Tutorial 10

Exercise 1

	$(A, y=0)$	$(X, y=0)$	$(U, y=0)$	$(U', y=0)$
$(A, y=0)$	$\sim / \sim_u$	—	—	—
$(X, y=0)$	① $\not\sim / \sim_u$	$\sim / \sim_u$	—	—
$(U, y=0)$	② $\not\sim / \not\sim_u$	④ $\not\sim / \not\sim_u$	$\sim / \sim_u$	
$(U', y=0)$	③ $\not\sim / \not\sim_u$	⑤ $\not\sim / \not\sim_u$	⑥ $\sim / \sim_u$	$\sim / \sim_u$

① $\not\sim$: $(X, y=0) \xrightarrow{2} (X, y=2) \xrightarrow{a}$ but

$(A, y=0) \xrightarrow{2} (A, y=2) \not\xrightarrow{}$

$\sim_u$:

$R = \{\langle (A, y=d), (X, y=e) \rangle \mid d \leq 1 \Leftrightarrow e \leq 2\} \cup$

$\{\langle (B, y=d), (Y, y=e) \rangle \mid d \leq 1 \Leftrightarrow e \leq 2\} \cup$

$\{\langle (C, y=d), (Z, y=e) \rangle \mid d, e \in \mathbb{R}_{\geq 0}\}$

is a time abstracted bisimulation.

② $\not\sim_u$ (and hence $\not\sim$)

$(A, y=0) \xrightarrow{3} (A, y=3) \overset{a}{\not\rightarrow}$ but

$(U, y=0) \xrightarrow{d} (U, y=d) \xrightarrow{a}$ for all d.

④ Similar

(6) $\sim$ (and hence $\sim_u$)

$$R = \{ \langle (U, y=d), (U', y'=d) \rangle \mid d \in \mathbb{R}_{\geq 0} \} \cup$$
$$\{ \langle (V, y=d), (W, y'=d) \rangle \mid d > 2 \} \cup$$
$$\{ \langle (V, y=d), (V', y'=d) \rangle \mid d \in \mathbb{R}_{\geq 0} \} \cup$$
$$\{ \langle (W, y=d), (W', y'=d) \rangle \mid d \in \mathbb{R}_{\geq 0} \}.$$

is a timed bisimulation.

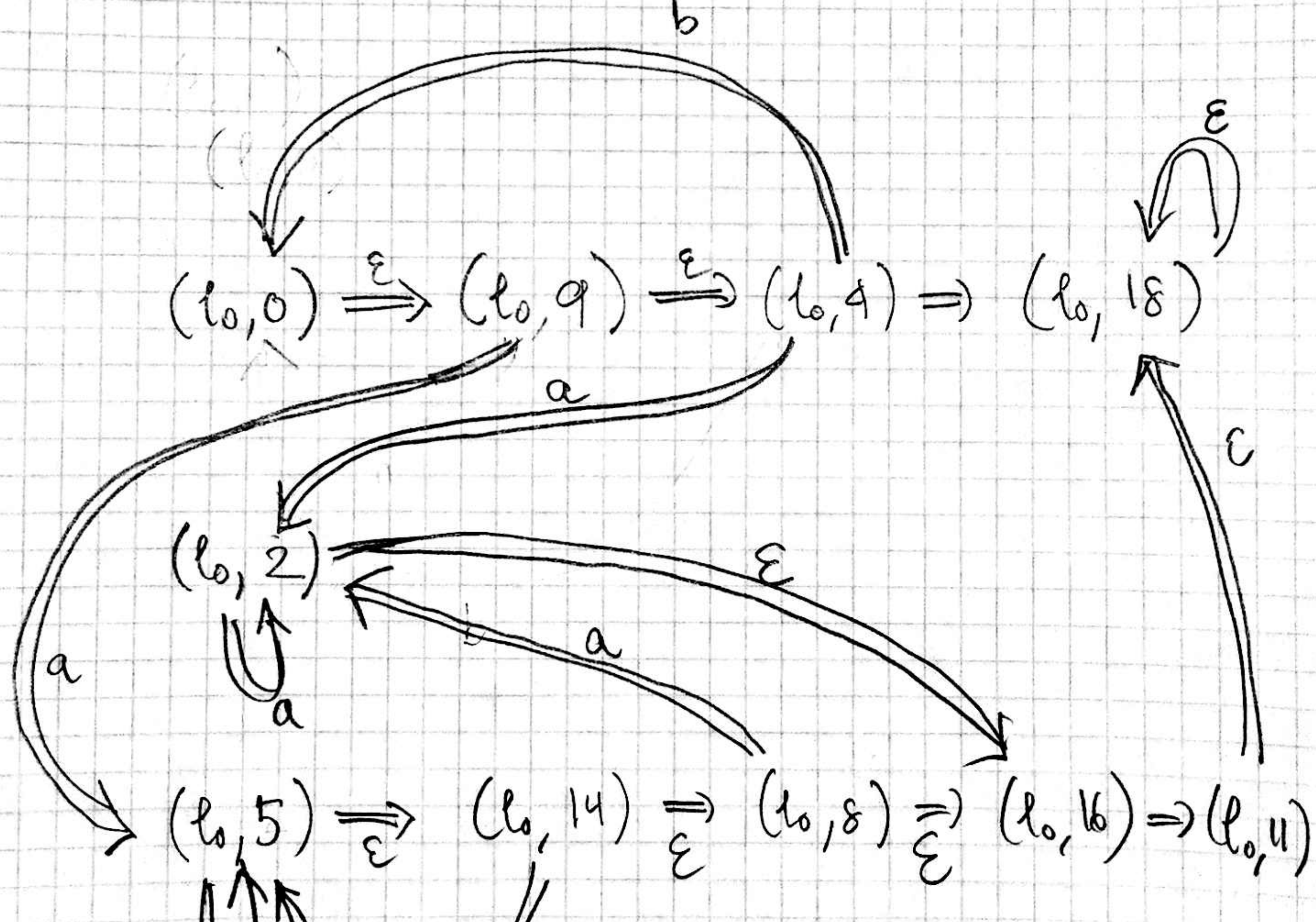
(5) Follows from (6) and (4)

(3) Follows from (6) and (2).

Exercise 4

We number the regions given the automaton considered has 2 clocks both with 1 being maximum constant as in example 11 in the book (page 212)

The following gives the reachable part of the region graph (we omit transitive & reflexive closure of $\xrightarrow{\varepsilon}$):



$(l_0, x=0.7, y=0.61)$ is reachable as $(0.2, 0.61) \in$ Region 14.

$(l_0, x=0.2, y=0.61)$ is NOT reachable as $(0.2, 0.61)$ is in region 15 which is NOT reachable.