

Programming Paradigms 2023

Session 11: Monads

Problems for solving and discussing

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Problems that we will definitely talk about

1. (*Work in pairs – 15 minutes*)

An influencer on Instagram has become famous for his frequent updates about the List monad. Yesterday the influencer presented a new function called `fourfirst`:

```
fourfirst xs = do
    x <- xs
    return (4,x)
```

”This function takes a list and gives us a pair $(4, x)$ where x is the first element of the list”, the influencer concluded. Soon after a heated discussion had begun among 4 million teenage followers: Was the influencer right? It is your task to find out. Explain, using the definition of the List monad *but without executing this piece of code*, what the code actually does and how it does it.

2. (*Work in pairs – 30 minutes*)

Here is a piece of Haskell code.

```
data W x = Bingo x deriving Show

instance Functor W where
    fmap f (Bingo x) = Bingo (f x)

instance Monad W where
    return x = Bingo x
    Bingo x >>= f = f x
```

- For this to make sense, a definition of `W` as an applicative functor is missing. Write such a definition.
- Use do-notation to define a function `wrapadd :: Num b => b -> W b -> W b` which satisfies that

`wrapadd x (Bingo y) = Bingo (x+y)`

Warning! Do not use pattern matching inside do-blocks. Only use monadic notation.

- Use do-notation to define a function `h` which satisfies that

`h (Bingo x) (Bingo y) = Bingo (x*y)`

And find its type without asking Haskell. **Warning!** Do not use pattern matching inside do-blocks. Only use monadic notation.

3. (*Everyone at the table – 30 minutes*)

Consider trees whose elements are values of some type in the type class `Ord`. The type `Tree a` is defined by

```
data Tree a = Leaf a | Node (Tree a) (Tree a)
```

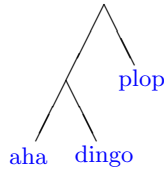


Figure 1: The ordered mytree1

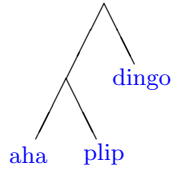


Figure 2: The unordered mytree2

Use a *monad* to write a function `minorder` that takes such a tree and checks if the numbers in the structure are in non-decreasing order when read from left to right. If it is, the function should return the smallest number in the tree, otherwise it should return `Nothing`. The tree `mytree` shown in Figure 1 is ordered, so `minorder mytree1` should return `Just "aha"`, but the tree in Figure 2 is not ordered, so `minorder mytree2` should return `Nothing`.

First define another function `minmax` that finds the minimal and the maximal element in a tree under the assumption that the tree is ordered. Then use `minmax` to define `minorder`.

Hint: Which monad should you use?

Warning! Do not use pattern matching inside `do`-blocks. Only use monadic notation.

More problems to solve at your own pace

a) Define a `foldM` function whose type should be

```
foldM :: Monad m => (t1 -> t2 -> m t2) -> [t1] -> t2 -> m t2
```

The idea is that the function works like `foldl` but folds over a monad.

Here is an example that shows what will happen if we fold over the `IO` monad. If we let

```
dingo x = do
    putStrLn (show x)
    return x
```

then we should see the following behaviour.

```
*Main> foldM (\x y -> (dingo (x+y))) [1,2,3,4] 0
1
3
6
10
```