULL approach



- One relation that includes everything
  - Movies( ID, name, Is\_3D, Is\_Adoption )
- For non-cartoon movies, its “Is\_3D” is set to NULL
- For non-sci-fi movies, its “Is\_Adoption” is set to NULL

# Which Approach is the Best?

- It depends
- The NULL approach
  - Advantage: Needs only one relation
  - Disadvantage: May have many NULL values
- The OO approach
  - Advantage: Good for searching subclass combinations
  - Disadvantage: May have too many tables
- The ER approach
  - A middle ground between OO and NULL

# Questions?



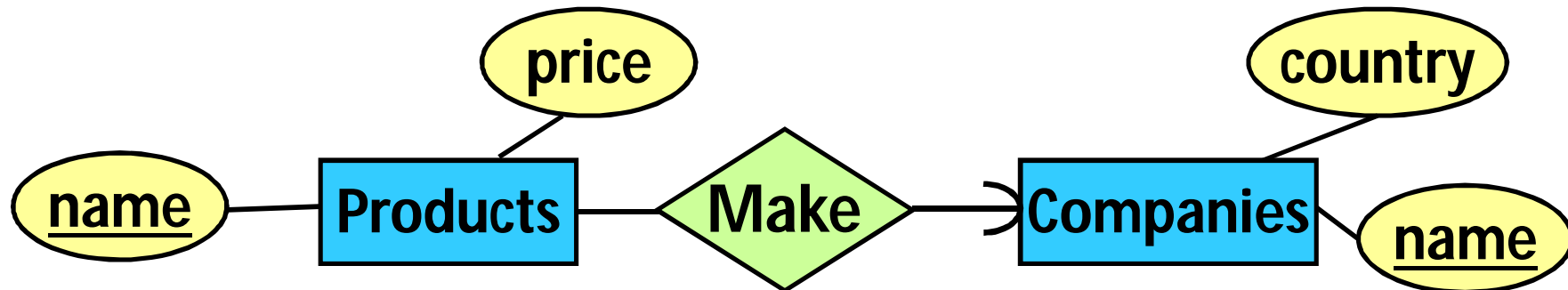
# ER Diagram → Relational Schema



- General rules:
  - Each entity set becomes a relation
  - Each many-to-many relationship becomes a relation
- Special treatment needed for:
  - Weak entity sets
  - Subclasses
  - Many-to-one and one-to-one relationships

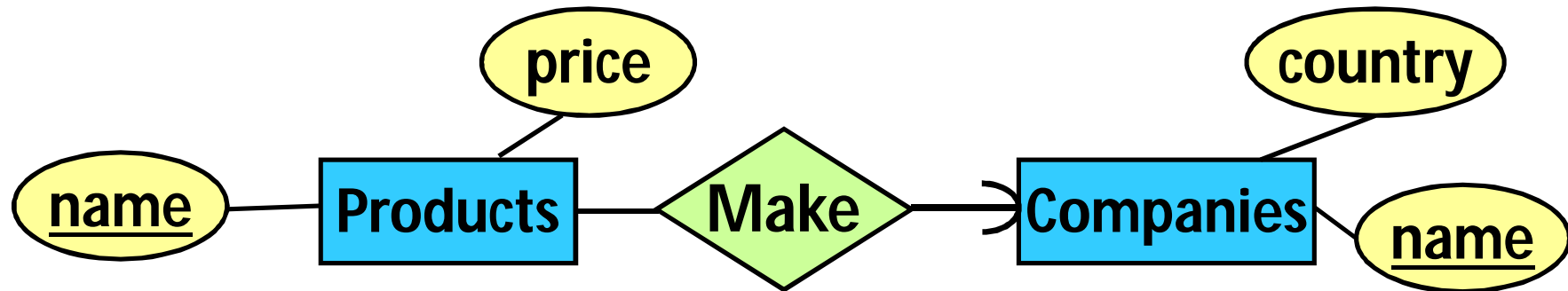


# Many-to-One Relationship → Relation



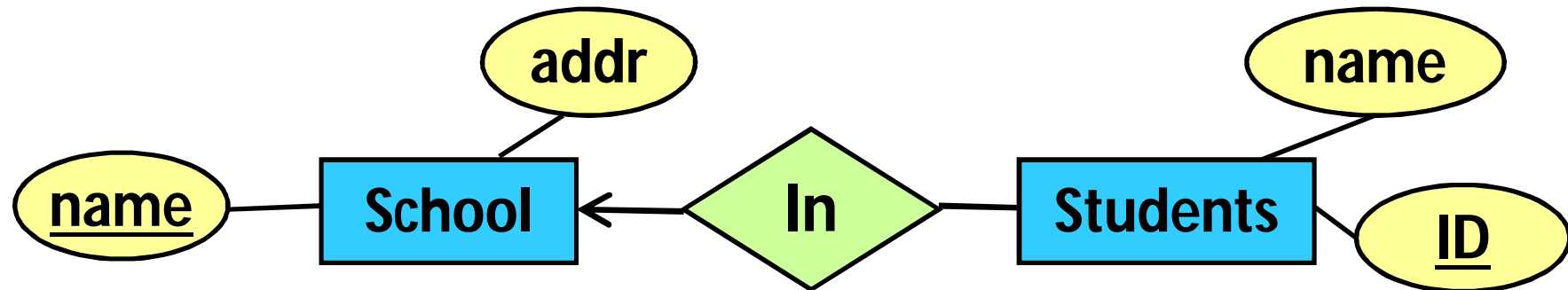
- Intuitive translation:
  - Products( Pname, price )
  - Companies( Cname, country )
  - Make( Pname, Cname )
- Observation: in “Make”, each Pname has only one Cname
- Simplification: Merge “Make” and “Products”
- Results:
  - Products( Pname, price, Cname )
  - Companies( Cname, country )

