

An Introduction to Quantum Computing

v. 1.0

February 12, 2024

Torben Larsen, tol@cs.aau.dk
Computer Science

Rafal Wisniewski, raf@es.aau.dk
Electronic Systems

Aalborg University, Denmark



AALBORG UNIVERSITY
DENMARK



1 Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

Course Information

Preliminary Course Information . . .

- ▶ In Denmark only few master courses exist – several mainly by new combinations of existing courses from e.g. mathematics, physics, and chemistry with few dedicated quantum courses. Some started with single master courses (as we do) a few years ago as we do now.
- ▶ This course is an introduction to the field of Quantum Computing that provides you with an overview and the necessary steps and their details to make and implement quantum algorithms and software.
- ▶ You will all have access to a quantum simulator – this could be on your own laptop and you could use a company provided simulator where the latter do not always come for free. We have applied for and been granted with 10,000 USD for execution time on Microsoft Azure.

2 Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

Preliminary Course information . . .

- ▶ The course is brand new and we also have to learn things ourselves – we have designed the course ourselves after studying many lecturing and reference books and did not find any of them directly suitable for you. Quantum Computing is still a fundamental research
- ▶ The downside is that we know, that we will have to adjust some of the material. The first 3 lectures and exercises are known but we need to have your feedback along the way to be able to adjust the lectures and exercises.

Course ...

- ▶ We are currently in the midst of what is known as the *second quantum revolution* where we take the rules from the and use them to develop new technologies *first quantum revolution*.
- ▶ Since around 2000 we have with increasing speed advanced from basic knowledge and rules to developing physical qubits, and gradually apply quantum in e.g. chemistry and secure communication.
- ▶ NISQ = Noisy Intermediate-Scale Quantum ... Meaning: Many physical qubits to approach one logical qubit due to noise (electronic noise, coupling to the surroundings, ..).
- ▶ Quantum Computing are for now and likely years to come still a research field. Right now Classical Computing easily outperforms Quantum Computing – but Quantum Computing supremacy may (will?) happen.

Material ...

- ▶ The course slides deliberately contain quite substantial text – this is to enable you to use the material later as standalone material.
- ▶ As reference books we recommend:
 - ▶ Michael A. Nielsen & Isaac L. Chuang: “Quantum Computation and Quantum Information – 10th Anniversary Edition”. Cambridge University Press, 2010.
 - ▶ Chris Bernhardt: “Quantum Computing for Everyone”. The MIT Press, 2019.
 - ▶ Ray LaPierre: “Introduction to Quantum Computing”. Springer, 2021. The Materials Research Society Series.

You can get pdf versions of the books via [AUB](#).

- ▶ Qiskit code examples.

Course ...

- ▶ Prerequisites:
 - ▶ Basics of linear algebra and statistics.
 - ▶ Basic Python programming skills.
- ▶ Objectives:
 - ▶ Providing a basic understanding of the key elements in Quantum Computing.
 - ▶ Overview of some of the most important vendors of quantum computers and simulators/emulators.
 - ▶ Provide examples of places to get low cost access to quantum computers and simulators/emulators.
- ▶ Exam:
 - ▶ Oral exam based on a mini-project and exercises done during the course.

Course ...

1. Course Information and Quantum Background.

- ▶ Course Information:
 - ▶ Course Plan.
 - ▶ Lecturers Scientific Background.
- ▶ Quantum Background:
 - ▶ Theory, companies, and applications.
 - ▶ Brief primer on states and qubits.
 - ▶ Core quantum principles.
 - ▶ Classical and Quantum Computing.

2. Qubits (1/2) and Quantum Simulator.

- ▶ Classical bits and Quantum qubits.
- ▶ Notation and properties.
- ▶ Bloch sphere.
- ▶ Quantum registers.
- ▶ Qubits and states.

Course ...

