Exercise 1*

Draw a graphical representation of the complete lattice $(2^{\{a,b,c\}}, \subseteq)$ and compute supremum and infimum of the following sets:

1. $\sqcap\{\{a\},\{b\}\} = ?$
2. $\sqcup\{\{a\},\{b\}\} = ?$
3. $\sqcap\{\{a\},\{a,b\},\{a,c\}\} = ?$
4. $\sqcup\{\{a\},\{a,b\},\{a,c\}\} = ?$
5. $\sqcap\{\{a\},\{b\},\{c\}\} = ?$
6. $\sqcup\{\{a\},\{b\},\{c\}\} = ?$
7. $\sqcap\{\{a\},\{a,b\},\{b\},\emptyset\} = ?$
8. $\sqcup\{\{a\},\{a,b\},\{b\},\emptyset\} = ?$

Exercise 2

Prove that for any partially ordered set $(D, \sqsubseteq)$ and any $X \subseteq D$, if supremum of X ($\sqcup X$) and infimum of X ($\sqcap X$) exist then they are uniquely defined. (Hint: use the definition of supremum and infimum and antisymmetry of $\sqsubseteq$.)

Exercise 3

Let $(D, \sqsubseteq)$ be a complete lattice. What are $\sqcup\emptyset$ and $\sqcap\emptyset$ equal to?

Exercise 4*

Consider the complete lattice $(2^{\{a,b,c\}}, \subseteq)$. Define a function $f : 2^{\{a,b,c\}} \rightarrow 2^{\{a,b,c\}}$ such that f is monotonic.

- Compute the greatest fixed point by using directly the Tarski's fixed point theorem.

- Compute the least fixed point by using the Tarski's fixed point theorem for finite lattices (i.e. by starting from $\perp$ and by applying repeatedly the function f until the fixed point is reached).

Exercise 5

Consider the following labelled transition system.

$$\begin{tikzcd} s & s_1 \arrow[l, "b"'] \arrow[loop above, "b"] \arrow[r, "b"] & s_2 \arrow[loop above, "a"] \end{tikzcd}$$

Compute for which sets of states $[\![X]\!] \subseteq \{s, s_1, s_2\}$ the following formulae are true.

- $X = \langle a \rangle tt \vee [b]X$
- $X = \langle a \rangle tt \vee ([b]X \wedge \langle b \rangle tt)$