# Many-to-One Relationship → Relation



- In general, we do not need to create a relation for a many-to-one relationship
- Instead, we only need to put the key of the "one" side into the relation of the "many" side

# Many-to-One Relationship → Relation

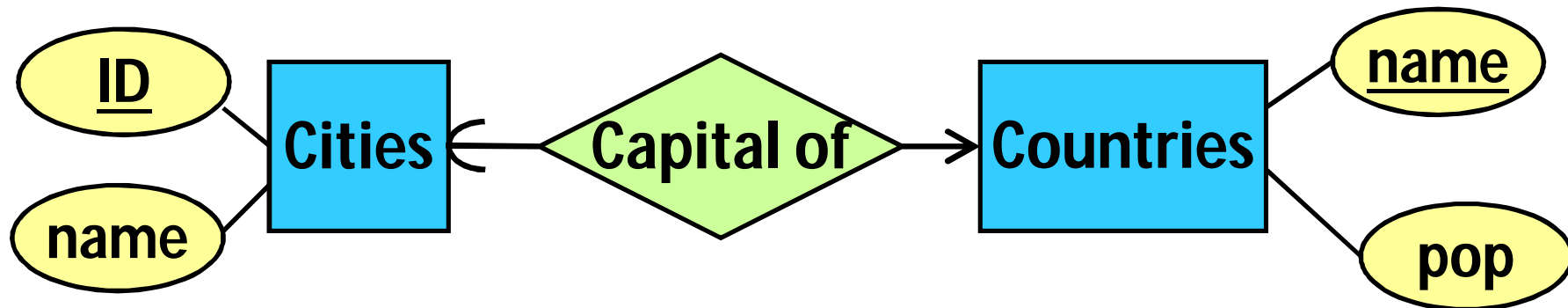


## ■ Translation:

- ❑ School( Hname, addr )
- ❑ Students( ID, Sname, Hname )

- Only need to put the key of the “one” side into the relation of the “many” side

# One-to-One Relationship → Relation



- No need to create a relation for a one-to-one relationship
- Only need to put the key of one side into the relation of the other
- Solution 1
  - Cities( TID, Tname )
  - Countries( Cname, pop, TID)
- Solution 2
  - Cities( TID, Tname, Cname )
  - Countries( Cname, pop )