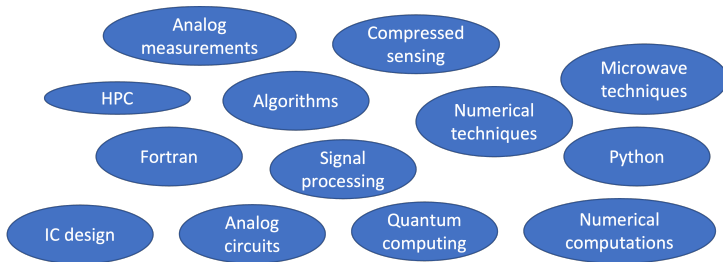
3. Simulation Tools:

- ▶ Types of simulation.
- ▶ Local (e.g. laptop), on-prem (e.g. Uni-servers) and cloud (e.g. company offered like Azure) solutions.
- ▶ Installing Qiskit.
- ▶ Simulating qubits and qregisters.

Torben's scientific background:

- ▶ Telecom engineering degree from 1988 — 35 years academic experience.
- ▶ Dr.Techn. in nonlinear noisy networks and systems from 1998.
- ▶ Substantial leadership experience – mainly as associate dean.
- ▶ After 12 years in leadership positions, I returned to research/teaching by 1 Nov. 2022.
- ▶ Since returning to research/teaching I have had 100% focus on quantum computing.

Torben's scientific background:



Torben's activities in Classical-HPC and QC:

- ▶ Close collaboration with Jørgen Ellegaard Andersen who is leading the Center for Quantum Mathematics at SDU.
- ▶ Further collaborates with: Julia Feddersen (QDNL, Holland), Peter Elias-van den Berg (TNO, Holland), Venkatesh Kannan (ICHEC, Ireland), Andrés Gómez Tato (CESGA, Spain) etc.
- ▶ Works around 2 days/week for DeiC as lead to apply for a Danish Quantum CoE in quantum algorithms and software.
- ▶ Advisor to the ministry on HPC and Quantum Computing.
- ▶ Involved in DeiC and DIREC activities on Quantum Computing.



Rafal's scientific background:

- ▶ PhD EE, PhD Math
- ▶ Control specialist at Danfoss
- ▶ Professor in control theory
- ▶ Head of Learning and Decision research group in Dept. of Electronic Systems



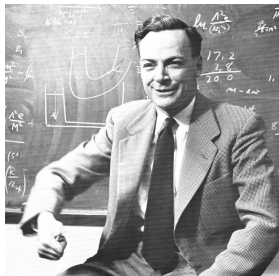
Rafal's activities in Classical-HPC and QC:

- ▶ Stabilisation of quantum systems
- ▶ Optimal control for quantum systems
- ▶ Parameters estimation for quantum systems
- ▶ Noisy Intermediate Scale Quantum (NISQ) devices - variational quantum algorithms

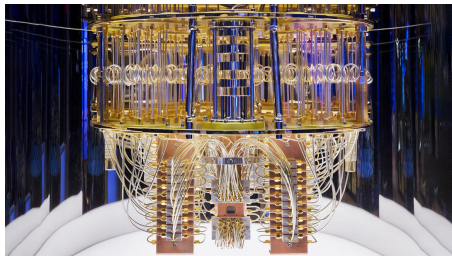


Qbackground

From invention to current implementation:



Feynman, 1958



IBM, 2023

Despite the nice looking IBM quantum computer, the area is still highly immature, far from complex and really useful applications → Research area .. but with fast growth, industrial interest, and billions of USD, technology advancement is fast.

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

15

Theory and Applications

States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

The End

Quantum Background

Information sources



AALBORG UNIVERSITY
DENMARK

Links:

- ▶ The Quantum Insider → <https://thequantuminsider.com>.
- ▶ Danish Quantum Committee → <https://qdc>
- ▶ Quantum Algorithm Zoo → <https://quantumalgorithmzoo.org>

Books:

1. Michael A. Nielsen & Isaac L. Chuang: "Quantum Computation and Quantum Information". Cambridge University Press, *10th Anniversary Edition*, 2010.
2. Noson S. Yanofsky & Mirco A. Mannucci: "QUANTUM COMPUTING FOR COMPUTER SCIENTISTS". Cambridge University Press, 2008.
3. N. David Mermin: "Quantum Computer Science – An Introduction". Cambridge University Press, 2007.
4. Bahman Zohuri and Farhang Mossavar Rahmani: "What is Quantum Computing and How it Works". Journal of Material Sciences & Manufacturing Research, 2020.

