An Introduction to Quantum Computing

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Quantum Background
Exercise 1.1

The system:

- We have a 2-state quantum system with basis states $|0\rangle$ and $|1\rangle$, and joint state:

$$|\psi\rangle = \alpha\,|0\rangle \;+\; \beta\,|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1,\ \alpha, \beta \in \mathbb{C} \tag{1}$$

  where $|\alpha|^2$ is the probability for measuring state $|0\rangle$, and $|\beta|^2$ is the probability for measuring state $|1\rangle$.
- In vector form the states are $|0\rangle = [1, 0]^{\mathsf{T}}$ and $|1\rangle = [0, 1]^{\mathsf{T}}$.
- We define two states as:

$$|+\rangle \;\equiv\; \frac{1}{\sqrt{2}}\,|0\rangle \;+\; \frac{1}{\sqrt{2}}\,|1\rangle \tag{2}$$

$$|-\rangle \;\equiv\; \frac{1}{\sqrt{2}}\,|0\rangle \;-\; \frac{1}{\sqrt{2}}\,|1\rangle \tag{3}$$

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Questions:

1. What are α and β in the system described in (1) above when $\alpha = \beta$?
2. What are $|+\rangle + |-\rangle$ and $|+\rangle - |-\rangle$ in (2)–(3) expressed by $|0\rangle$ and $|1\rangle$ in the general case of α and β?
3. And finally, what is then $|\psi\rangle$ expressed by $|0\rangle$ and $|1\rangle$?
4. How can the arbitrary state $|\psi\rangle$ in (1) be expressed by α, β, $|+\rangle$, and $|-\rangle$?