# Questions?

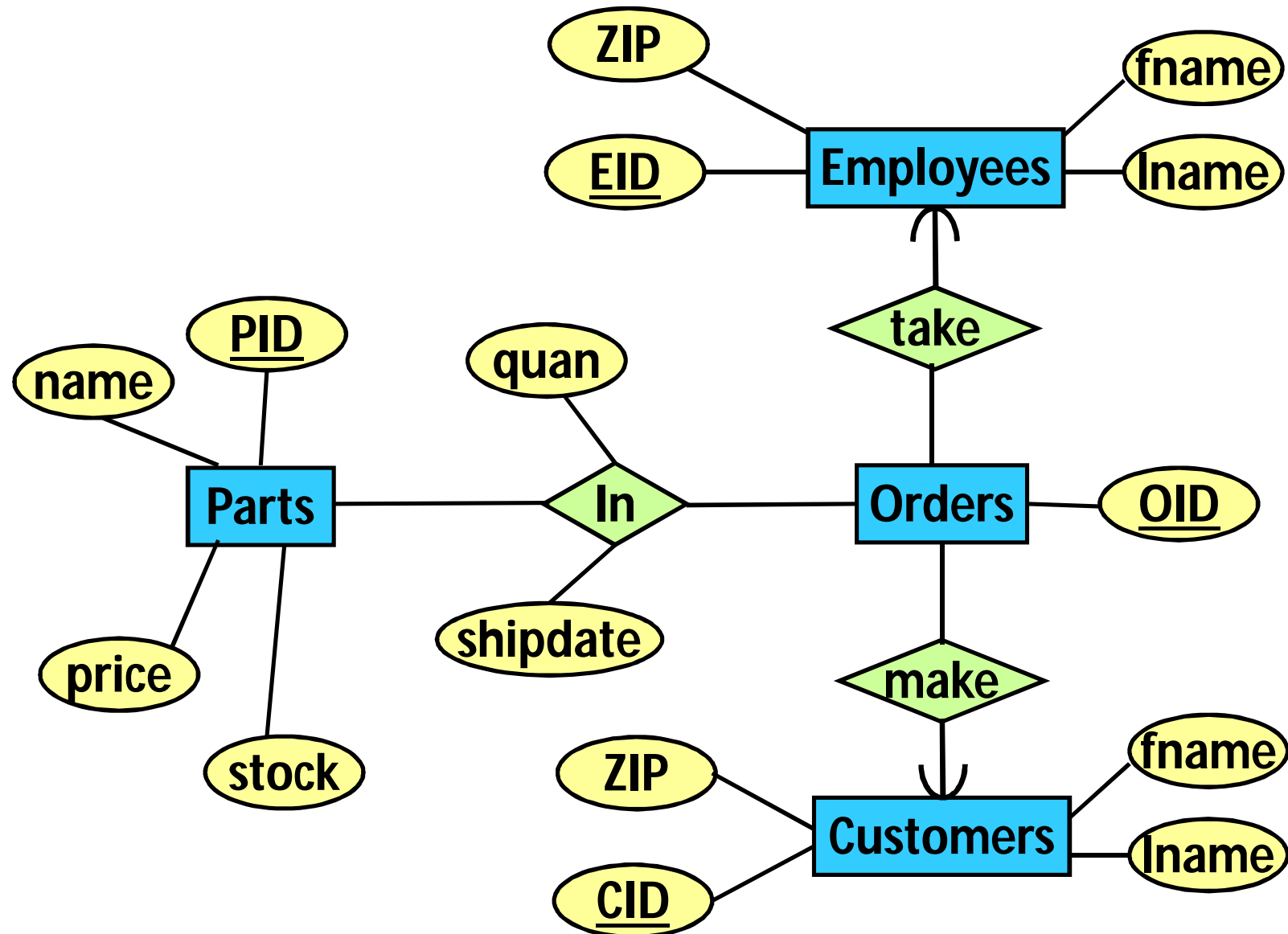


# Classroom Exercise-1



- Consider a mail order database in which employees take orders for parts from customers. The requirements are:
- Each employee is identified by a unique employee number, and has a first name, a last name, and a zip code.
- Each customer is identified by a unique customer number, and has a first name, last names, and a zip code.
- Each part being sold is identified by a unique part number. It has a part name, a price, and a quantity in stock.
- Each order placed by a customer is taken by one employee and is given a unique order number. Each order may contain certain quantities of one or more parts. The shipping date of each part is also recorded.

# Classroom Exercise-1



# Classroom Exercise-1: Solution



- Parts( PID, name, price, stock )
- Employees( EID, fname, lname, ZIP )
- Customers( EID, fname, lname, ZIP )
- Orders( OID, EID, CID )
- In( OID, PID, quan, shipdate )