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

16

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

Quantum Background

The 4 Pillars of Quantum Technology



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

17

Theory and Applications

States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

The End

The European “Quantum Flagship” defines 4 pillars:

- ▶ Quantum computing.
- ▶ Quantum simulation.
- ▶ Quantum communication.
- ▶ Quantum metrology and sensing.

The European Commission launched the major “Quantum Flagship” in 2018 with a minimum lifetime of 10 years, and a budget of at least 1 b€.

The goal is to consolidate and expand European scientific leadership and excellence in this research area, to kick-start a competitive European industry in Quantum Technologies and to make Europe a dynamic and attractive region for innovative research, business and investments in this field.

► Vendors of Quantum Computer Systems:

- IBM. <https://www.ibm.com/quantum>
- Quantinuum. <https://www.quantinuum.com>
- Strangeworks. <https://strangeworks.com>
- D-Wave Systems. <https://www.dwavesys.com>
- Rigetti Computing. <https://www.rigetti.com>
- Oxford Ionics. <https://www.oxionics.com>
- IonQ. <https://ionq.com>
- 1QBit. <https://1qbit.com>
- Algorithmic. <https://algorithmiq.fi>
- Xanadu. <https://www.xanadu.ai>
- Intel. <https://www.intel.com/content/www/us/en/research/quantum-computing.html?wapkw=quantum>
- Google. <https://quantumai.google>
- Microsoft. <https://azure.microsoft.com/en-us/solutions/quantum-computing/>

Quantum Background

Theory and Applications



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

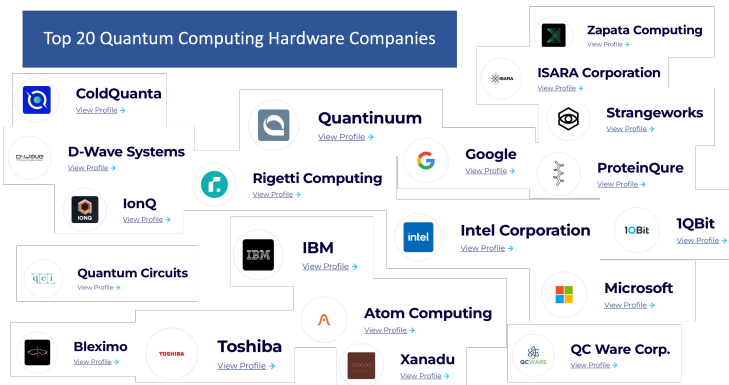
Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

Top 20 Quantum Computing Hardware Companies



19

39

Application industries:

- ▶ Defense sector.
- ▶ Encryption and data security.
- ▶ Weather prediction and climate models.
- ▶ Construction and discovery of new materials and medicine.
- ▶ Energy sector.
- ▶ Logistics.
- ▶ Financial sector.
- ▶ AI and Neural networks.
- ▶ Digital twins.
- ▶ Gaming industry.

Quantum Computer State Description:

- ▶ The smallest unit of information in Quantum Computing is the Quantum-bit or Qubit.
- ▶ A Qubit represents the state of the wavefunction $|\phi\rangle$ in Schrödingers equation at a specific time.
- ▶ A single Qubit may be in the “on” state ($|1\rangle$) or it may be in the “off” state ($|0\rangle$) or any linear combination thereof.
- ▶ Schrödingers equation, which describes how the state of a quantum mechanical system evolves in time is linear. Hence, linear combinations of solutions are also valid solutions.

Quantum Computer State Description:

- ▶ If a qubit has the state $|0\rangle$ and $|1\rangle$, a superposition of these also describe the same state.
- ▶ The general superposition form of the joint state of a 2-state quantum system is:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1, \quad \alpha, \beta \in \mathbb{C}$$

- ▶ The two core states $|0\rangle$ (ground state) and $|1\rangle$ (excited state) are orthonormal¹ in Hilbert space.
- ▶ The joined state $|\psi\rangle$ is a superposition of the core states each multiplied by a constant complex number at the given time instant.

