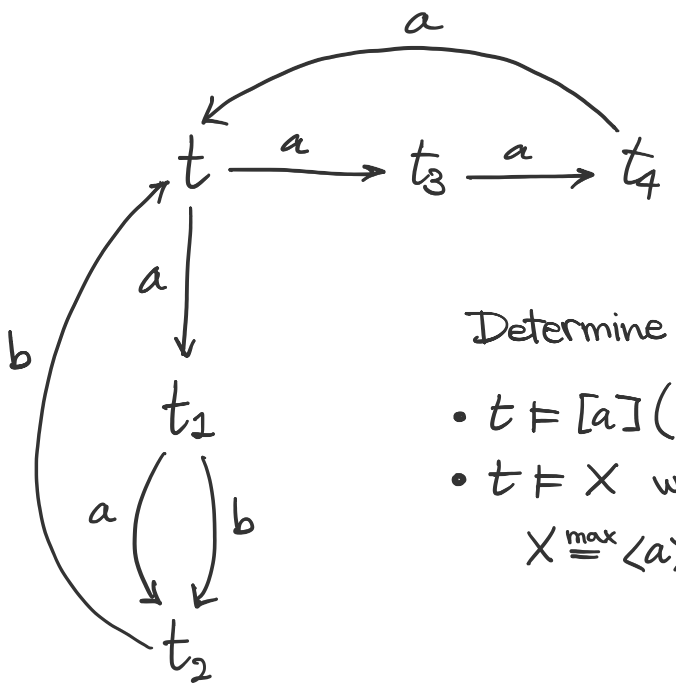
Example exercise 4

Tuesday, 28 April 2020 17.26



Determine whether:

- $t \models [a](\langle b \rangle tt \vee [a][b]ff)$
- $t \models X$ where
  $X \overset{max}{=} \langle a \rangle tt \wedge [Act]X$

Solution: To solve the exercise we can equivalently choose to use the denotational semantics, or play a game.

- DENOTATIONAL SEMANTICS:

Let $\psi = [a](\langle b \rangle tt \vee [a][b]ff)$. It's denotational semantics is given by

$$
\begin{aligned}
[\![\psi]\!] &= [\cdot a \cdot](\langle \cdot b \cdot \rangle Proc \cup [\cdot a \cdot][\cdot b \cdot]\emptyset) \\
&= [\cdot a \cdot](\{t_1, t_2\} \cup [\cdot a \cdot][\cdot b \cdot]\emptyset) \\
&= [\cdot a \cdot](\{t_1, t_2\} \cup [\cdot a \cdot](\{t, t_3, t_4\})) \\
&= [\cdot a \cdot](\{t_1, t_2\} \cup \{t_3, t_4\}) \\
&= [\cdot a \cdot](\{t_1, t_2, t_3, t_4\}) \\
&= \{t, t_1, t_2, t_3, t_4\}.
\end{aligned}
$$

Since $t \in [\![\psi]\!]$, then $t \models \psi$.

- GAME CHARACTERIZATION:

We show that $t \not\models X$ by providing a universal winning strategy for attacker, starting from the configuration (t, X).

$$
\begin{aligned}
(t, X) &\longrightarrow (t, \langle a \rangle tt \wedge [Act]X) \\
&\xrightarrow{a} (t, [Act]X) \\
&\xrightarrow{a} (t_1, X) \\
&\longrightarrow (t_1, \langle a \rangle tt \wedge [Act]X) \\
&\xrightarrow{a} (t_1, [Act]X) \\
&\xrightarrow{a} (t_2, X) \\
&\longrightarrow (t_2, \langle a \rangle tt \wedge [Act]X) \\
&\xrightarrow{a} (t_2, \langle a \rangle tt)
\end{aligned}
$$

But defender has no available transitions. So attacker wins.

This trivially a universal strategy because defender never gets a chance to play a move.