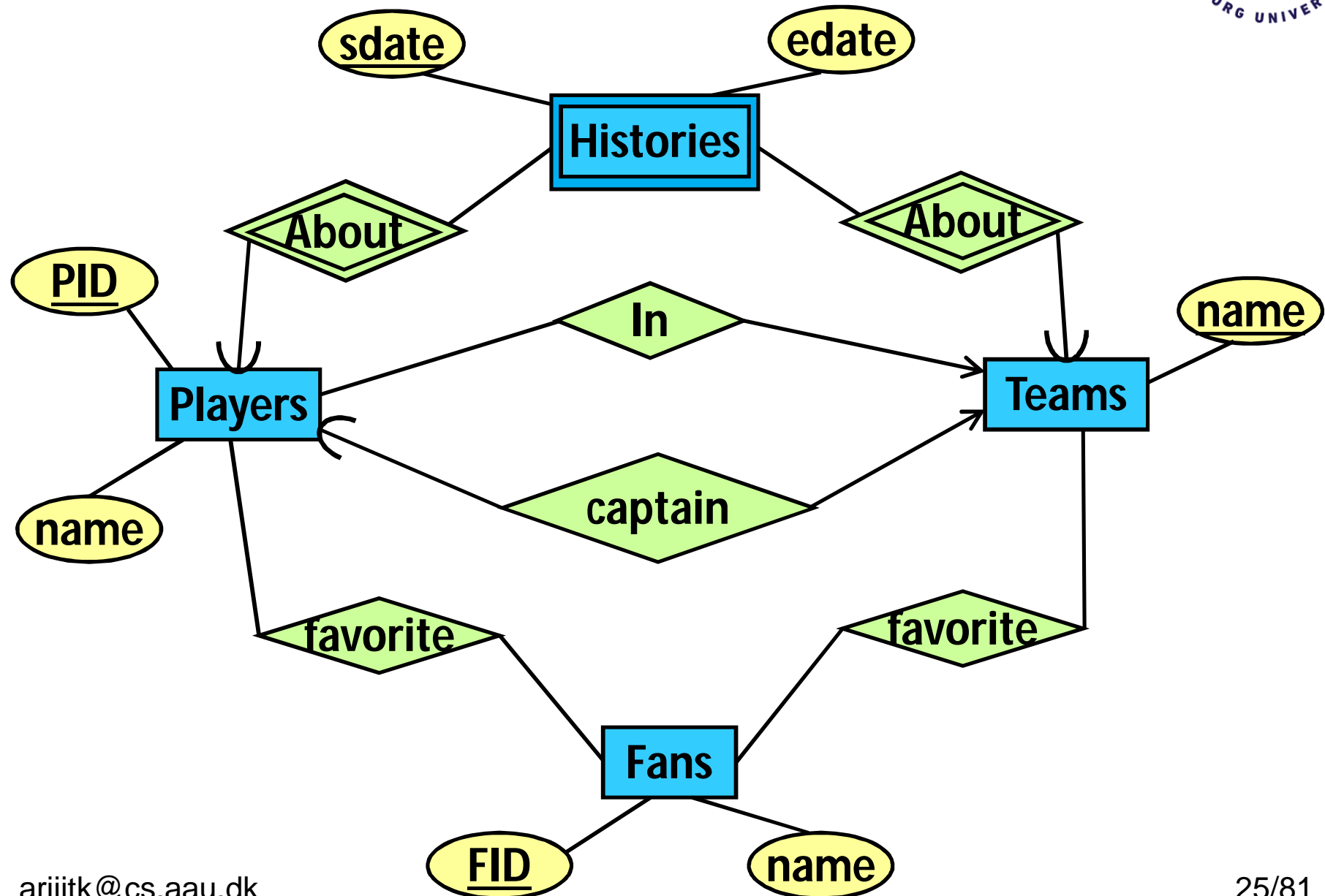


# Classroom Exercise-2



- Record info about teams, players, and their fans, including:
  - For each team, its name, its players, its team captain (who is also a player)
  - For each player, his/her name, and the history of teams on which he/she has played, including the start and ending dates for each team
  - For each fan, his/her name, favorite teams, favorite players
- Additional information:
  - Each team has at least one player, and exactly one captain
  - Each team has a unique name
  - Two players (or two fans) may have the same name
  - Each fan has at least one favorite team and at least one favorite player

# Classroom Exercise-2



# Classroom Exercise-2: Solution



- Player( PID, name, team-name )
- Fans( FID, name )
- Teams( team-name, PID )
- Histories( PID, team-name, sdate, edate )
- FavoriteTeam( FID, team-name )
- FavoritePlayer( FID, PID )

# 5 Minutes Interval



The important thing is not to  
stop questioning.

Albert Einstein

# Database Design

- Applications → ER diagrams → tables
- It works in general, but sometimes things might go wrong
  - The ER diagrams might not be well designed
  - The conversion from ER diagrams to tables might not be done properly
- As a consequence, the resulting tables might have various problems

# Data Anomalies

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:  
(CPR, PhoneNumber)
- There are several **anomalies** in the table
- First, redundancy:
  - Alice's address is duplicated

# Data Anomalies

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:  
(CPR, PhoneNumber)
- Second, update anomalies:
  - We may accidentally update one of Alice's addresses, leaving the other unchanged

# Data Anomalies

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:  
(CPR, PhoneNumber)
- Third, deletion anomalies:
  - Bob no longer uses a phone
  - Can we remove Bob's phone number?
  - No. (Note: Primary key attributes cannot be NULL)



# Data Anomalies

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Primary key of the table:  
(CPR, PhoneNumber)
- Fourth, insertion anomalies:
  - Name = Cathy, CPR = 9394, HomeAddress = YiShun
  - Can we insert this information into the table?
  - No. (Note: Primary key attributes cannot be NULL)

# Normalization

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- How do we get rid of those anomalies?
- **Normalize** the table (i.e., decompose it)

Name	<u>CPR</u>	HomeAddress
Alice	1234	Jurong East
Bob	5678	Pasir Ris

<u>CPR</u>	<u>PhoneNumber</u>
1234	67899876
1234	83848384
5678	98765432

# Effects of Normalization

Name	<u>CPR</u>	HomeAddress
Alice	1234	Jurong East
Bob	5678	Pasir Ris

<u>CPR</u>	<u>PhoneNumber</u>
1234	67899876
1234	83848384
5678	98765432

- Duplication?
  - No. (Alice's address is no longer duplicated.)
- Update anomalies?
  - No. (There is only one place where we can update the address of Alice)
- Deletion anomalies?
  - No. (We can freely delete Bob's phone number)
- Insertion anomalies?
  - No. (We can insert an individual with a phone)

# Questions?



# Functional Dependencies: Intuition

Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- Why was this table bad?
- It has a lot of anomalies
- Why does it have those anomalies?
- Intuitive answer: It contains a bad combination of attributes

# Functional Dependencies: Intuition



Name	<u>CPR</u>	<u>PhoneNumber</u>	HomeAddress
Alice	1234	67899876	Jurong East
Alice	1234	83848384	Jurong East
Bob	5678	98765432	Pasir Ris

- In general, how do we know whether a combination of attributes is bad?
- We need to check the **correlations** among those attributes
- What kind of correlations?
- **Functional dependencies (FD)**

# Functional Dependencies (FD)



- Consider two attributes in practice: CPR, Name
- Given an CPR, can we always uniquely identify the name of the person?
- Theoretically yes -- We just need help from Govt.
- Given a Name, can we always uniquely identify the CPR of the person
- In general no -- Different people can have the same name
- Therefore
  - CPR decides Name, but not vice versa
  - Functional dependencies:  
CPR  $\rightarrow$  Name, but not Name  $\rightarrow$  CPR

# Functional Dependencies (FD)



- Consider three attributes in practice:
  - StudentID, Name, Address, PostalCode
- Functional Dependencies:
  - StudentID  $\rightarrow$  Name, Address, PostalCode
  - Address  $\rightarrow$  PostalCode
  - PostalCode  $\rightarrow$  Address



# Classroom Exercise-3



Name	Category	Color	Department	Price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office Supplies	59