¹Two vectors in a inner product space are orthonormal if they are orthogonal unit vectors.

Quantum Background

Core Quantum Principles



AALBORG UNIVERSITY
DENMARK

An Introduction to Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

23 Core Quantum Principles

Classical and Quantum
Computing

The End

- ▶ Quantum Computing uses qubits as the basis unit of information.
- ▶ Physical quantum computers are based on quantum elements such as photons, ions, electrons, and protons.
- ▶ Relevant quantum mechanical concepts:
 - ▶ Superposition.
 - ▶ Entanglement.
 - ▶ Decoherence.
 - ▶ Measurement.

Superposition:

- ▶ A quantum state can be any linear combination of states:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1, \quad \alpha, \beta \in \mathbb{C}$$

Note, we cannot determine α and β based on the quantum. After measurement we get either 0 or 1.

Entanglement:

- ▶ A state where particles are so tightly correlated that gaining information about one will give immediate information about the other.

Decoherence:

- ▶ Loss of superposition due to the fragile quantum system spontaneously couples to the environment.

Measurement:

- ▶ Collapsing a superposition to yield state $|0\rangle$ or $|1\rangle$ based on probabilities $|\alpha|^2$ and $|\beta|^2$, respectively.

Noisy Intermediate Scale Quantum (NISQ) computing

- ▶ Noisy Intermediate Scale Quantum (NISQ) computing is a term coined by John Preskill in 2018, which noted that current quantum computers at the time (and indeed still in 2024) are prone to considerable error rates and limited in size by the number of logical qubits (or even physical qubits) in the system.

Towards low-noise and fault tolerant quantum computers

- ▶ Noise in quantum gates limits the size of reliable quantum circuits.
- ▶ 100-qubit quantum computer will not change the world ... it is an important step though.
- ▶ Technology development aims for low-noise and later for fault tolerant quantum computing.

Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

The End

26

Out→

- ▶ Moore's law is coming to an end with doubling in transistor density every 18 months.
- ▶ Some problems are so large that they can not be computed on even the largest supercomputers that exist.

→In

- ▶ A computer that uses the laws of quantum mechanics to perform massively parallel computing through superposition and entanglement.

Quantum Computing relies on quantum mechanics and quantum theory. Possibilities for extreme speed in some areas but lacks the generality of classical computers in other areas. Well suited for 'combinatorics', optimization, chemistry, AI/ML, statistics, secure communication, ...

Schrödingers Equation and Impact:

► **Schrödingers Equation:**

$$\frac{\partial |\psi(t)\rangle}{\partial t} = -i \frac{\hat{H} |\psi(t)\rangle}{\hbar} = -i \frac{2\pi \hat{H} |\psi(t)\rangle}{h} \quad (1)$$

where \hbar is the reduced Planck constant $\hbar = h/(2\pi)$, and $\hat{H}(t)$ is the Hamiltonian of the system.

- Schrödingers Equation describes how the state changes in time depending on the Hamiltonian.
- Eq. (1) can be slightly rewritten as:

$$i \hbar \frac{\partial}{\partial t} |\psi(t)\rangle = \hat{H} |\psi(t)\rangle \quad (2)$$

which is a first order linear differential equation.

Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

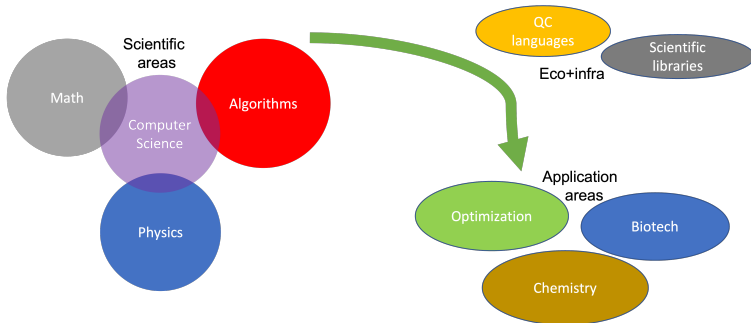
Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

From basic scientific competences in various areas
via middleware etc. on to applications ...



