41. Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers. What kind of proof did you use?

$$\frac{a_1 + a_2 + \ldots + a_n}{n} = \mu$$

$$a_1 + a_2 + \ldots + a_n = n \cdot \mu$$

Lets say that every number is less than μ which is the average then we calculate it like above. now we get that that when you sum n numbers that are all less than μ is equal to n times μ which is not possible. This this has been poven with proof by contradiction.

43. Prove that if n is an integer, these four statements are equivalent: (*i*) n is even, (*ii*) $n+1$ is odd, (*iii*) $3n+1$ is odd, (*iv*) $3n$ is even.

$i) n = 2k$

$ii) n = 2k + 1$

$iii) 3\ (2k + 1) + 1 = 6k + 4 \Rightarrow n \text{ is even};$

$iiii) 3(2k) = 6k$

check solutions **in** *book*

43. Find counterexamples to each of these statements about congruences.

a) If $ac \equiv bc \pmod{m}$, where a, b, c, and m are integers with $m \geq 2$, then $a \equiv b \pmod{m}$.

43. a) Let $m = c = 2$, $a = 0$, and $b = 1$.

tried to avoid using zero but that's what you are supposed to do

44. Show that if n is an integer then $n^2 \equiv 0$ or $1 \pmod 4$.

if n is even

$n = 2k \Rightarrow n^2 = (2k)^2 = 4k^2$

if n is odd

$n = 2k + 1 \Rightarrow n^2 = (2k + 1)^2 = 4k^2 + 4k + 1 = 4\left(k^2 = k\right) + 1$