- Find the functional dependencies are definitely not true on the above table
  - Category  $\rightarrow$  Department
  - Category, Color  $\rightarrow$  Price
  - Price  $\rightarrow$  Color
  - Name  $\rightarrow$  Color
  - Department, Category  $\rightarrow$  Name
  - Color, Department  $\rightarrow$  Name, Price, Category

# Classroom Exercise-3: Solution



Name	Category	Color	Department	Price
Gizmo	Gadget	Green	Toys	49
Tweaker	Gadget	Black	Toys	99
Gizmo	Stationary	Green	Office Supplies	59

- Find the functional dependencies are definitely not true on the above table
  - Category  $\rightarrow$  Department
  - Category, Color  $\rightarrow$  Price
  - Price  $\rightarrow$  Color
  - **Name  $\rightarrow$  Color** ✗
  - **Department, Category  $\rightarrow$  Name** ✗
  - Color, Department  $\rightarrow$  Name, Price, Category

# Where Do FDs Come From?

- From common sense
- From the application's requirements
- Example
  - Purchase( CustomerID, ProductID, ShopID, Price, Date )
  - Requirement: Each shop can sell at most one product
  - FD implied: ShopID  $\rightarrow$  ProductID

# Where Do FDs Come From?

- Example
  - Purchase( CustomerID, ProductID, ShopID, Price, Date )
  - Requirement: No two customers buy the same product
  - FD implied: ProductID  $\rightarrow$  CustomerID

# Where Do FDs Come From?

- Example
  - Purchase( CustomerID, ProductID, ShopID, Price, Date )
  - Requirement: No shop will sell the same product to the same customer on the same date at two different prices
  - FD implied:  
CustomerID, ProductID, ShopID, Date  $\rightarrow$  Price

# Reasoning with FDs

- Example:
  - Given:  
 $CPR \rightarrow Address, \quad Address \rightarrow PostalCode$
  - We have:  $CPR \rightarrow PostalCode$
- In general
  - Given  $A \rightarrow B, B \rightarrow C$
  - We always have  $A \rightarrow C$
- Any others rules? Armstrong's Axioms

# Armstrong's Axioms

- Axiom of Reflexivity
  - A set of attributes  $\rightarrow$  A subset of the attributes
- Example
  - CPR, Name  $\rightarrow$  CPR
  - StudentID, Name, Age  $\rightarrow$  Name, Age
  - ABCD  $\rightarrow$  ABC
  - ABCD  $\rightarrow$  BCD
  - ABCD  $\rightarrow$  AD

# Armstrong's Axioms

- Axiom of Augmentation
  - Given  $A \rightarrow B$
  - We always have  $AC \rightarrow BC$ , for any  $C$
- Example
  - If  $CPR \rightarrow Name$
  - Then  $CPR, Age \rightarrow Name, Age$
  - and  $CPR, Salary, Weight \rightarrow Name, Salary, Weight$
  - and  $CPR, Addr, Postal \rightarrow Name, Addr, Postal$



# Armstrong's Axioms

- Axiom of Transitivity
  - Given  $A \rightarrow B$  and  $B \rightarrow C$
  - We always have  $A \rightarrow C$
- Example
  - If  $CPR \rightarrow Addr$ , and  $Addr \rightarrow Postal$
  - Then  $CPR \rightarrow Postal$

# Reasoning with FDs

- Given  $A \rightarrow B$ ,  $BC \rightarrow D$
- Can you prove that  $AC \rightarrow D$ ?
- Proof
  - Given  $A \rightarrow B$ , we have  $AC \rightarrow BC$  (Augmentation)
  - Given  $AC \rightarrow BC$  and  $BC \rightarrow D$ , we have  $AC \rightarrow D$  (Transitivity)

# Reasoning with FDs

- Given  $A \rightarrow B$ ,  $D \rightarrow C$
- Can you prove that  $AD \rightarrow BC$ ?
- Proof
  - Given  $A \rightarrow B$ , we have  $AD \rightarrow BD$  (Augmentation)
  - Given  $AD \rightarrow BD$ , we have  $AD \rightarrow B$  (Reflexivity)
  - Given  $D \rightarrow C$ , we have  $AD \rightarrow AC$  (Augmentation)
  - Given  $AD \rightarrow AC$ , we have  $AD \rightarrow C$  (Reflexivity)
  - In other words,  $AD$  decides  $B$  and  $C$
  - Therefore,  $AD \rightarrow BC$