28

39

Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

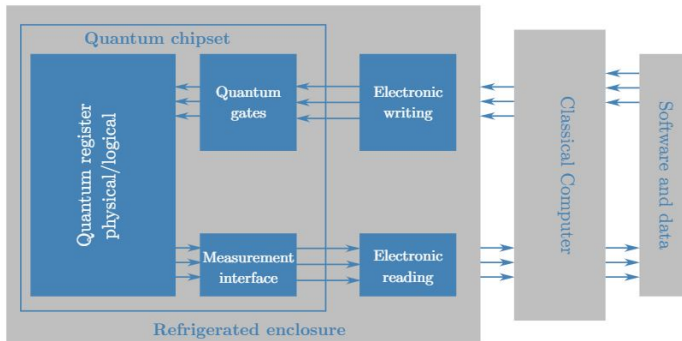
Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

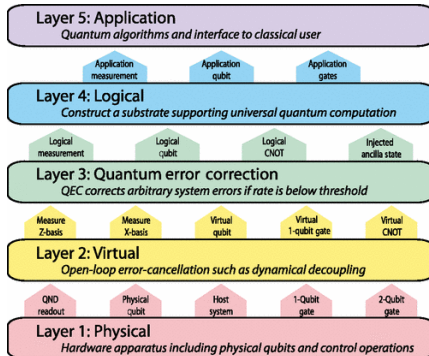
Quantum Computer architecture:



29

39

5-level stack:



Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

The End

31

Classical Computer:

- ▶ A computer that uses voltages/currents across/through circuits and gates, which can be controlled and manipulated entirely by classical mechanics.
- ▶ Building blocks: bits, registers, and logic gates.

Quantum Computer:

- ▶ A computer that uses the laws of quantum mechanics to perform massively parallel computing through superposition and entanglement.
- ▶ Building blocks: qubits, quantum registers, and reversible gates.

39

A technology platform is a combination of mathematical quantum processing, a mapping to realizable hardware, and coupling this to a software stack for further processing.

► **Quantum Computer Technology Platforms²:**

- Trapped Ion-Based Superconductor
Using ions trapped in magnetic fields and manipulating them using electromagnetic waves and/or lasers.
- Linear Optical Quantum Computer (LOQC)
Realization of qubits by processing different modes of light as quantum states (photonic qubits).
- Quantum Dot Quantum Computer
A type of nanoscale atomic/molecular structure allowing control of the flow of electrons using small voltages.
- Topological Quantum Computer (TQC)
Based on the braiding of anyons (quasi-particles) in a 2D lattice providing a high degree of error protection from decoherence.

²Edited descriptions from: R.L. Amoroso: "Brief Primer on the Fundamentals of Quantum Computing".

Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

Classical:

- ▶ In classical computing, parallelism refers to the computation of multiple calculations across multiple computational units simultaneously.

Quantum:

- ▶ Quantum parallelism refers to the ability of quantum computers to evaluate a function for multiple input values simultaneously.
 - ▶ This can be achieved by preparing a quantum system in a superposition of input states, and applying a unitary transformation that encodes the function to be evaluated.
 - ▶ The resulting state encodes the function's output values for all input values in the superposition, allowing for the computation of multiple outputs simultaneously.
 - ▶ This property is key to the speedup of many quantum algorithms.

33

39

Number of qubits for IBM quantum computers:

Year	Vendor	Name	qubits
2019	IBM	Falcon	27
2020	IBM	Hummingbird	65
2021	IBM	Eagle	127
2022	IBM	Osprey	433
2023	IBM	Condor	1,121
2024	IBM	Flamingo	$\geq 1,386$
2025	IBM	Kookaburra	$\geq 4,158$

Although the number of (physical) qubits is only one metric to describe the quantum computer capability. So far the predictions have been followed,

