Exercise 1 :

a. Let π be the interpretation that assigns the following truth values:

$$\pi(a) = \textit{true}, \pi(b) = \textit{false}, \pi(c) = \textit{false}, \pi(d) = \textit{true}$$

Determine the truth values for the following propositions:

$$\begin{array}{l} \neg a \rightarrow b \\ (\neg b \vee c) \wedge (d \rightarrow a) \\ (a \rightarrow c) \rightarrow c \end{array}$$

b. For the following propositions, find an interpretation in which they are true:

$$\begin{array}{l} (a \vee (a \rightarrow c)) \rightarrow b \\ (a \wedge (\neg b \vee c)) \wedge (a \rightarrow (c \rightarrow b)) \end{array}$$

Exercise 2 :

Consider the experiment of flipping a coin, and if it lands heads, rolling a four-sided die, and if it lands tails, rolling a six-sided die.

(i) Suppose that the coin and both dice are fair. As we are interested only in the number rolled by the die, and the possible worlds $\mathcal{S}_A$ for the experiment could thus be the numbers from 1 to 6. Another set of possible worlds could be $\mathcal{S}_B = \{t1, \ldots, t6, h1, \ldots, h4\}$, with for example $t2$ meaning "tails and a roll of 2" and $h4$ meaning "heads and a roll of 4." Choose either $\mathcal{S}_A$ or $\mathcal{S}_B$ and associate probabilities with it. According to your chosen set of possible worlds and probability distribution, what is the probability of rolling either 3 or 5.

(ii) Let $\mathcal{S}_B$ be defined as above, but with a loaded coin and loaded dice. A probability distribution is given in Table 1. What is the probability that the loaded coin lands "tails"? What is the conditional probability of rolling a 4, given that the coin lands tails? Which of the loaded dice has the highest chance of rolling 4 or more?

$t1$	$\frac{5}{18}$	$t6$	$\frac{1}{18}$
$t2$	$\frac{1}{9}$	$h1$	$\frac{1}{24}$
$t3$	$\frac{1}{9}$	$h2$	$\frac{1}{24}$
$t4$	$\frac{1}{18}$	$h3$	$\frac{1}{8}$
$t5$	$\frac{1}{18}$	$h4$	$\frac{1}{8}$

Table 1: Probabilities for $\mathcal{S}_B$.

Exercise 3 :

Calculate $P(A)$, $P(B)$, $P(A|B)$, and $P(B|A)$ from the joint probability distribution $P(A, B)$ given in Table 2.

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	b_1	b_2	b_3
a_1	0.05	0.10	0.05
a_2	0.15	0.00	0.25
a_3	0.10	0.20	0.10

Table 2: The joint probability distribution for $P(A, B)$.

	b_1	b_2	b_3
a_1	0.1	0.7	0.6
a_2	0.9	0.3	0.4

Table 3: The conditional probability distribution $P(A|B)$.

Exercise 4 :

Consider the binary variable A and the ternary variable B. Assume that B has the probability distribution $P(B) = (0.1, 0.5, 0.4)$ and that A has the conditional probability distribution given in Table 3.

Questions:

(i) Verify that Table 3 specifies a valid conditional probability distribution.

(ii) Calculate $P(B|A)$. *Hint:* Consider Bayes rule illustrated by the temperature-sensor example that we discussed in the lecture

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Exercise 5 :

Table 4 describes a test T for an event A. The number 0.01 is the frequency of *false negatives*, and the number 0.001 is the frequency of *false positives*.

(*i*) The police can order a blood test on drivers under the suspicion of having consumed too much alcohol. The test has the above characteristics. Experience says that 20% of the drivers under suspicion do in fact drive with too much alcohol in their blood. A suspicious driver has a positive blood test. What is the probability that the driver is guilty of driving under the influence of alcohol?

(*ii*) The police block a road, take blood samples of all drivers, and use the same test. It is estimated that one out of 1,000 drivers have too much alcohol in their blood. A driver has a positive test result. What is the probability that the driver is guilty of driving under the influence of alcohol?

Hint: Structure-wise this exercise is closely connected to the temperature-sensor example that we discussed in the lecture.

	$A = \textit{yes}$	$A = \textit{no}$
$T = \textit{yes}$	0.99	0.001
$T = \textit{no}$	0.01	0.999

Table 4: Table for Exercise 5. Conditional probabilities $P(T \mid A)$ characterizing test T for A.

	b_1	b_2
a_1	$(0.006, 0.054)$	$(0.048, 0.432)$
a_2	$(0.014, 0.126)$	$(0.032, 0.288)$

Table 5: $P(A, B, C)$ for Exercise 7.

Exercise 6 :

A routine DNA test is performed on a person (this exercise is set in the not too distant future!). The test T gives a positive result for a rare genetic mutation M linked to Alzheimer's disease. The mutation is present in only 1 in a million people. The test is 99.99% accurate, i.e. it will give a wrong result in 1 out of 10000 tests performed. Should the person be worried, i.e., what is the probability that the person has the mutation given that the test showed a positive result?

Hint: This is partly a modeling exercises and partly a calculation exercise. First you need to formalize the problem:

- What are the relevant variables and what states do they have?
- Based on the description above, what probability distributions can you infer for the variables?

Based on this formalization, you need to find the rules required to answer the question about the probability of a mutation given a positive test result.

Exercise 7 :

In Table 5, a joint probability table for the binary variables A, B, and C is given.

- Calculate $P(B, C)$ and $P(B)$.
- Are A and C independent given B?

Exercise 8 :

In a university far, far away… every year a lot of students take the Artificial Intelligence (AI) lecture. From years and years of experience it is known that there are three (distinct) types of students at the university:

- *H(ard-working)*, who solve all exercises and qualify for the exam,
- *L(azy)*, who get enough points to qualify for the exam, but then stop working on the exercises,
- *U(nfortunate)*, who do not qualify for the exam.

Typically, 90% of the H-students pass the AI exam with a good grade, but only half of the L-students get a good grade. Every student that qualifies for an exam takes it.

a) Last year, 7 out of 10 students belonged to the L category and there were 10% U-students. What was the probability that a student who got a good grade in the AI exam was of $Type\ H$?

b) In another year 400 students took the AI course. We know that 36 H-students got a good grade, and that 80% of those who got a good grade were L-students. What's the probability that a student in that year was a U-student?

c) Assume there exists a second lecture at the university, namely Machine Learning (ML). This year, we expect 35% of the students to be working hard and 27% of the students to be lazy. Overall, 40% of the students get a good grade in ML and 45% of the students get a good grade in AI. Assume the latter two events to be independent. Furthermore, 80% of the students that get a good grade in ML are H-students, and 5% of the students with a good grade in both ML and AI are lazy.

   Are the events of getting a good grade in AI and getting a good grade in ML conditionally independent given the type of student? Justify your answer by checking if the condition from the definition of conditional independence holds.

Exercise 9 :

Solve Exercise 1(a-d) in PM.