# Reasoning with FDs

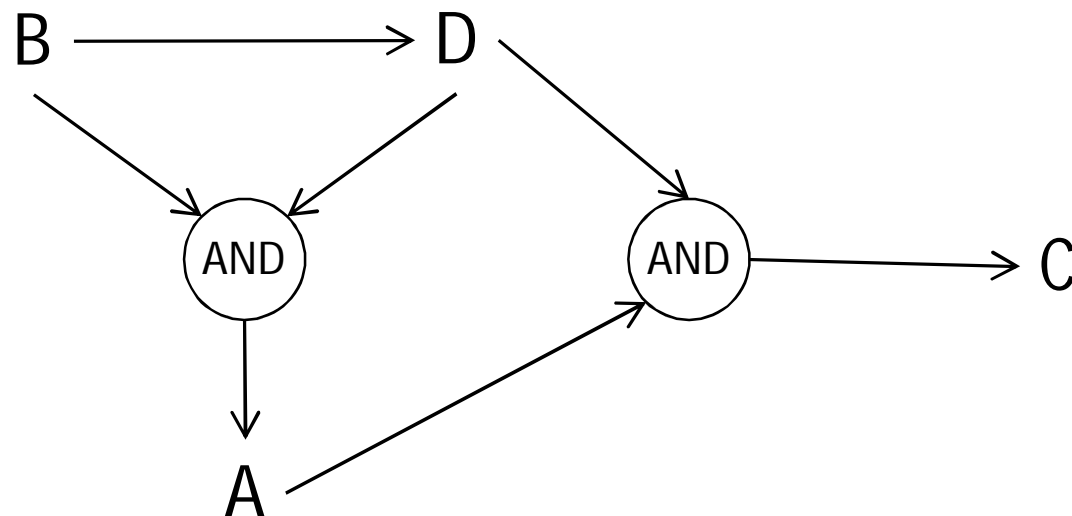
- Given  $A \rightarrow C$ ,  $AC \rightarrow D$ ,  $AD \rightarrow B$
- Can you prove that  $A \rightarrow B$ ?
- Proof
  - Given  $A \rightarrow C$ , we have  $A \rightarrow AC$  (Augmentation)
  - Given  $A \rightarrow AC$  and  $AC \rightarrow D$ , we have  $A \rightarrow D$  (Transitivity)
  - Given  $A \rightarrow D$ , we have  $A \rightarrow AD$  (Augmentation)
  - Given  $A \rightarrow AD$  and  $AD \rightarrow B$ , we have  $A \rightarrow B$  (Transitivity)

# Reasoning with FDs

- Four attributes: A, B, C, D
- Given:  $B \rightarrow D$ ,  $DB \rightarrow A$ ,  $AD \rightarrow C$
- Can you prove  $B \rightarrow C$ ?
- Doable with Armstrong's axioms, but troublesome
- We will discuss a more convenient approach

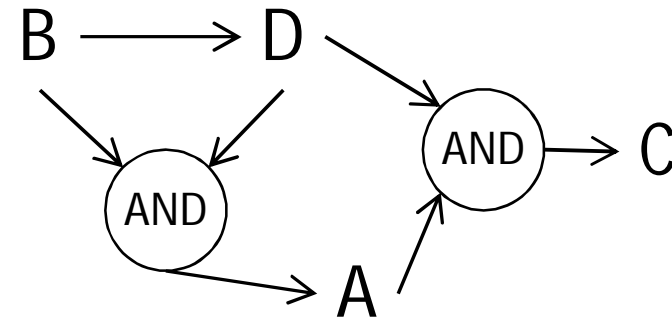
# An Intuitive Solution

- Four attributes: A, B, C, D
- Given:  $B \rightarrow D$ ,  $DB \rightarrow A$ ,  $AD \rightarrow C$
- Can you prove  $B \rightarrow C$ ?



# Steps of the Intuitive Solution

- Four attributes: A, B, C, D
- Given:  $B \rightarrow D$ ,  $DB \rightarrow A$ ,  $AD \rightarrow C$
- Can you prove  $B \rightarrow C$ ?



- 
- First, activate B
    - Activated set = { B }
  - Second, activate whatever B can activate
    - Activated set = { B, D }, since  $B \rightarrow D$
  - Third, use all activated elements to activate more
    - Activated set = { B, D, A }, since  $DB \rightarrow A$
  - Repeat the third step, until no more activation is possible
    - Activated set = { B, D, A, C }, since  $AD \rightarrow C$ ; done

# Questions?





# Classroom Exercise-4

- Given:  $A \rightarrow C$ ,  $C \rightarrow B$ ,  $B \rightarrow D$ ,  $D \rightarrow E$ ,  $E \rightarrow A$
- Can you prove  $C \rightarrow ABE$ ?
- We start with  $\{C\}$
- Since  $C \rightarrow B$ , we have  $\{C, B\}$
- Since  $B \rightarrow D$ , we have  $\{C, B, D\}$
- Since  $D \rightarrow E$ , we have  $\{C, B, D, E\}$
- Since  $E \rightarrow A$ , we have  $\{C, B, D, E, A\}$
- $A, B, E$  are all in the set, so  $C \rightarrow ABE$  holds

# Classroom Exercise-5

- Given:  $C \rightarrow D$ ,  $AD \rightarrow E$ ,  $BC \rightarrow E$ ,  $E \rightarrow A$ ,  $D \rightarrow B$
- Can you prove  $C \rightarrow A$ ?
- We start with  $\{C\}$
- Since  $C \rightarrow D$ , we have  $\{C, D\}$
- Since  $D \rightarrow B$ , we have  $\{C, D, B\}$
- Since  $BC \rightarrow E$ , we have  $\{C, D, B, E\}$
- Since  $E \rightarrow A$ , we have  $\{C, D, B, E, A\}$
- $A$  is in the set, so  $C \rightarrow A$  holds

# Classroom Exercise-6

- Given:  $C \rightarrow D$ ,  $AD \rightarrow E$ ,  $BC \rightarrow E$ ,  $E \rightarrow A$ ,  $D \rightarrow B$ ,  $B \rightarrow F$
- Can you prove  $D \rightarrow C$ ?
- We start with  $\{D\}$
- Since  $D \rightarrow B$ , we have  $\{D, B\}$
- Since  $B \rightarrow F$ , we have  $\{D, B, F\}$
- What else?
- No more.
- $\{D, B, F\}$  is all what can be decided by D
- We refer to  $\{D, B, F\}$  as the **closure** of D