Quantum Background

Classical and Quantum Computing



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications

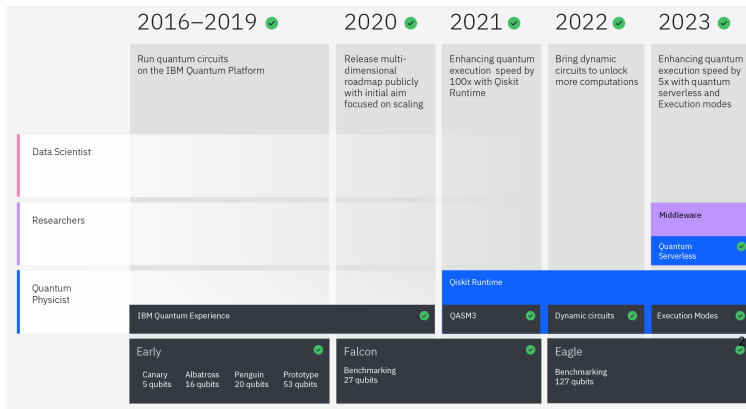
States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

The End

IBM quantum development roadmap:



35

39

IBM quantum development roadmap:

2024	2025	2026	2027	2028	2029	2033+
Improving quantum circuit quality and speed to allow 5K gates with parametric circuits	Enhancing quantum execution speed and parallelization with partitioning and quantum modularity	Improving quantum circuit quality to allow 7.5K gates	Improving quantum circuit quality to allow 10K gates	Improving quantum circuit quality to allow 15K gates	Improving quantum circuit quality to allow 100M gates	Beyond 2033, quantum-centric supercomputers will include 1000's of logical qubits unlocking the full power of quantum computing
Platform						
Code assistant	Functions	Mapping Collection	Specific Libraries			General purpose QC libraries
Transpiler Service	Resource Management	Circuit Knitting x P	Intelligent Orchestration			Circuit libraries
Heron (5K)	Flamingo (5K)	Flamingo (7.5K)	Flamingo (10K)	Flamingo (15K)	Starling (100M)	Blue Jay (1B)
Error Mitigation	Error Mitigation	Error Mitigation	Error Mitigation	Error Mitigation	Error correction	Error correction
5k gates 133 qubits	5k gates 156 qubits	7.5k gates 156 qubits	10k gates 156 qubits	15k gates 156 qubits	100M gates 200 qubits	1B gates 2000 qubits
Classical modular	Quantum modular	Quantum modular	Quantum modular	Quantum modular	Error corrected modularity	Error corrected modularity
133x3 = 399 qubits	156x7 = 1092 qubits	156x7 = 1092 qubits	156x7 = 1092 qubits	156x7 = 1092 qubits		

36

39

The End



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific
Background

Qbackground

Theory and Applications
States and Qubits

Core Quantum Principles
Classical and Quantum
Computing

The End

37

The End

39

Aalborg University, Denmark

Literature of general type:

1. Michael A. Nielsen & Isaac L. Chuang: “Quantum Computation and Quantum Information – 10th Anniversary Edition”. Cambridge University Press, 2010.
2. Noson S. Yanofsky: “An Introduction to Quantum Computing”. arXiv:0708.0261v1, 2007.
3. Chris Bernhardt: “Quantum Computing for Everyone”. The MIT Press, 2019.
4. Ray LaPierre: “Introduction to Quantum Computing”. Springer, 2021. The Materials Research Society Series.
5. N. David Mermin: “Quantum Computer Science – An Introduction”. Cambridge University Press, 2007.

The End

Contact Information



AALBORG UNIVERSITY
DENMARK

An Introduction to
Quantum Computing

Torben Larsen

Course Information

Course Plan

Lecturers Scientific

Background

Qbackground

Theory and Applications

States and Qubits

Core Quantum Principles

Classical and Quantum
Computing

In case you have any comments, suggestions or have found a bug, please do not hesitate to contact us. You can find our contact details below:

Torben Larsen, tola@cs.aau.dk
Computer Science

Rafal Wisniewski, raf@es.aau.dk
Electronic Systems

Contact:

Torben Larsen

Department of Computer Science

Selma Lagerlöfs Vej 300, Office 4.2.16,

9220 Aalborg, Denmark

Mail: tola@cs.aau.dk ; Mobile: +45 2020 6856

39

The End

39