

**Exercise 1\***

Draw a graphical representation of the complete lattice  $(2^{\{a,b,c\}}, \subseteq)$  and compute supremum and infimum of the following sets:

1.  $\sqcap\{\{a\}, \{b\}\} = ?$
2.  $\sqcup\{\{a\}, \{b\}\} = ?$
3.  $\sqcap\{\{a\}, \{a, b\}, \{a, c\}\} = ?$
4.  $\sqcup\{\{a\}, \{a, b\}, \{a, c\}\} = ?$
5.  $\sqcap\{\{a\}, \{b\}, \{c\}\} = ?$
6.  $\sqcup\{\{a\}, \{b\}, \{c\}\} = ?$
7.  $\sqcap\{\{a\}, \{a, b\}, \{b\}, \emptyset\} = ?$
8.  $\sqcup\{\{a\}, \{a, b\}, \{b\}, \emptyset\} = ?$

## Exercise 2

Prove that for any partially ordered set  $(D, \sqsubseteq)$  and any  $X \subseteq D$ , if supremum of  $X$  ( $\sqcup X$ ) and infimum of  $X$  ( $\sqcap X$ ) exist then they are uniquely defined. (Hint: use the definition of supremum and infimum and antisymmetry of  $\sqsubseteq$ .)

## Exercise 3

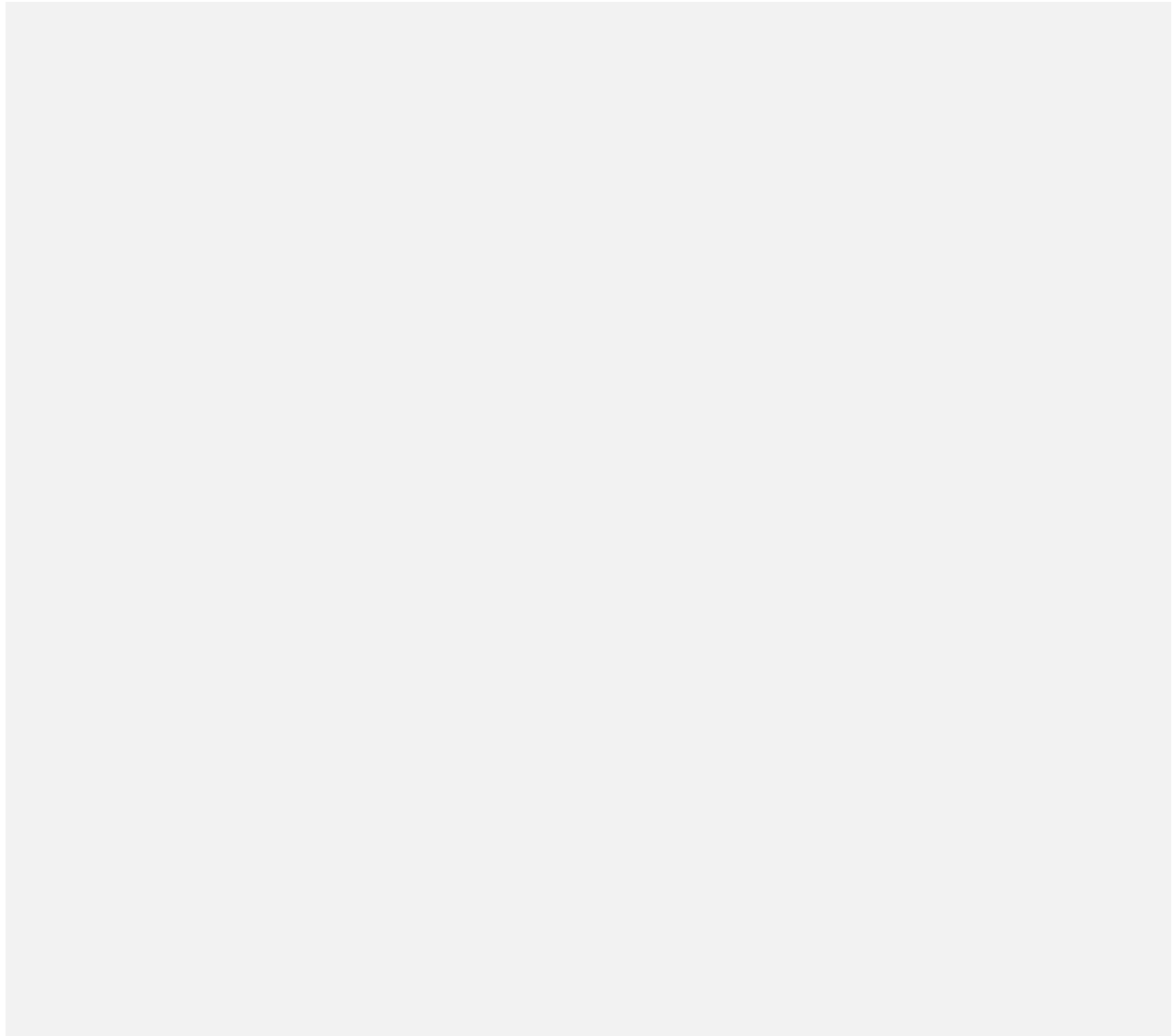
Let  $(D, \sqsubseteq)$  be a complete lattice. What are  $\sqcup \emptyset$  and  $\sqcap \emptyset$  equal to?

## Exercise 4\*

Consider the complete lattice  $(2^{\{a,b,c\}}, \subseteq)$ . Define a function  $f : 2^{\{a,b,c\}} \rightarrow 2^{\{a,b,c\}}$  such that  $f$  is monotonic.

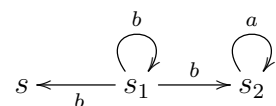
- Compute the greatest fixed point by using directly the Tarski's fixed point theorem.

- Compute the least fixed point by using the Tarski's fixed point theorem for finite lattices (i.e. by starting from  $\perp$  and by applying repeatedly the function  $f$  until the fixed point is reached).



### Exercise 5

Consider the following labelled transition system.



Compute for which sets of states  $\llbracket X \rrbracket \subseteq \{s, s_1, s_2\}$  the following formulae are true.

- $X = \langle a \rangle t \vee [b]X$
- $X = \langle a \rangle t \vee ([b]X \wedge \langle b \rangle t)$