# Classroom Exercise-7

- A table with five attributes A, B, C, D, E, F
- $AB \rightarrow C$ ,  $AD \rightarrow E$ ,  $B \rightarrow D$ ,  $AF \rightarrow B$
- Compute the following closures
  - $\{B, C\}^+ =$
  - $\{A, B\}^+ =$
  - $\{A, F\}^+ =$

# Classroom Exercise-7: Solution



- A table with five attributes A, B, C, D, E, F
- $AB \rightarrow C$ ,  $AD \rightarrow E$ ,  $B \rightarrow D$ ,  $AF \rightarrow B$
- Compute the following closures
  - $\{B, C\}^+ = \{B, C, D\}$
  - $\{A, B\}^+ = \{A, B, C, D, E\}$
  - $\{A, F\}^+ = \{A, F, B, C, D, E\}$

# Closure & FD

- To prove that  $X \rightarrow Y$  holds, we only need to show that  $\{X\}^+$  contains  $Y$
- $AB \rightarrow C, AD \rightarrow E, B \rightarrow D, AF \rightarrow B$
- Prove that  $AF \rightarrow D$
- $\{AF\}^+ = \{AFBCDE\}$ , which contains  $D$
- Therefore,  $AF \rightarrow D$  holds

# Closure & FD

- To prove that  $X \rightarrow Y$  **does not** hold, we only need to show that  $\{X\}^+$  **does not** contain  $Y$
- $AB \rightarrow C, AD \rightarrow E, B \rightarrow D, AF \rightarrow B$
- Prove that  $AD \rightarrow F$  does not hold
- $\{AD\}^+ = \{ADE\}$ , which does not contain  $F$
- Therefore,  $AF \rightarrow F$  does not hold

# Questions?





# Superkeys of a Table

Name	<u>CPR</u>	Postal	Address
Alice	1234	939450	Jurong East
Bob	5678	234122	Pasir Ris
Cathy	3576	420923	Yishun

- Definition: A set of attributes in a table that decides all other attributes
- Example:
  - {CPR} is a superkey
  - Since  $CPR \rightarrow Name, Postal, Address$
  - {CPR, Name} is a superkey
  - Since  $\{CPR, Name\} \rightarrow Postal, Address$

# Keys of a Table

Name	<u>CPR</u>	Postal	Address
Alice	1234	939450	Jurong East
Bob	5678	234122	Pasir Ris
Cathy	3576	420923	Yishun

- Definition: A superkey that is **minimal**
- i.e., if we remove any attribute from the superkey, it will not be a superkey anymore
- Example:
  - {CPR} is a superkey
  - Since  $\text{CPR} \rightarrow \text{Name, Postal, Address}$
  - {CPR, Name} is a superkey
  - Since  $\{\text{CPR, Name}\} \rightarrow \text{Postal, Address}$
  - CPR is a key, but {CPR, Name} is not a key

# Keys of a Table

Name	<u>CPR</u>	Postal	Address
Alice	1234	939450	Jurong East
Bob	5678	234122	Pasir Ris
Cathy	3576	420923	Yishun

- Note: Not to be confused with the keys of entity sets

# Candidate Keys

Name	CPR	StudentID	Postal	Address
Alice	1234	1	939450	Jurong East
Bob	5678	2	234122	Pasir Ris
Cathy	3576	3	420923	Yishun

- A table may have multiple keys
- In that case, each key is referred to as a candidate key
- Example:
  - {CPR} is a key
  - Since  $\text{CPR} \rightarrow \text{Name, StudentID, Postal, Address}$
  - {StudentID} is a key
  - Since  $\text{StudentID} \rightarrow \text{Name, CPR, Postal, Address}$
  - Both {CPR} and {StudentID} are candidate keys

# Primary and Secondary Keys



Name	CPR	StudentID	Postal	Address
Alice	1234	1	939450	Jurong East
Bob	5678	2	234122	Pasir Ris
Cathy	3576	3	420923	Yishun

- When a table have multiple keys...
- We choose one of them as the primary key
- The others are referred to as secondary keys
- Example:
  - {CPR} is a key
  - {StudentID} is a key
  - If we choose {CPR} as the primary key
  - Then {StudentID} is the secondary key

# Summary

- ER Diagram → Relational Schema



- Functional Dependency (FD)



- Keys



## Study-at-Home Slides

Finding the (Candidate) Keys

Will be in the syllabus  
of Exercise, self-study,  
and Exam

# Finding the (Candidate) Keys



- To check whether a table is “good”, we need to find the keys of the table
- How do we do that?
- Use functional dependencies (FDs) and closures

# Example

- Definition of a Key: A minimal set of attributes that decides all other attributes
- A table  $R(A, B, C)$
- FDs given:  $A \rightarrow B, B \rightarrow C$
- Is A a key?
- $\{A\}^+ = \{ABC\}$ , i.e.,  $A \rightarrow ABC$ . Yes.
- Is B a key?
- $\{B\}^+ = \{BC\}$ , i.e., B does not decide A. No.
- Is C a key?
- $\{C\}^+ = \{C\}$ . No.
- Is AB a key?
- No, since A is already a key.
- What about BC, AC, ABC?



# Example

- Definition of a Key: A minimal set of attributes that decides all other attributes
- A table  $R(A, B, C)$
- FDs given:  $A \rightarrow B$
- Is A a key?
- $\{A\}^+ = \{A, B\}$ . No.
- Is B or C a key?
- $\{B\}^+ = \{B\}$ .  $\{C\}^+ = \{C\}$ . No.
- Is AB or BC a key?
- $\{AB\}^+ = \{AB\}$ .  $\{BC\}^+ = \{BC\}$ . No.
- Is AC a key?
- $\{AC\}^+ = \{ABC\}$ . Yes.
- Is ABC a key?

# Finding the Keys: Algorithm

- Check all possible combinations of attributes in the table
  - Example: A, B, C, AB, BC, AC, ABC
- For each combination, compute its closure
  - Example:  $\{A\}^+ = \dots$ ,  $\{B\}^+ = \dots$ ,  $\{C\}^+ = \dots$ , ...
- If a closure contains all attributes, then the combination might be a key (or superkey)
  - Example:  $\{A\}^+ = \{ABC\}$
- Make sure that you select only keys
  - Example:  $\{A\}^+ = \{ABC\}$ ,  $\{AB\}^+ = \{ABC\}$ , don't select AB

# Example

- A table  $R(A, B, C, D)$
- $AB \rightarrow C, AD \rightarrow B, B \rightarrow D$
- First, enumerate all attribute combinations:
  - $\{A\}, \{B\}, \{C\}, \{D\}$
  - $\{AB\}, \{AC\}, \{AD\}, \{BC\}, \{BD\}, \{CD\}$
  - $\{ABC\}, \{ABD\}, \{ACD\}, \{BCD\}$
  - $\{ABCD\}$

# Example

- A table  $R(A, B, C, D)$
- $AB \rightarrow C, AD \rightarrow B, B \rightarrow D$
- Second, compute the closures:
  - $\{A\}^+ = \{A\}, \{B\}^+ = \{BD\}, \{C\}^+ = \{C\}, \{D\}^+ = \{D\}$
  - $\{AB\}^+ = \{ABCD\}, \{AC\}^+ = \{AC\}, \{AD\}^+ = \{ABCD\}$
  - $\{BC\}^+ = \{BCD\}, \{BD\}^+ = \{BD\}, \{CD\}^+ = \{CD\}$
  - $\{ABC\}^+ = \{ABD\}^+ = \{ACD\}^+ = \{ABCD\}$
  - $\{BCD\}^+ = \{BCD\}$
  - $\{ABCD\}^+ = \{ABCD\}$
- Finally, output the keys

# A Small Trick

- Always check small combinations first
- A table  $R(A, B, C, D)$
- $A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A$
- Compute the closures:
  - $\{A\}^+ = \{ABCD\}, \{B\}^+ = \{ABCD\}, \{C\}^+ = \{ABCD\}, \{D\}^+ = \{ABCD\}$
  - No need to check others
  - The others are all superkeys but not keys
- Keys:  $\{A\}, \{B\}, \{C\}, \{D\}$

# Another Small Trick

- A table  $R(A, B, C, D)$
- $AB \rightarrow C, AD \rightarrow B, B \rightarrow D$
- Notice that A does not appear in the right hand side of any functional dependencies
- In that case, A must be in every key
- Keys of R: AB, AD (From the previous slide)
- In general, if an attribute that does not appear in the right hand side of any FD, then it must be in every key

# Exercise (Find the Keys)

- A table  $R(A, B, C, D)$
- $A \rightarrow B, A \rightarrow C, C \rightarrow D$
- A must be in every key
- Compute the closures:
  - $\{A\}^+ = \{ABCD\}$
  - No need to check others
- Keys:  $\{A\}$

# Exercise (Find the Keys)

- A table  $R(A, B, C, D, E)$
- $AB \rightarrow C, C \rightarrow B, BC \rightarrow D, CD \rightarrow E$
- A must be in every key
- Compute the closures:
  - $\{A\}^+ = \{A\}$
  - $\{AB\}^+ = \{ABCDE\}$
  - $\{AC\}^+ = \{ACBDE\}$
  - $\{AD\}^+ = \{AD\}, \{AE\}^+ = \{AE\}$
  - $\{ADE\}^+ = \{ADE\}$



# Exercise (Find the Keys)

- A table  $R(A, B, C, D, E, F)$
- $AB \rightarrow C, C \rightarrow B, CBE \rightarrow D, D \rightarrow EF$
- A must be in every key
- Compute the closures:
  - $\{A\}^+ = \{A\}$
  - $\{AB\}^+ = \{ABC\}$
  - $\{AC\}^+ = \{ACB\}$
  - $\{AD\}^+ = \{ADEF\}$
  - $\{AE\}^+ = \{AE\}, \{AF\}^+ = \{AF\}$
  - $\{ABC\}^+ = \{ABC\}$
  - $\{ABD\}^+ = \{ABE\}^+ = \{ACD\}^+ = \{ACE\}^+ = \{ABCDEF\}$
  - $\{ADE\}^+ = \{ADEF\}$

# Questions ??

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Thank You !